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Application of two parameter groups of the Knothe–Budryk theory in subsidence prediction

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ABSTRACT

The research includes geodetic measurements carried out along eight observation lines located in the area of Upper Silesian Coal Basin in order to determine the possible usage of the bimodal method in subsidence prediction. The article contains information about principles of the proposed method, description of observation material and applied software and comparison of calculations results of deformation indicators by the bimodal method (including three parameters: a, tgβ_1 and tgβ_2) and the classic one (including two parameters: a and tgβ). The article includes initial results in terms of parameters' values determination of the bimodal method on the basis of mining and geological conditions, and it presents an accuracy evaluation of the proposed method.

1. Introduction

The Knothe–Budryk theory is commonly used for impact prediction of underground mining exploitation, both in Poland and abroad. Due to a well-defined mathematical model and low number of parameters, the method describes with adequate accuracy the phenomena of surface and rock mass deformation at different mining and geological conditions (Kwiatek, 1998).

The research carried out by J. Białek indicates that there are some limitations of the geometric and integral theories in terms of subsidence description. Predicted vertical displacements in a distance greater than the radius of the main influences range r are usually smaller than ones measured along observation lines. J. Białek in order to solve the issue presented possible usage of the influence function as a linear combination of a number of Gaussian functions (Białek, 1991a, 1991b, 1993).

J. Zych also emphasized systematic divergences between the observed and calculated deformation indicators using geometric and integral theories. The researcher proposed to use two non-linear functions that define vertical and horizontal displacement. He based his studies on his own research (Zych, 1987, 1998) and on research of other authors (Greń, Popiołek, & Ostrowski, 1985; Popiołek & Ostrowski, 1981).

S. Knothe proposed to include in the own theory that has been used since 1951 the asymmetric influence function composed of two formulae and assuming division of rock mass into two areas over a longwall face where features of rock strata are different. This means that the radius of the main influences range r_1 over a coal seam is greater than the radius of the main influences range r_2 over goaf of an extracted seam (Knothe, 1953, 2005, 2006).

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The article presents a method of subsidence prediction based on formulae of the Knothe–Budryk theory. The method involves two components of the influence function with various radii of the main influences range *r* according to the assumptions proposed by J. Białek. The article contains information about principles of the proposed method, description of observation material and applied software and comparison of calculations results of deformation indicators by the bimodal method (including three parameters: *a*, $tg\beta_1$ and $tg\beta_2$) and the classic one (including two parameters: *a* and tgβ). Additionally, an initial accuracy evaluation of the bimodal method was carried out.

2. Principles of the method

The aim of the method is an attempt to improve the description of predicted deformation indicators using two influence functions of the Knothe–Budryk theory. In 1991 J. Białek (Białek, 1991a, 1991b, 1993) presented a possibility to define the influence function for subsidence as a linear combination of a number of Gaussian functions with varied parameters *r* which can be defined as the following:

$$F(\rho;r) = A_1F(\rho;r_1) + A_2F(\rho;r_2) + \ldots + A_nF(\rho;r_n) \tag{2.1}$$

where:

$F(\rho,r)$ – the influence function of Gaussian distribution,
$F(\rho,r_1)$ – the function component for the radius of the main influences range r_1,
$F(\rho,r_2)$ – the function component for the radius of the main influences range r_2,
A_1,A_2 – coefficients of proportionality.

The method includes a two-component description, later referred to as bimodal:

$$F = A_1F(\rho;r_1) + A_2(\rho;r_2) \tag{2.2}$$

According to the Knothe–Budryk theory, the formulae for components of the influence function are as follows:

$$F(\rho;r_1) = \frac{1}{r_1}\exp\left[\frac{-\pi\rho^2}{r_1^2}\right] \tag{2.3}$$

$$F(\rho;r_2) = \frac{1}{r_2}\exp\left[\frac{-\pi\rho^2}{r_2^2}\right] \tag{2.4}$$

where:

r_1 – the radius of the main influences range over an extracted seam [m],
r_2 – the radius of the main influences range over a coal seam [m],
ρ – the horizontal distance between point *x* on the surface and an elementary area of an extracted coal seam *s*; $\rho = x - s$ [m].

The method assumes virtual division of an extracted seam into two layers. The surface deformation indicators will be calculated with the exploitation coefficient *a* and two parameters: $tg\beta_1$ and $tg\beta_2$, for each of seam layers. Due to an experimental character of the method (a combination of the influence functions), it is necessary to conduct an analysis of parameters values, as well as determination of correlation between them on the basis of gathered data.

The basic deformation indicator, which will be the subject of analysis, is subsidence. After applying the two-component influence function the formula of the indicator is as follows:

$$w(x) = ag_1F(\rho;r_1) + ag_2F(\rho;r_2) \tag{2.5}$$

where:

$$g = g_1 + g_2 \tag{2.6}$$

$$r_1 = \frac{H}{tg\,\beta_1} \tag{2.7}$$

$$r_2 = \frac{H}{tg\,\beta_2} \tag{2.8}$$

a – exploitation coefficient,
g – extracted seam thickness [m],
g_1,g_2 – height of virtual extracted seam layer [m],
H – average exploitation depth [m],
β_1 – angle of the main influences range over an extracted seam [°],
β_2 – angle of the main influences range over a coal seam [°].

