Journal of Machine Engineering, Vol. 13, No. 3, 2013

machining system, dynamic parameters.
white noise excitation, self-excited vibration

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DYNAMIC PARAMETER IDENTIFICATION IN NONLINEAR MACHINING SYSTEMS

The demand for enhanced performance of production systems in terms of quality, cost and reliability is ever increasing while, at the same time, there is a demand for shorter design cycles, longer operating life, minimisation of inspection and maintenance needs. Experimental testing and system identification in operational conditions still represent an important technique for monitoring, control and optimization. The term identification refers in the present paper to the extraction of information from experimental data and is used to estimate operational dynamic parameters for machining systems. Such an approach opens up the possibility of monitoring the dynamics of machining systems during operational conditions, and can also be used for control and/or predictive purposes. The machining system is considered nonlinear and excited by random loads. Parametric and nonparametric techniques are developed for the identification of the nonlinear machining system and their application is demonstrated both by numerical simulations and in actual machining operations. Discrimination between forced and self-excited vibrations is also presented. The ability of the developed methods to estimate operational dynamic parameters ODPs is presented in practical machining operations.

1. INTRODUCTION

The demand for enhanced performance of production systems in terms of quality, cost and reliability is ever increasing while, at the same time, there is a necessity for shorter design cycles, longer operating life, minimisation of inspection and maintenance needs. Through the employment of advanced computing systems it has become less expensive, both in terms of cost and time, to perform numerical simulations, than to run time and material consuming experiments. The consequence has been a shift toward computer-aided design and numerical trials, where virtual models are employed to simulate experiments, and to perform accurate and reliable predictions of system behaviour.

Even if the technology of virtual prototyping is steadily growing in the manufacturing environment, experimental testing and system identification still play a key role because they provide valuable information to production engineers concerning the influence of process modification on the system's performance, the prediction of failures in the system and support for the maintenance of the production systems.

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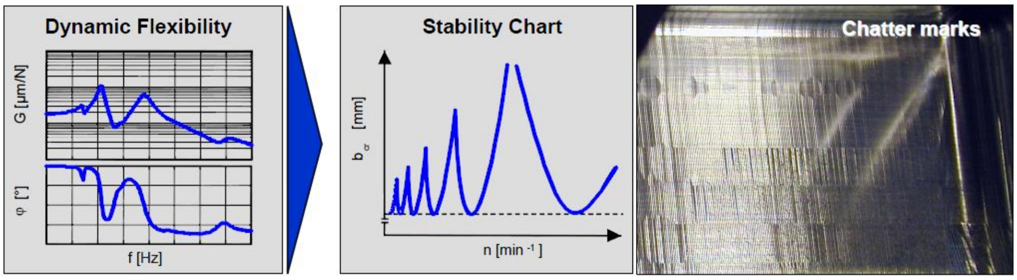
In the classical machining theory and practice, a large body of research has been dedicated to study the machining system's dynamics. Tlusty [13] has a major contribution in developing basic chatter theories. At a closer examination there are some phenomenological and technical shortcomings in the classical methodology. One critical issue is that the parameters describing dynamic behaviour of machining systems are extracted independently from the structure and from the process before connecting them together to study the system's (see Fig. 1) [27].



a) b) c)

Fig. 1. Analysis of the dynamic behaviour of the machining system: a) Extraction of dynamic parameters of the elastic structure, b) Stability charts, c) Chatter marks as a result of dynamically unstable machining

Machining system (MS) represents the interaction between the elastic structure and the chip formation process. This interaction is controlled at interface tool – chip – workpiece by a tiny elasto-plastic material volume. Separate extraction of modal parameters using off-operational experiments, e.g. experimental modal analysis (EMA), and controlled experiments for extraction of cutting process coefficients is not an efficient way of characterizing the machining system in operational conditions. From a theoretical point of view, using the direct analysis of the elastic structure and cutting process, based on the fundamental physical laws, may be insufficient for the accurate description of the behaviour of the system in real operational conditions. Regarding the cutting process, estimation of e.g. cutting force coefficients can be done only with barely simplified hypotheses, for few particular operations, and only in laboratory controlled conditions.

The machining system is excited by loads that are unmeasurable and unpredictable. In addition, the dynamics of the system is changing continuously during machining due to variation of the load magnitude, orientation and position. Consequently, the dynamic behaviour of a machining system is not the same in open loop i.e. in off-operational conditions, and in closed loop i.e. in operational conditions. In this paper, the excitation forces are considered as stochastic processes. Lightly damped structures of machine tools have a response in a narrow frequency range and therefore the excitation can be, at least at the first approximation, considered an ideal white noise. When the dynamic response of machining systems is analysed, linear theory is often used. For linear machining systems, a large body of theory has been developed for the identification and modelling of system parameters in various dynamic configurations [16]. One of the main concerns in these theories is the study of self-excited vibration and the related stability analysis. For nonlinear machining systems with broad band excitation, closed form solutions are not available.

In this paper, the main objective is to investigate the response of the nonlinear machining system and to develop a procedure for dynamic parameter identification.

1.1. NONLINEAR MACHINING SYSTEM

In the problem inv the dynamic behaviour of machining systems, one major source of uncertainty is the excitation. The chip-formation process takes place through the intricate closed-loop interaction between the tool/toolholder and the workpiece/fixture with the machine tool structure in the primary loop (open-loop) and the chip formation process in the feedback. The static and dynamic behaviour of a machining systems is governed by mass, stiffness and damping. In a nonlinear system, damping and stiffness characteristics depend on the energy levels of the excitation [24-26]. The excitation of the machining system is mainly created by the cutting force. On the cutting force, other loads, of a thermal or a mechanical nature, are superimposed to generate the complex system's response. The nature of these forces is nonlinear conservative and dissipative. Nonlinear conservative forces are restoring forces that arise from sources such as gravitational field and internal stresses generated in deformed structural elements. Apart from the cutting force, during the cutting process there are dynamic forces generated due to dynamic unbalance of rotating parts, inertia forces of reciprocating movable elements, bearing irregularities, and geometric imperfections in structural elements.

Ito [12] has pointed out that nearly all the theories of elasticity available at present can deal with the problem of the monolithic elastic body, i.e., elastic body without any joints. In reality, the machine tool's elastic structure consists of a relatively large number of structural elements. These elements are connected to each other by fixed and movable joints. Consequently, the elastic structure can be represented by a model with lumped masses connected by damping and 'springs-like' elements (see Fig. 2). The joint is one of the structural body components within a machine tool which causes that the static stiffness reduces and damping capacity increases [12].

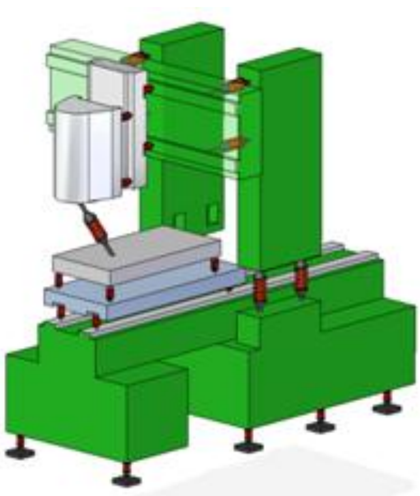


Fig. 2. Machine tool with represented joints [4]

The static stiffness and the damping capacity of the joints determine to a large extent the overall stiffness and damping of machining systems and by this the stability margin of the latter. In classical machining dynamics theory, regenerative chatter is considered the major cause for instability [6],[13]. The analysis method of system's stability is commonly implemented in three steps: (1) the modal parameters of the elastic structure are computed. This step is either experimentally performed e.g. using experimental modal analysis (EMA), or the corresponding modal parameters are obtained from FEM models. (2) the process stiffness and damping are estimated from cutting experiments and (3) stability diagrams are computed. The dynamic stability of machining system depends on overall stiffness and damping of the join elastic structure and process [45]. Regarding the elastic structure, for linear systems, the stiffness is obtained without difficulties from the material and geometric properties. If the structure is nonlinear the stiffness is not easily evaluated. Damping is normally not possible to analytically compute, even in linear cases. Concerning process stiffness and damping, their accurate estimation is difficult since they depend on the operational conditions and the estimation is possible only for very simple operations. Complications arise also due to the fact that for estimating the dynamic stiffness coefficient of one cutting variable the other variables must be maintained constant which normally is not trivial [27].

1.1.1. STRUCTURAL STIFFNESS AND DAMPING

Structural stiffness and damping control the tolerances and surface finish of machined parts. By way of the cutting force, they affect productivity and energy efficiency. One of the basic rules of machine tool design is the principle of compliance [10]. According to this, the stiffness has to be carefully optimized. Stiffness is the capability of the structure to resist deformation or to hold a position under the applied loads. Static stiffness in machine tools refers to the performance of structures under the static or quasi-static loads. Static loads in machine tools normally come from gravity and cutting force, etc. Apart from the static loads, machine tools are subjected to constantly changing dynamic forces and the machine tool structure will deform according to its dynamic stiffness and the amplitude and frequency of the dynamic loads. Dynamic stiffness is related to damping. Damping defines the ability of a system or structure to dissipate energy. The need for high dynamic stiffness in a machining system results mainly from two separate aspects; in the first case inadequate dynamic stiffness will result in a poor surface finish quality of the machined parts due to relatively high levels of vibration occurring during machining processes. In the second case, low dynamic stiffness can lead to chatter and even damage the cutting tool and machine structures [10]. As a machine tool's elastic structure is over dimensioned in terms of strength, and as its damping is relatively low, the deformation and energy dissipation will primary occur in the joints. Ito [12] presented an extensive survey of knowledge about machine tool joints and meantime provided a classification and a detailed analysis of the static, dynamic and thermal characteristics of stationary and sliding joints.

Nonlinear and even random behaviour of machine tool structure was observed in experimental tests. Joints loaded normally to the contact surface are characterized by a hardening nonlinearity due to the increase of effective contact area with the increasing load. The source of nonlinear behaviour is the change of contact area with the load magnitude. Although deformations are very small in relation to the overall model size, they significantly change the overall model stiffness and thus require a nonlinear treatment. Tangential contact deformation exhibits a softening nonlinearity [14]. At low loads, the clearance in the kinematic couplings causes slight displacement to take place in joints. The nature of the damping mechanism is very complex and depends on numerous factors, which makes the understanding of damping very difficult. The primary sources of damping for large flexible structures could fit into three categories:

- material damping due to internal friction,
- damping at joints and interconnections,
- artificially introduced damping (dashpots).

Compared to other damping types material damping is very low and can sometimes be ignored. Joint damping exists between joint surfaces. The more joint surfaces a machine has, the more damping exists. However the stiffness will be reduced because of joint surfaces. The damping at joints in turn depends on types of interconnection, joint loads, macroslippage and microslippage [12],[31]. It was shown by Rivin [16] that damping characteristics of the tool at large vibration amplitudes are due, largely, to normal contact deformations and at small amplitudes due to tangential contact deformations. Damping dissipation is basically a nonlinear and still not fully understood phenomenon because it is difficult to identify the major mechanisms responsible for energy dissipation. Dry friction effects (bodies in contact, sliding with respect to each other) and hysteretic damping are examples of nonlinear damping (Sherif and Abu Omar [32] Al-Bender [33]). It is important to note that dry friction affects the dynamics especially for small-amplitude motion.

The Coulomb friction model is a phenomenological description of friction and has experimentally been validated for global sliding between two rigid bodies. For a sliding velocity v and a normal force F_N the frictional force F_f is:

$$F_f = -\text{sgn}(v) \cdot \mu \cdot |F_N| \tag{1}$$

Coulomb friction is the most widely used model for sliding but also for micro-slip, and several modified friction models have been derived from it. Stribeck investigated roll bearings and noted that the coefficient of friction decreases at the onset of sliding until a certain velocity before it increases again. This behaviour is called the Stribeck effect [33].

1.1.2. PROCESS STIFFNESS AND DAMPING

The cutting process can be represented as a system with cutting parameters acting at the input in order to generate a cutting force at the output.

$$F(t) = F(a_p(t), v_c(t), h(t), \ldots) \tag{2}$$

The dynamics of the cutting process is determined by various parameters, such as depth of cut, $a_p(t)$, cutting speed, $v_c(t)$, uncut chip thickness, h(t), workpiece and tool materials, tool geometry, etc. Simplified models are used to represent stiffness and damping of the cutting process. One such method represents the process by a linear spring and damper.

If the cutting process is to be represented by the causal relationship between input cutting parameters and output force or torque, then some functions equivalent to structural stiffness and damping functions may be defined [35]. In a simpler approximation these functions can be resolved as coefficients describing the input-output causality. Under static conditions the output cutting force can be considered mainly as a function of uncut chip thickness and cutting speed. The chip thickness and cutting speed coefficients may be determined from the characteristics of the cutting force plotted as a function of chip thickness and cutting speed, respectively. Under dynamic conditions, a third coefficient has to be added, the penetration coefficient, to account for variations of the rate of penetration [17] and [18]. Estimation of the three dynamic coefficients is not trivial since each of them must be independently determined from dynamic experiments.

Nonlinearities also characterize the cutting process due to boundary conditions in the chip formation. Self-excited vibration or chatter is itself a nonlinear phenomenon. An example of nonlinear model describes the nonlinear cutting force in the following expression [27].

$$F = K_1 \Delta h - K_2 \frac{\dot{x}(t)}{v_c} - K_3 \left(\frac{\dot{x}(t)}{v_c} \right)^2 + K_3 \left(\frac{\dot{x}(t)}{v_c} \right)^3 \tag{3}$$

where $\dot{x}(t)$ is the vibration's velocity and v_c is the cutting speed. A large number of research studies on dynamics of machining and chatter are in particular focused on the estimation of dynamic stability by independently evaluating the dynamic characteristics of elastic structure and the cutting process, respectively. The two subsystems are considered as linear and deterministic. In addition, the contribution of process damping is rarely considered. Even in those conditions when process damping is considered, it is difficult to compare it to the structural damping as the damping is very sensitive to the testing method. The energy dissipation in the dynamic machining systems due to the cutting process is often neglected or simplified. For theoretical and practical interests, an appropriate model of process damping is indispensable and an effective identification procedure is required. There have been a number of studies considering the process damping and its identification.

The main source of process damping has been previoulsy recognized as indentation of the tool edge and flank face into workpiece surface undulations [8]. A ploughing force model based on the interference between the tool and workpiece has been developed, the concept introduced by Elbestawi [7]. Based on the ploughing force analysis, a small volume of work material is pressed by the tool during wave cutting. Meantime, a resistance force is generated by the stress field inside the displaced work material [11]. Budak [1] and Tunc [2] have developed an approach to identify the process damping from the chatter tests using experimental and analytical stability limits. The process damping coefficient was then related to the instantaneous indentation volume. The determined coefficient is then used for the stability limit and process damping prediction in different cases. Kurata [5] presented the identification method of the process damping from turning cutting tests. Tyler [3] developed an analytical solution for machining stability that included process damping effects. This approach refers to a velocity-dependent process damping model that describes the process damping force, F_D, in the y direction normal to the machined surface, as a function of velocity, chip width, b, cutting speed, V_c, and a constant C

$$F_D = -C\frac{b}{V_c}\dot{y} \quad (4)$$

The process damping coefficient was identified experimentally and it was shown that a smaller relief angle or higher wear results in increased process damping and improved stability at lower spindle speeds [15]. The model was inspired by Altintas [9] who developed a cutting force model with three dynamic cutting force coefficients related to regenerative chip thickness, velocity and acceleration terms, respectively. The dynamic cutting force coefficients are identified from controlled orthogonal cutting tests with a fast tool servo oscillated at the desired frequency to vary the phase between inner and outer modulations.

Stability charts are determined for simple cutting operations mostly in orthogonal cutting and performing simpler cutting paths. Therefore, their practical usability is limited. Though the range of machining operations has not been increased, the mathematical formulations used to evaluate stability diagrams has been. Notable are recently developed studies using chaos theory and various approaches for mathematical representation and solving differential equations with delay, which describe one of the most conventional types of chatter [28]. Nonlinear models for treating regenerative chatter in machine tools were developed in [36]. Based on chaos theory, experimental and theoretical results were reported for a deeper understanding of nonlinear regenerative chatter [38].

A serious limitation in practical validation of stability diagrams originates from the lack of a rigorous criterion to unambiguously detect the stability limit, i.e., impending chatter. In other words, formulation of a discrimination criterion to distinguish between stability and instability will be much more useful from both a theoretical and especially a practical point of view. As the interest is to avoid chatter, developing robust discrimination criteria will help in solving the increasing dynamic problems on the shop floor. A reason for difficulties in studying the dynamic stability of machining systems is the lack of an explicit scheme for representing the join interaction of physical processes and structural systems in a unified way based on a common concept. Many of the methods used to analyze, control and optimize machining systems are based on off-line procedures or test environments that do not represents the actual machining conditions.