3. Characteristics of observation material and applied software

3.1. Characteristics of acquired data

The research included geodetic measurements carried out along eight observation lines located in the area of Upper Silesian Coal Basin in order to determine the possible usage of the bimodal method in prediction of surface deformation indicators.

Gathered data concerns the exploitation of coal seams using longwall system with caving (7 instances) and with hydraulic filling (1 instance). Deposits were extracted with single, two and four longwall faces at depth of 400 m–1080 m. The height of the extracted coal seam or layer is 2.1–3.4 m. The analysed longwall panels were in the disturbed and undisturbed rock mass by earlier workings. The dip of the coal beds does not exceed 10°. The thickness of overburden is 5–290 m.

The examples of chosen longwall panels do not involve mining operations in other seams at the same time that would have an influence on geodetic measurements results.

Table 1 presents data of longwalls with basic geological conditions.

3.2. Description of the applied software

The parameters of the Knothe–Budryk theory were determined by the bimodal and the classic methods using

Table 1 – Mining and geological data of investigated longwall panels.

No.	Mine	Seam	Longwall no.	g_{avg}, m	H_{avg}, m	W × l, m	α, °	H_{ovb}, m	Rock mass type
1	Pokój	502 t.l.	225	2.1	520	270 × 307	10	5–20	Compact disturbed
2	Pokój	502 b.l.	225/II	2.1	520	255 × 300	10	5–20	Compact disturbed
3	Piekary	510 IIIth l.	534	3.0	425	175/235 × 745	4–8	185–200	Compact disturbed
4	Wujek	409	3	2.3	1030	260 × 805	6	40–290	Compact undisturbed
			5	2.4	1060	250 × 1090			
5	Kazimierz-Juliusz	510 Vth l.	255	3.2	440	40/150 × 600	6	30–50	Compact undisturbed
			256	3.2	440	80/150 × 960			
			257	3.2	435	190 × 1090			
			251	3.0	425	40/105 × 570			
6	Kazimierz-Juliusz	510 IVth l.	246	3.15	437	100/254 × 660	6	30–50	Compact disturbed
			247	3.15	435	130 × 940			
7	Bobrek-Centrum	510 b.l.	24	2.25	805	265/295 × 805	6	175–190	Compact highly disturbed
8	Wujek	405	10	3.40	710	221 × 970	3–5	30–90	Compact undisturbed

where:
g_{avg} – average thickness of the extracted coal seam,
H_{avg} – average height of exploitation,
w – longwall panel width,
l – longwall panel length,
α – dip of the coal seam,
H_{ovb} – overburden thickness,
t.l. – top layer of the extracted coal seam,
b.l. – bottom layer of the extracted coal seam,
nth – n-th layer of the extracted coal seam.

FitParams software written in the Central Mining Institute by E. Jędrzejec.

It is a 32-bit application written in Object Pascal programming language using the system Delphi 5.0. The software enables to calculate subsidence and horizontal deformations by determining the parameters values of multimodal model of the Knothe–Budryk theory. The bimodal method was limited to the two-component linear combination of Gaussian function with different parameters *r*.

The program was created in order to determine the values of a number of parameters (*a*, tgβ, *p* – operating rim). The following conditions were met:

$$\sum_i \left[w_{cal_1}^{(i)} + w_{cal_2}^{(i)} - w_{obs}^{(i)}\right]^2 = \min \quad (3.1)$$

where:

$w_{cal_1}^{(i)}$ – calculated partial subsidence at the *i*-th point for the first extracted seam layer [mm],
$w_{cal_2}^{(i)}$ – calculated partial subsidence at the *i*-th point for the second extracted seam layer [mm],
$w_{obs}^{(i)}$ – measured subsidence at the *i*-th point [mm].

The software enables to define:

- number of partial subsidence (m),
- number of determined parameters (*a*, tgβ, *p*),
- operating rims for panels (*p*),
- division of panels according to an exploitation method (*a*, tgβ),
- division of panels according to differentiation of operating rim (*p*),
- value range of defined parameters (*a*, tgβ, *p*).

3.3. *Calculation assumptions*

In order to match the profile of the subsidence trough as unknown quantities the research assumed the exploitation coefficient *a* and tangents of angles of the main influences range $tg\beta_1$ and $tg\beta_2$. The values of parameters $tg\beta_1$ and $tg\beta_2$ were searched between 0.5 and 5.0.

The calculations included equal operating rims *p* in order to get a reliable evaluation of both methods.

The selection of an optimal combination of parameters took into consideration a minimum value of the aim function and the lowest value of the mean squared error of matching σ_M.

4. Comparison and analysis of calculations results of deformation indicators by the classic and the bimodal methods

The analysis based on acquired observation data was conducted to determine the calculation accuracy of deformation indicators using the bimodal and the classic methods.