2. ESTIMATION OF MACHINING SYSTEM' S DYNAMIC PARAMETERS

The approach presented in this paper has the purpose of identifying the operational dynamic parameters (ODPs) of a machining system i.e., the equivalent stiffness and damping existing in a particular system configuration at a particular moment in time. Therefore, the approach introduces a probabilistic concept where both parametric and nonparametric identification models are employed. The machining system is considered inherently nonlinear. The contribution of the process stiffness and damping gives a new dimension to the system's nonlinearity when the machining system is considered as an entity.

A machining system is subjected to complex loads and it changes its configuration continuously as the tool moves along the workpiece or discretely as the system is reconfigured for various cutting operations. As a consequence both the excitation and the parameters characterizing the system may be considered random. An adequate description of excitation, and therefore of the response of the system to such loads has to be developed within the framework of the statistical dynamics theory [29],[39]. Embracing such a probabilistic point of view implies that some statistical characteristics of both the stochastic excitation and the systems response to this excitation have to be considered. In dynamic systems with random excitation and involving the interaction of coupled structures through a fluid or through other media, self-excited phenomena are frequently present [30]. The problem of discrimination between random forced vibration and self-excited oscillations is a key issue in the evaluation of the system stability boundary during operational conditions. The response of a dynamic system to a broadband random excitation will be a nonzero steady-state signal both in stable and in unstable states. In case of self-excited vibration, the excitation persists even in the absence of the random excitation. The discrimination between self-excited and forced vibrations of a dynamical system in operational conditions is also important for selecting the type and strategy of control that may be implemented to reduce or cancel the vibration [37].

The field of application of statistical dynamics, which is the focus of this paper, is related to two concepts: (i) identification and (ii) discrimination of the response of dynamic systems. The term identification refers to the formulation of a mathematical model of the dynamic system based upon on-line signal measurements, and belongs to a class of inverse dynamic problems encountered in various technological fields as suggested by Ljung [40]. The discrimination concept refers to a function to characterize the nature of the system's response, i.e., to determine whether x(t) represents a forced vibration response or a self-excited vibration.

The problem of interest in machining system dynamics is the discrimination between forced and self-oscillations in view of the following considerations:

- Formulation of a qualitative/semi-qualitative mathematical model of the machining system for subsequent quantitative analysis,
- Evaluation of the system's stability boundary,
- Implementation of a suitable design for chatter control.

The term "qualitative" implies that the model is based on a statistical analysis of the measured system's response. Although the model-based identification approach presented in this paper leads to the estimation of key dynamic parameters (hereby the term "semi-qualitative"), they are nevertheless meaningful only within a certain confidence interval.

The main contributions of present paper are: (1) the development of parametric and non-parametric models based on identification techniques with the purpose of integrating into a single step the estimation of dynamic parameters characterizing the machining system, (2) in non-parametric identification, implementing techniques for ODPs and random excitation estimation, (3) in parametric identification, the development of the recursive computational model of the machining system based on the data obtained during the actual operational regime. Through these contributions, a step is taken beyond the classical approach to analyse the dynamics of a machining system by separately identifying the structural and process parameters. With the process considered, the two substructures, tool/toolholder and workpiece/fixture, are coupled, in addition to the open loop (elastic structure), by a feedback loop closing the energy loop, through the thermoplastic chip formation mechanism [41]. The machining system can be completely analysed only in closed loop i.e. in operational conditions, since specially designed off-line experiments with controlled input, such as modal testing, give the response from only the open loop.

3. NONPARAMETRIC IDENTIFICATION OF STIFFNESS AND DAMPING

The nonparametric identification presented in this section follows the method developed by Roberts [19], Krenk [20] and Rüding [21]. The approach is implemented in three stages for generation of the response of a machining system using numerical simulation. At the first stage, a Gaussian white noise process is created. At second stage, the equation of motion is integrated with a suitably chosen time step, using a Runge-Kutta algorithm of 4th order which enables accurate response histories to be obtained. Finally, the response data are processed appropriately to estimate the system's parameters.

As already mentioned, the excitation of the machining system is considered as an external zero mean white-noise process with the covariance function

$$E[W_0(t)W_0(t+\tau)] = 2\pi S_0 \delta(\tau) \tag{5}$$

where E[o] is the ensemble average, S_0 is the intensity of white noise and δ is the Dirac's delta function. White noise process, labelled $W_0(t)$, consists of a train of Dirac's delta impulses at Δt time increment. The pulses are linearly interpolated by the algorithm during numerical integration. The PSD (power spectral density) of the white noise process is given by Roberts [25].

$$S_w(\omega) = S_0 \left[\frac{\sin\left(\frac{1}{2}\omega\Delta t\right)}{\frac{1}{2}\omega\Delta t} \right]^4 \quad , \quad S_0 = \sigma_w^2 \frac{\Delta t}{2\pi} \tag{6}$$

If the frequency bandwidth of the system is ω_{max}, the excitation will approximate white noise as long as $\omega_{max}\,\Delta t << 1$ as illustrated in Fig. 3 where the power spectrum density function is plotted.

The equation of motion of the machining system is described by a nonlinear stochastic differential equation with additive excitation

$$\ddot{X}+h(E)+u(X)=W_0(t) \tag{7}$$

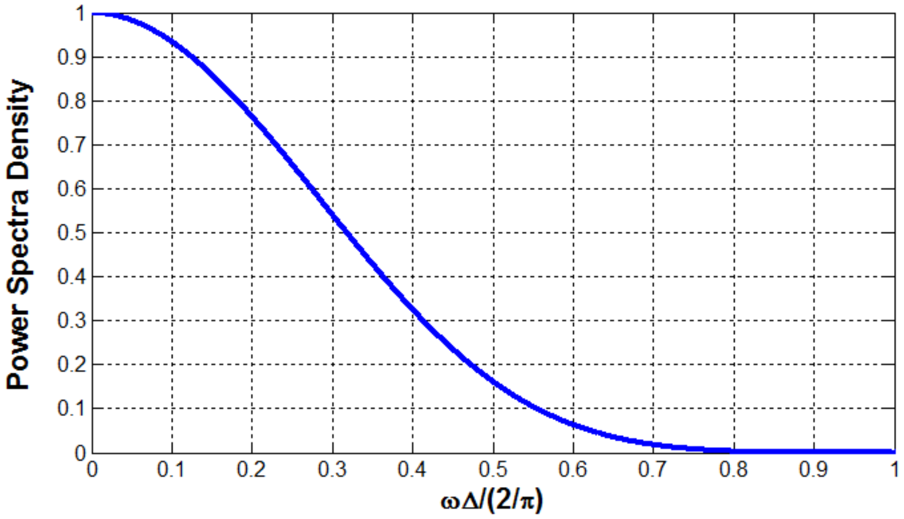


Fig. 3. Power spectrum of white noise generated for simulation purpose

where *h(E)* and *u(X)* are nonlinear damping and restoring forces of the machining system. If it is assumed that the state space variables X and $\dot{X}$ enter the equation only through the total energy function E, then the damping is a function of the energy only. The total energy is given by

$$E=\frac{1}{2}\dot{X}^2+U(X) \tag{8}$$

where

$$U(X)=\int_0^X u(\xi)d\xi \tag{9}$$

In Eq. (8) the right-hand side represents the sum of the kinetic energy (first term) and the potential energy (second term). With a change of variables, $X_1 = X$ *and* $X_2 = \dot{X}$, Eq. (7) is transformed into a set of two stochastic differential equations. Under the assumption of Gaussian white noise excitation, the state space vector (X_1, X_2) represents a Markov process and the probability density is the solution of Fokker-Plank equation

$$x_2\frac{\partial p}{\partial x_1}-\frac{\partial p}{\partial x_2}\quad h(E)x_2-u(x_1)\ \ p\ -\pi S_0\frac{\partial^2 p}{\partial x_2^2}=0 \tag{10}$$

where p(x_1, x_2; t) is the joint probability distribution of the state space vector (x_1, x_2). The initial conditions are given in the form: p(x_1, x_2; t_0) = $\delta(x_1 - x_{10})\,\delta(x_2 - x_{20})$ for $x_1(t_0) = 0$ and $x_2(t_0) = x_{20}$. Different boundary conditions are possible such as reflective boundary, absorbing boundary and periodic boundary as discussed by Risken [22].

The solution to the Eq. (10) was obtained by Caughey [23] in the form

$$p(x_1, x_2) = C\exp\left[-\frac{1}{\pi S_0}\int_0^E h(\zeta)d\zeta\right] \qquad (11)$$

where C is a normalizing constant.

The integral in the above expression is denoted damping potential. Eq. (12) shows that the damping function h(E) can be obtained from the derivative of the damping potential H(E).

$$\text{H(E)} = \int_0^E h(\zeta)d\zeta \qquad (12)$$

The damping potential is computed from Eq. (10) after the join probability $p(x_1, x_2)$ is evaluated. Because, the probability density distribution of the energy envelope process, E(t), can be estimated from system's response, it is required to relate the distribution p(E) to the distribution $p(x_1, x_2)$.

3.1. STIFFNESS ESTIMATION

The estimation of system stiffness from the stochastic response obtained from Eq. (7) follows the Rüding approach [19]. The response X is calculated at zero level where the energy is kinetic energy and at extremes where the energy is potential energy. A nonlinear system with linear-cubic stiffness was used to generate a stochastic response. The damping ratio in system is low, ξ = 0.1%. The results of stiffness identification are illustrated in Fig. 4. Each blue dot represents a sample ($1/2\,\dot{x}^2$, X^2). The values of the potential energy U(x), the target values, calculated from Eq. (8) are represented by the red line and the potential energy U(x) for an equivalent linear system is represented by the green dash line. There are two important observations: (1) the points are centred on the target line, which represents the analytical expression. (2) the departure from linearity is apparent.

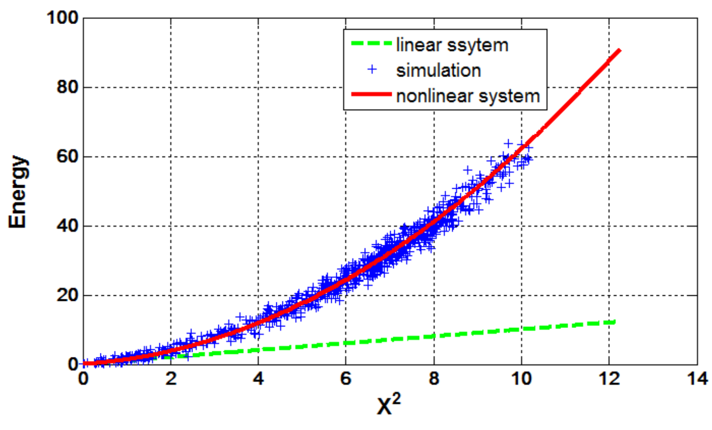


Fig. 4. Kinetic $1/2\,\dot{x}^2$ and potential U(x) energy for a light damped system, 0.1% and linear-cubic stiffness

By increasing the damping to 1% the level of scatter around the target value increases as illustrated in Fig. 5. This is due to the variation of the energy from one period to another. As damping tends to zero, the energy will also vary infinitely slow, and the points will be virtually located on the target line. Therefore the distance between a point and the line represents the energy variation during a half of period, i.e. the time between sampling a kinetic energy value and a potential energy value (or between a zero and an extreme value). By examining the plots in Figs. 4 and 5 it is reasonable to accept that an estimate of U(x) can be extracted from the dot cloud. The procedure is based on dividing the energy plane in zones of equal energy. This approach is illustrated in Fig. 6. As the potential function U(x) is unknown at the beginning of the procedure, the potential energy is approximated to the linear value. After that, the samples ($1/2\,\dot{x}^2$, x^2) are averaged in each zone and one average value is calculated for each zone as illustrated by the green dots in Fig. 6. The estimating procedure shows excellent results but increasing deviations from the target is apparent at higher energy levels. This is because of the smaller amount of samples at a higher energy level. Eliminating the samples above level 2 will considerably improve the estimation. The next step is to fit a polynomial to the estimates to compute the potential energy U(x). Then from Eq. 9 the stiffness function may be calculated.

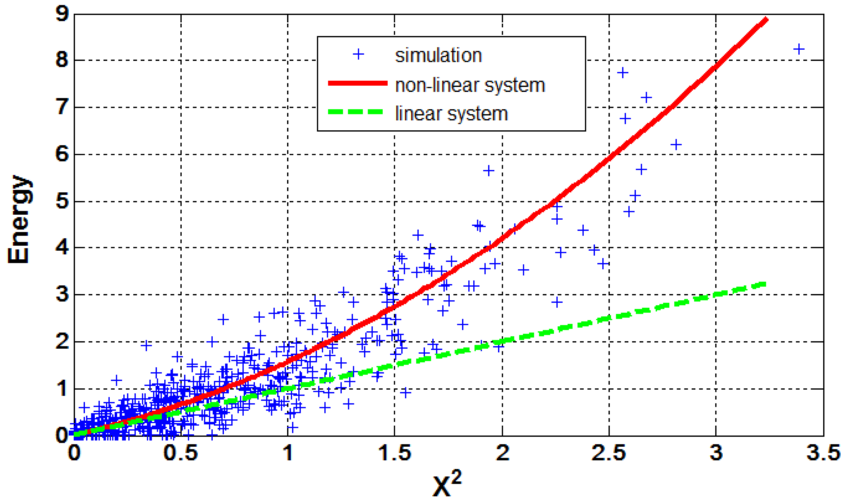


Fig. 5. Kinetic $1/2\,\dot{x}^2$ and potential U(x) energy for a heavier damped system, 1%

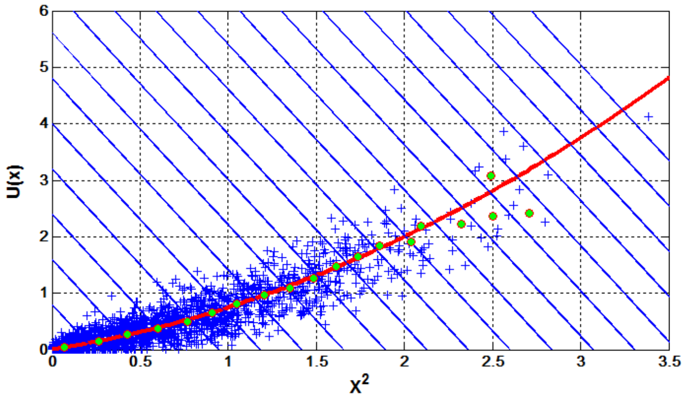


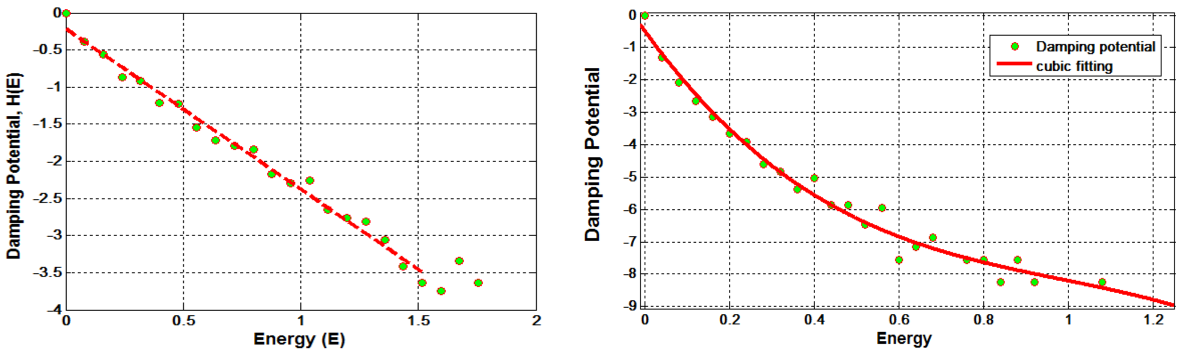
Fig. 6. Estimation of U(x) in zones of equal energy. The dots represents the average U(x) in each energy zone

3.2. DAMPING ESTIMATION

Damping estimation is treated in two steps. The first step starts from the damping potential. The equation of motion is represented by Eq. (7) and Eq. (11) can be rewritten as

$$\frac{H(E)}{\pi S_0} = -\ln\left(\frac{p_{x,\dot{x}}(x,\dot{x})}{C}\right) \tag{13}$$

where H(E) is the damping potential as described by Eq. (12). By estimating the probability density function from the system's response, the damping potential can be calculated. This function, for a linear system is represented in Fig. (7). By fitting a linear polynomial to the experimental data, the ratio between damping potential and excitation intensity, S_0 can be calculated.



a) b)

Fig. 7. Estimation of damping potential: a) - linear system and linear polynomial fitting, b) - nonlinear damping system, dry friction, and cubic polynomial fitting

In the second step, the equivalent damping function, h_{eq}, in Eq. (11) is estimated from the covariance function at various energy levels according to

$$h_{eq} = \frac{4}{c(E)T(E)}\ln\left(-\frac{X_1}{X_2}\right) \tag{14}$$

where c(E) is the participation factor in the covariance function, T(E) the natural period and X_1 and X_2 are the extreme values of the covariance function in the first period.

$$R_{\dot{x}}(\tau \mid E) = E\exp\left(-\tfrac{1}{2}h_{eq}\tau\right)\sum_{j=1}^{\infty} c_j(E)^2\cos(j\omega\tau) \tag{15}$$

For a linear system the covariance functions at different energy levels are shown in Fig. 8. Having calculated $H(E)/S_0$ and h_{eq} the excitation intensity S_0 can be estimated. For each energy level the extreme values (shown in Fig. 8 at X_1 for the potential energy, and at X_2 for kinetic energy) are extracted from the corresponding covariance functions and the equivalent damping computed according to Eq. (14). The calculated values are represented by dots in Fig. 9. As the system is linear the damping is independent of energy level and therefore constant. It can also be noticed in Fig. 8 that the natural period, and therefore the natural frequency, are constant for different energy levels. Each curve in Fig. 8 represents an energy level corresponding to the dots in Fig. 9.

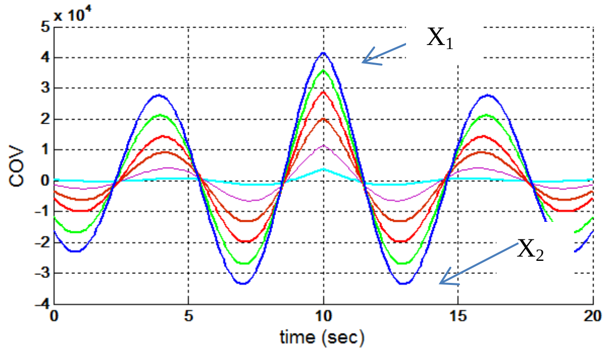


Fig. 8. Covariance function estimated at different energy levels

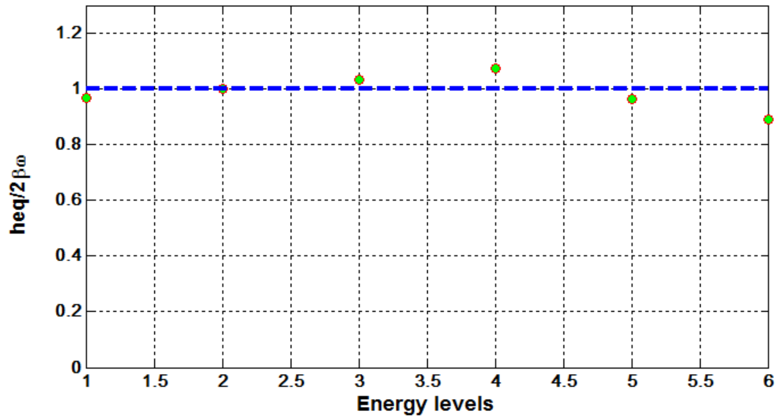


Fig. 9. Estimation of the equivalent damping function, h_{eq}, from measured autocovariance functions

After plotting the equivalent damping values, a line is fitted to represent the constant damping. As the energy level is increased, a larger deviation from the analytical value is apparent.

4. IDENTIFICATION OF MACHINING SYSTEM ODPs

The procedure developed in section 3 will be applied for the identification of the join characteristics of the machining system. Steel bars (C < 0.20%) with length of 1200 mm and initial diameter 42 mm were machined in longitudinal turning between tail and chuck at a cutting speed v_c= 180 m/min, feed rate f_r = 0.3 mm/rev and variable depth of cut (0.5 – 3 mm). Cemented carbide inserts with 1.2 mm nose radius were used in all experiments.

The vibration signals were measured by a pre-polarized microphone and sample at 12.8 kHz sampling rate. In Fig. 10 an example of stable machining is presented, while Fig. 11 illustrates an unstable machining.

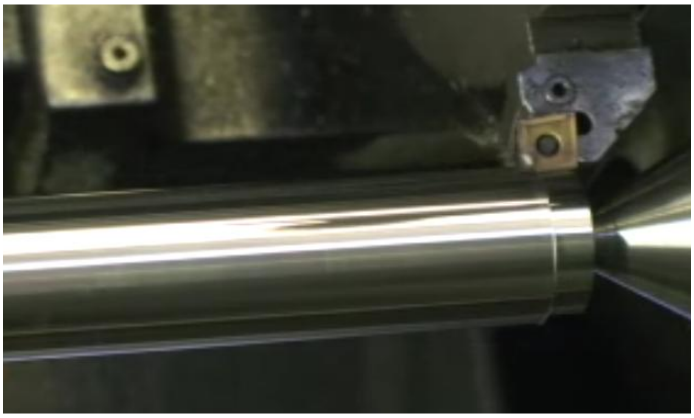


Fig. 10. Stable machining (D = 38 mm)

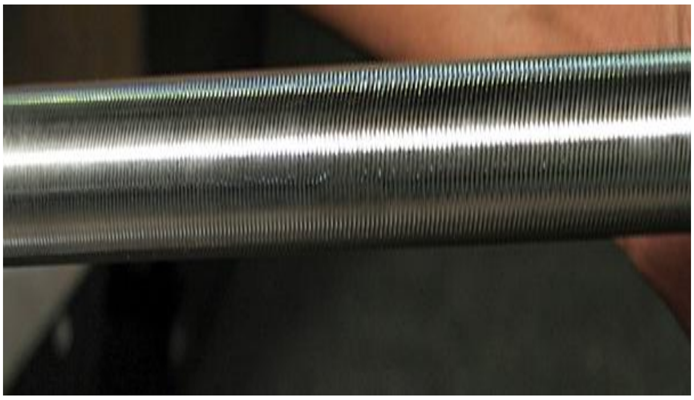


Fig. 11. Unstable machining, chatter

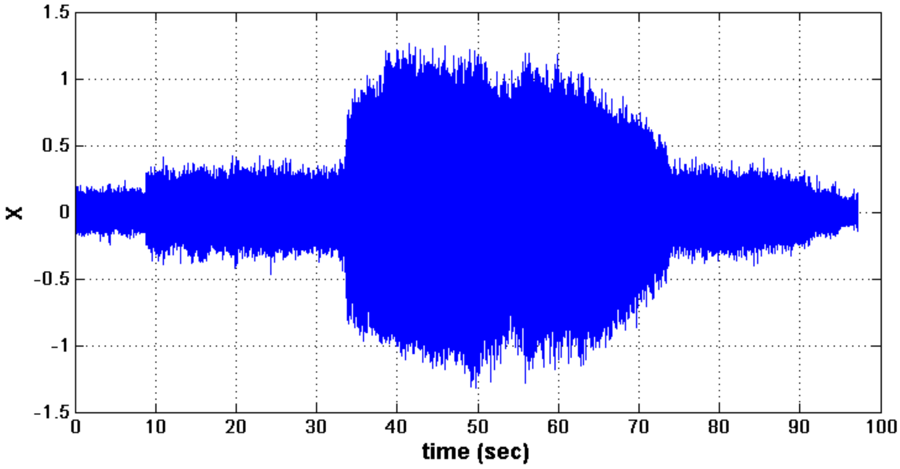


Fig. 12. Unstable process, as the tool approaches the centre of the bar heavy vibration sets on

Coming down to diameters below 34 mm, as the tool approaches centre of the bar, chatter vibration is generated. Fig.12 presents the time signal showing first a stable process then the chatter in the middle of the bar, then as the tool approaches the chuck, the system recovers stability. In Fig. 13, the unstable and stable signals are represented in frequency domain.

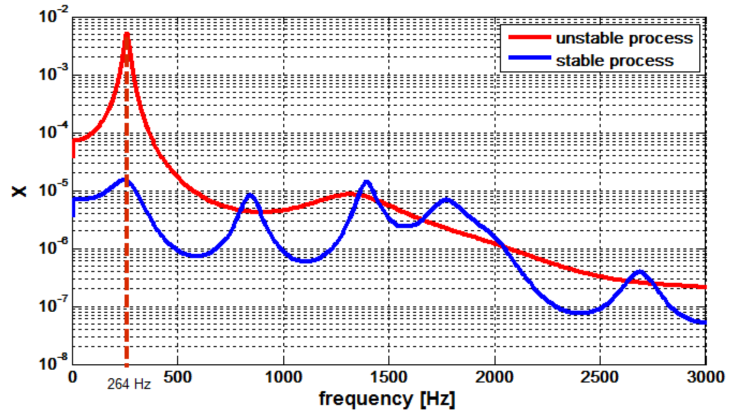


Fig. 13. The stable and unstable signals in frequency domain

4.1. OPERATIONAL STIFFNESS INDETIFICATION

Following the procedure described in section 3, the overall stiffness of the machining system is estimated. For a stable process, Fig. 14 illustrates the potential function U(x). The function shows a soft nonlinear characteristic. After fitting to a quadratic polynomial function, the stiffness function can be extracted from Eq. (9). The blue line represents the theoretical linear system at the system's natural frequency.

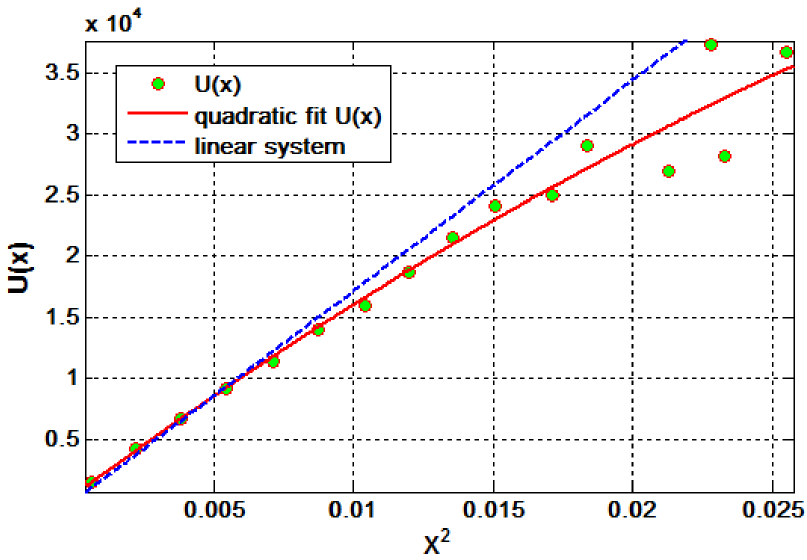


Fig. 14. Stiffness estimation - stable machining

The estimation of the overall stiffness of the machining system entering unstable behaviour is presented in Fig. 15. The potential function U(x) and consequently the stiffness function show a hard nonlinear behaviour which is characteristic for self-excited vibration. The deviation from a nonlinear behaviour is apparent by comparing to the linear characteristic shown by the blue dotted line. As these results show, if the stiffness of a machining system enters the inelastic range of the material, or the process becomes nonlinear, or a combination of both and the degree-of-nonlinearity are large, using a linearization method for the nonlinear stiffness may yield large errors in response estimation.

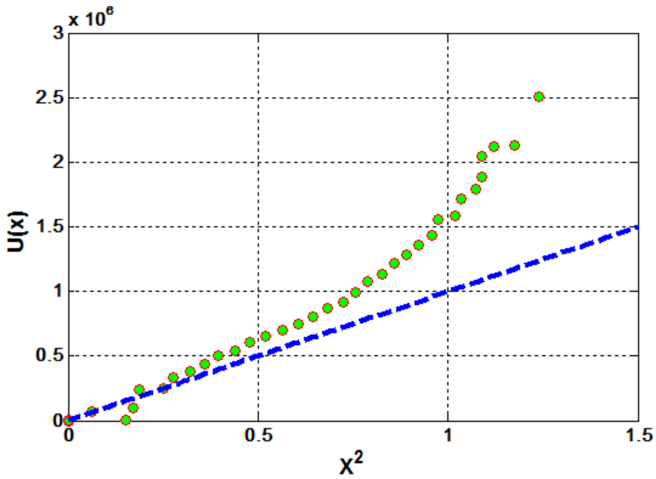


Fig. 15. Stiffness estimation - unstable machining

4.2. OPERATIONAL DAMPING IDENTIFICATION

Following the approach described in section 3, the damping potential and equivalent damping are estimated from the system's response both in stable and unstable condition. However, the results are presented only for unstable machining system.

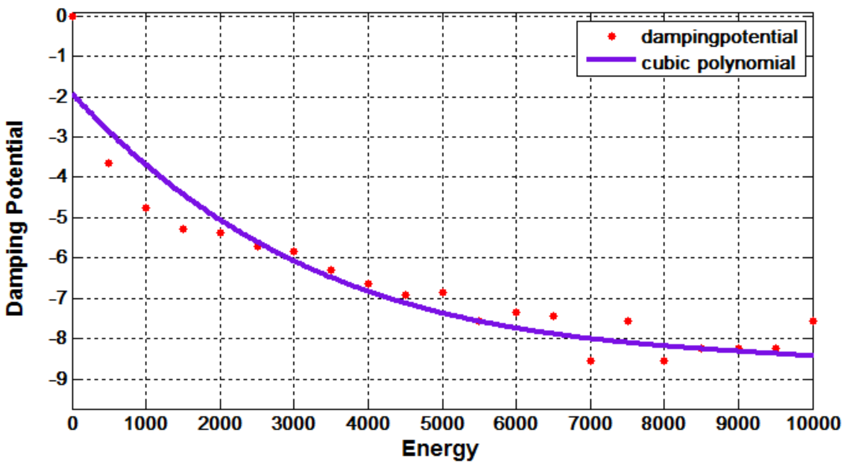


Fig. 16. Estimation and fitting of damping potential for unstable machining

The estimation procedure starts with computation of the damping potential H(E) from the experimental computation of probability density function at different levels of energy. From Eq. (13), the function H(E) is computed and then fitted to a polynomial. As in the case of stiffness, the damping in an unstable machining system shows a nonlinear behaviour and a cubic polynomial is then employed. In the second step, the covariance function is calculated for gradually increasing levels of energy. Some of these functions are represented in Fig. 17. It is worth noticing that, as the stiffness is nonlinear, the natural period depends on the energy level. The same can be stated for the equivalent damping. A modified form of Eq. (14) is used to take into account the changes in the natural period. The total energy is calculated as a sum of the kinetic and potential energies. The kinetic energy is extracted from the derivative of the response while potential energy from an iterative procedure applied to the identified potential function U(x).

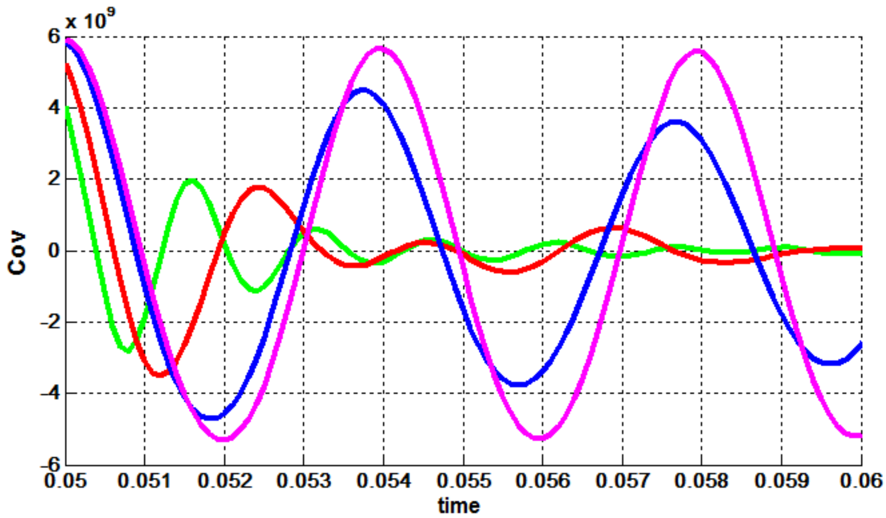


Fig. 17. Covariance function calculated for various energy levels

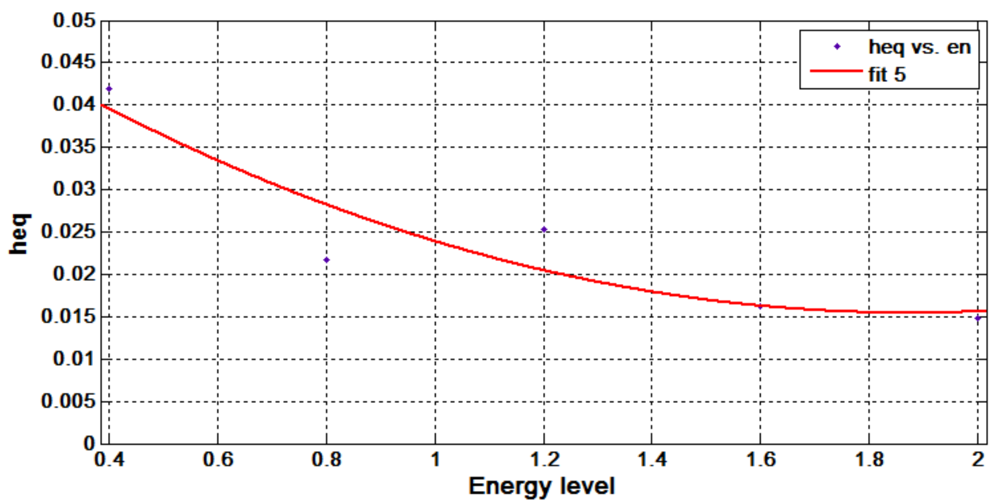


Fig. 18. Computation of equivalent damping function

5. ODP PARAMETRIC IDENTIFICATION

In section 4, a non-parametric identification procedure has been used to estimate the ODPs from the response of the machining system in stable and unstable conditions. The main benefit of this approach is that the estimated dynamic parameters are extracted in operational conditions directly from the interaction between the elastic structure and the cutting process. Knowledge of ODP opens entire new opportunities for optimizing the machining system. Strategies for improving, if necessary, the system may be straightforward implemented by comparing different solutions. Non-parametric identification has however, the limitation that requires long sample sequences. In addition, as the non-parametric models developed in this paper are based on SDOF system, special treatment of data is required. Another issue is that this approach requires knowledge of the response probability distribution.

In the remainder of this section, the identification technique based on parametric models is presented. Parametric models can be applied to any numbers of DOF and in their recursive implementation can take into account the nonlinear nature of the system. A parametric model is a special class of representation of a system, where the input in the model is driven by white noise processes and the model is described by rational system functions, including autoregressive (AR) (Burg, least square, Yule Walker, geometric lattice, instrumental variable), ARX (autoregressive with eXogeneous variables, iv4), moving average (MA), autoregressive-moving average (ARMA), Box Jenkins, Output Error models [42-44]. The process output of this class of models has power spectral density (PSD) that is entirely described in terms of model parameters and the variance of the white noise process.

The response generated by a process can be identified in a parametric model. The model is a synthetic one since the parameters in the model do not have any physical meaning. It will later be shown how the synthetic model can be converted in a physical one. The modelling of a stationary time series as the output of a dynamic system whose input is white noise a(t), can be carried out in several ways. One way is to use the parsimonious parameterization which is employing ARMA(p,q) representation [34]. Given a time series of data X(t), the ARMA model is an identification technique for predicting future values in this series [35],[36]. The model consists of two parts, an autoregressive (AR) part and a moving average (MA) part [38]. The combined model is usually then referred to as the ARMA(p, q) model were p is the order of the autoregressive part and q is the order of the moving average part. The input excitation in an ARMA process is not observable but can be assumed to be random and broadband compared with the measured output sequence for the reasons explained above. In milling for instance the intermittent engagement of multi tooth cutters excites the structure with impulse like forces. The model for an ARMA process can be expressed as

$$Y(z) = H(z)U(z) \tag{16}$$

where Y(z), U(z) and H(z) are the z-transforms (the z-transform is the discrete-time counterpart to the Laplace transform for continuous-time systems) of the output sequence, input sequence and the system impulse response (transfer function), respectively, and

$$H(z)=\frac{b_0+b_1z^{-1}+b_2z^{-2}+\ldots+b_qz^{-q}}{1-a_1-a_1z^{-1}-a_2z^{-2}-\ldots-a_pz^{-p}} \tag{17}$$

where the b_i and a_i are coefficients of the polynomials of the MA part and AR part, respectively. As mentioned earlier, the ARMA model consists of two parts, an AR part and an MA part. If the polynomial of the AR part ≡ 1 the ARMA model represents a MA model. The properties of such models allow for an analysis of the frequency spectrum with deep nulls, but without any sharp peaks. If the polynomial of the MA part ≡ 1 the ARMA model represents an AR model. The properties of such models allow for an analysis of the frequency spectrum with sharp peaks, but without any deep troughs. Thus, the ARMA model can be used to represent spectra with both kinds of behaviour. Using Eqs. (16) and (17) the ARMA model can be expressed

$$\sum_{i=0}^{p}a_i y(t-i)=\sum_{i=0}^{q}b_i y(t-i), a_0=1 \tag{18}$$

5.1. RECURSIVE PARAMETERS IDENTIFICATION

The model-based identification method used in this paper is based on the recursive prediction-error method (RPEM) [42]. As before, the model structure is based on a parametric process where the input to the model is driven by white noise processes and the model is described by a rational system function and represented by the recursive autoregressive moving average (RARMA) model structure. The process output of this model has the power spectral density (PSD) that is entirely described in terms of model parameters and the variance of the white noise process. By definition, a non-conservative mechanical system with positive damping is said to be dynamically stable, whereas one with negative damping is considered unstable. This gives a robust criterion for discrimination between forced and self-excited vibrations which is not related to de vibration amplitude criteria.

Assuming that the machining system excited by a random excitation e(t) can be represented by an n degree of freedom nonlinear equation of motion

$$M\ddot{x}+C\dot{x}+Kx+g(x,\dot{x})=e(t) \tag{19}$$

where M, C and K represent n x n mass, damping and stiffness matrices respectively; e(t) is a vector of external excitation. Matrices C and K contain both structural and process damping and stiffness respectively. The expressions $x(t), \dot{x}(t), \ddot{x}(t)$ are (nx1) vectors of displacement, velocity and acceleration for the n degrees of freedom system, and $g(x,\dot{x})$ is a nonlinear function. The system of equations (19) can be recast in

$$\dot{z}(t) = F(z) + f(t) \tag{20}$$

where

$$z = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}; \quad f(t) = \begin{bmatrix} 0 \\ M^{-1}e(t) \end{bmatrix}$$

and

$$\mathrm{F(z)}=\begin{bmatrix} \dot{x} \\ -M^{-1}C\dot{x} - M^{-1}Kx - M^{-1}g(x,\dot{x}) \end{bmatrix}$$

Let $y_j(k\Delta T)$, k = 0, 1, 2 …n be the discrete samples of the measurement response of the displacement of the j-mass. ΔT is the sampling interval. Then the observations y_j (kΔT) can be represented by an ARMA model described by Eq. (16). The measurement equation is then of the form

$$\mathrm{Yi} = \mathrm{H}(k\Delta t, X_j; \Theta) + e_k \tag{21}$$

The purpose of RARMA is to recursively identify the join parameters Θ in Eq. (21) from the response measurements in the time domain

$$\begin{aligned} \widehat{\Theta}_k &= \widehat{\Theta}_{k-1} + \mu_k R_k^{-1} \psi_{k;\Theta_{k-1}} \varepsilon_{k;\Theta_{k-1}} \\ \varepsilon_{k;\Theta} &= y_k - \widehat{y}_{k;\Theta} \\ R_k &= R_{k-1} + \mu_k \left[\psi_{k;\Theta_{k-1}} \psi^T_{k;\Theta_{k-1}} - R_{k-1} \right] \end{aligned} \tag{22}$$

where ψ_k is the gradient of y. Thus, model parameter estimation refers to the recursive determination, for a given model structure, parameter vector Θ [a_1, a_2 … a_p, b_1, b_2, … b_q] and the residual variance $\sigma e^2(t)$ at every sample time instant k = 1, 2, … n. The AR characteristic equation of (19) can be written [4]

$$\sum_{i=0}^{p} a_i y(t-i) = \prod_{j=1}^{n} (\mu - \mu_j)(\mu - \mu_j^*) \tag{23}$$

where μ_j^* is the complex conjugate of μ_j. From Eq. (23) the operational damping, ξ_j and frequency ω_j are recursively calculated at each time instant t

$$(\xi_{\mathrm{mod}})_j = \frac{\ln(\mu_j \mu_j^*)}{\sqrt{\ln(\mu_j \mu_j^*)^2 - 4\left[\tan^{-1}\left(\frac{\mu_j - \mu_j^*}{\mu_j + \mu_j^*}\right)\right]^2}} \tag{24}$$

$$(\omega_{\mathrm{mod}})_j = -\frac{1}{2\Delta T}\sqrt{\ln(\mu_j \mu_j^*)^2 - 4\left[\tan^{-1}\left(\frac{\mu_j - \mu_j^*}{\mu_j + \mu_j^*}\right)\right]^2} \tag{25}$$

One of the major benefits of implementing the damping criterion in RARMA is the fast tracking of the instantaneous ODP under actual machining. In industrial applications it is often the case that cutting conditions are changing due to variations in workpiece geometry, cutting parameters, clamping device position relative cutting position, and machine tool position etc. For instance, when end-milling the top plane of a cylinder block for car and truck engines, the cutting parameters are changing due to variations in cutting conditions such as position on cylinder block [46]. In such a case the position in respect to clamping devices is essential.

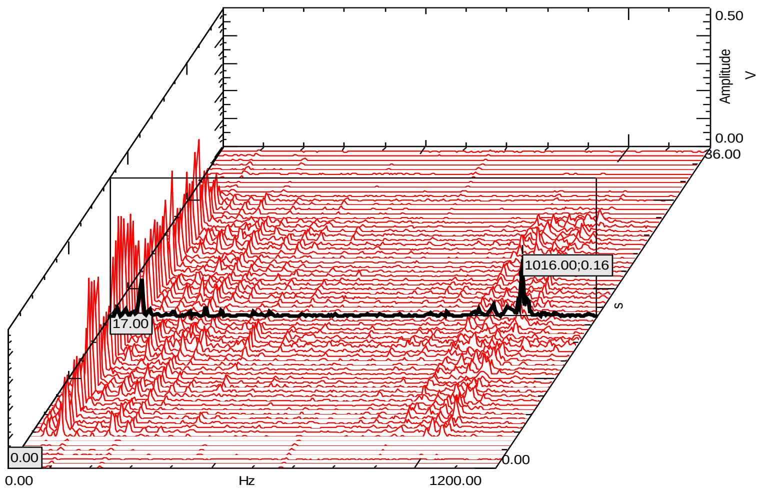


Fig. 19. PSD of the vibration signal

To verify the recursive parametric identification, a workpiece was prepared with slots and pockets (see Fig. 20). Between each section of slots and pockets sections with homogenous material are machined, enabling two inserts simultaneously to be engaged in workpiece. The chosen operational conditions are believed to highly affect the cutting stability as the discontinuity in the tooth passing frequency, caused by varying multiple entries each section, leads to a strong variability in the cutting operation. The response was measured by a microphone and a triaxial accelerometer. The microphone was placed inside the machine tool work area and the accelerometer was placed on the spindle casing at the location corresponding to the front bearings [42]. The variation in cutting condition can clearly be seen by studying the PSD (in waterfall representation) of the acquired microphone and accelerometer signals (see Fig. 19). The response amplitude corresponding to the frequency representing the forced vibrations is varying depending on where on the workpiece the cutting is performed. The forced vibration frequency is normally correlated to the cutting frequency of the system. In this experiment, due to holes and slots, the cutting frequency is varying depending on the position on the workpiece. Over the inhomogeneous sections, the cutting frequency increases. This can clearly be seen by studying the microphone signal and accelerometer signal in X-direction. When machining over homogeneous sections the cutting frequency decreases. Milling over the second inhomogeneous section of the workpiece leads to a change in cutting frequency (multiple entries each revolution). The amplitude of the PSD increases over the whole frequency range particularly two vibration concentrations can be seen, 2 kHz and 3 kHz. At these frequencies two structural modes are located (identified with EMA). The RARMA-ODP algorithm identifies two dominant operational frequencies and related damping ratios. The first operational frequency $(f_{op})_1$ is approximately 95 Hz, and relates to the workpiece-table system (verified by EMA). This frequency is mainly correlated to cutting of multiple tooth entries (due to slots and pockets). As chatter is generated close to a structural natural frequency the instability is likely to occur in the weakest mode or modes of the structure. Normally these modes can be related to machine tool structure such as tool, tool holder and spindle. Regenerative self-excited vibrations occurs on the second operational frequency $(fop)_2$ which is approximately 990 Hz and is related to the tool-spindle system (verified with EMA). In this case, when the operational damping ratio $(\xi_{op})2$ is locally decreasing to zero (or close to zero) then chatter occurs (verified by surface roughness analysis) due to regenerative effects (Figure 19). The model order was RARMA(4,3).

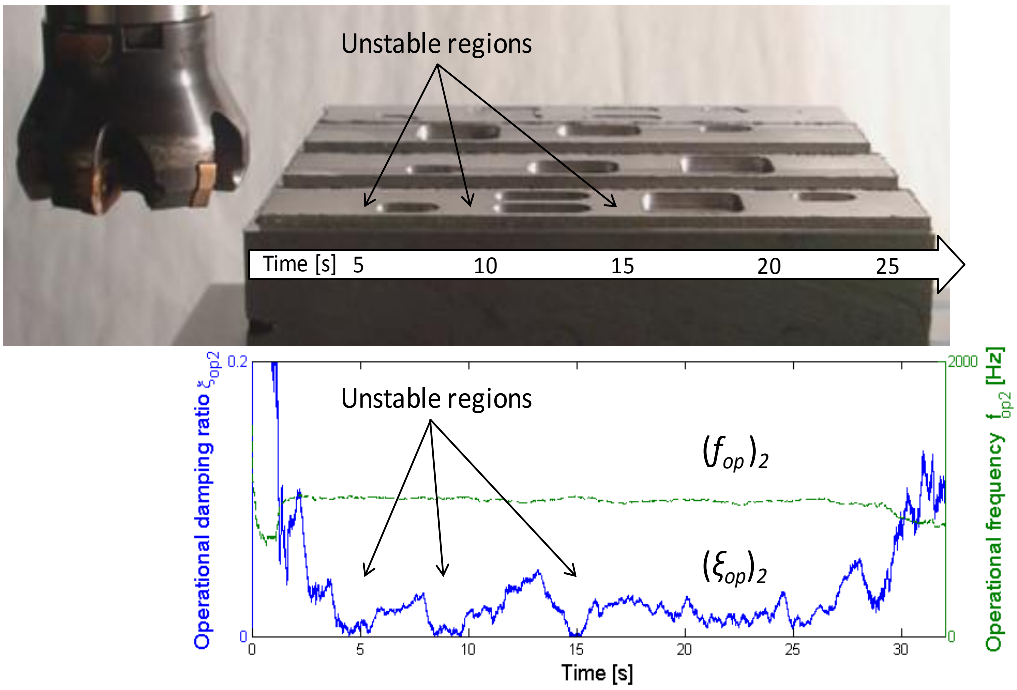


Fig. 20. Recursive identification of ODP in a milling operation

6. CONCLUSION

The present paper considers the identification methods of the machining system. The paper gives an assessment, in the first section, of the nonlinearities encountered in the machine tool structure and the cutting process. The machining system can be completely analysed only in closed loop i.e. in operational conditions, since specially designed off-line experiments with controlled input, such as modal testing, give the response from only the open loop. Also, as the system's parameters are nonlinear, they depend on the energy level.

In the subsequent sections, the application of parametric and nonparametric models for ODPs identification is described. Both methods are used in conditions where input excitation is unknown, which is the case of machining systems. The non-parametric identification approach is best suited in systems with low damping. The non-parametric identification technique provides possibility of estimating the departure from linearity of system parameters. The experimental results show that close to chatter the nonlinearity of the system increases. The non-parametric identification technique also provides the possibility of estimating ODPs at different energy levels as nonlinearity of the system requires. Identification procedure in the case of non-parametric technique follows three steps: (1) potential energy estimation, and fitting to a polynomial. Then the stiffness function is calculated from the derivative of the polynomial. (2) the damping potential is calculated from the response probability density function. (3) the equivalent damping is calculated for different levels of energy and fitted to a polynomial. From the polynomial and with knowledge of potential function the operational damping can be calculated. Simulations demonstrate very good results in stiffness estimation, especially for light damping. Damping estimates show a certain scattering effect especially at higher energy levels and for nonlinear systems. A longitudinal turning operation is used to demonstrate the capability to estimate ODPs both in stable and unstable systems.

Regarding parametric identification, a recursive model is developed allowing real time identification of the ODPs. The ability of the recursive estimation to track fast changes in operation conditions is demonstrated in the case of a milling operation. A stable condition alternates with an unstable. There is a good correlation between the behaviour of the system and variations of ODPs. The ODP values are extracted from an ARMA model’s parameters. There is a compromise however between the accuracy of the identified parameters and the execution speed. Parametric identification is a well suited technique for detecting impending chatter since the criteria for detection is based on the damping variation rather than the relative change in the amplitude of vibration. This also gives a robust criteria for discrimination between forced and chatter vibration. The distinction is important since chatter represents an inherent unstable system and forced vibration represents a stable system that could work closed to one of the natural frequencies of the system. Therefore, the ways avoiding these phenomena are different. The recursive parametric identification method can be straightforward implemented in various approaches for the control of a machining system.

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