The following accuracy indexes were defined in order to carry out the analysis:

– mean error:

$$v_{avg} = \frac{\sum\left(w_{obs}^{(i)} - w_{cal}^{(i)}\right)}{n}, \text{mm} \quad (4.1)$$

– mean absolute error:

$$v_0 = \frac{\sum\left(\left|w_{obs}^{(i)} - w_{cal}^{(i)}\right|\right)}{n}, \text{mm} \quad (4.2)$$

– mean squared deviation:

$$\sigma = \sqrt{\frac{\sum\left(w_{obs}^{(i)} - w_{cal}^{(i)}\right)^2}{n-1}}, \text{mm} \quad (4.3)$$

– percent error:

$$O = \frac{\sigma}{w_{obs}^{max}} \quad (4.4)$$

– percent error at the bottom of a subsidence trough:

$$O_B = \frac{w_{obs}^{max} - w_{cal}^{max}}{w_{obs}^{max}} \quad (4.5)$$

– percent error at the edge of a subsidence trough:

$$O_E = \frac{w_{obs} - w_{cal}}{w_{obs}^{max}} \quad (4.6)$$

where:

w_{obs} – observed subsidence [mm],
w_{cal} – calculated subsidence [mm],
n – the number of observation points.

Tables 2 and 3 present results of accuracy evaluation for vertical displacements using the bimodal and the classic methods. Fig. 1–3 show charts for chosen examples of observed and calculated subsidence, tilts, curvatures and horizontal deformations along measurement lines and an outline of the longwalls over observation fields.

The data in Table 2 indicates that the exploitation coefficient a increases when the calculation includes two radii of the main influences range r_1 and r_2 in the bimodal method. Five in eight examples the difference is very small and ranges from 0.01 to 0.08. However, for three examples (point 1, 3, 4 in Table 2) it amounts to about 0.2. An increase of the coefficient values is related to calculations which include surface mining influences over an extracted panel and over a coal seam.

There are two parameters $tg\beta_1$ and $tg\beta_2$ in the proposed method. They describe surface mining influences over an extracted panel and over a coal seam. Data in Table 2 shows that the divergence between those two parameters is fairly large. The $tg\beta_1$ values range from 1.8 to 4.5 and the $tg\beta_2$ values range from 0.6 to 1.9. The radii ratio r_2/r_1 for gathered examples of mining operations is between 1.7 and 3.8. The results are consistent with assumption presented by J. Białek (1991a, 1991b, 1993).

It can be concluded, on the basis of the analysis of acquired data (Tables 2 and 3), that the $tg\beta_2$ value is smaller than 1.0 in the case of mining operation in undisturbed rock mass (Wujek Mine, items 4 and 8, Table 2) or in the case of its resumption (Pokój Mine, item 1, Table 2). However the parameter value increases in the case of mining operations in disturbed rock mass. The maximum value of $tg\beta_2$ for the studied examples amounts to 1.9 (Bobrek-Centrum Mine, item 7, Table 2).

The values of the $tg\beta_1$ parameter obtained by the bimodal method are higher than the ones calculated by the classic method. The values may exceed 2.0 when the level of rock mass disturbance is high and the exploitation areas are large. The highest value of $tg\beta_1$ was determined for Kazimierz-Juliusz and Bobrek-Centrum mines (item 5–7, Table 2).

Calculated parameters of the Knothe–Budryk theory by the bimodal method for acquired examples of the mining operations may be defined as reliable. If the values of $tg\beta_1$ and $tg\beta_2$ are averaged, the value is similar to $tg\beta$ calculated by the classic method. Greater data set will allow to verify the above presented conclusion and to specify relationship between the parameters.

Comparing the values of the indicators presented in Tables 2 and 3 it is possible to conclude that the bimodal method enables to obtain greater accuracy of vertical displacement calculations, and as a result higher level of accuracy for tilts and curvatures.

The mean error ν_{avg} for subsidence obtained by the bimodal method is from −21 mm to 17 mm. However the indicator does not always emphasise the differences of compared data as a result of occurring negative values. More accurate indicator is the mean absolute error ν_0 which ranges between 8 mm and 61 mm. This indicator is smaller in comparison to the results of the classic method. The differences range from 4% to 46%.

The mean squared deviation σ is a significant indicator to assess the differences between the measured and calculated vertical displacements along the observation lines. The bimodal method enables to improve description of subsidence up to 46%.

There are no significant differences while comparing the rest of the indicators of accuracy evaluation. However, generally the values of percent errors in the bimodal method are smaller than the ones in the classic method (the differences are up to 3% for the whole subsidence trough – indicator O and up to 4% for the edge sides of the subsidence trough – indicator O_E).

The percent error for the bottom part of the subsidence trough O_B with two radii r_1 and r_2 ranges from −2% to 4%, and with radius r ranges from −2% up to 8%.

A greater level of conformity may be obtained for the observed and calculated profile of the subsidence trough by determining the parameters of the Knothe–Budryk theory with two radii r_1 and r_2. However, if the asymmetry of the subsidence trough profile is high, the difference between the results obtained by the bimodal and the classic method is minor (items 7 and 8, Tables 2 and 3).

5. Conclusion

The analysis of the determined parameters by the bimodal and the classic methods for acquired examples of mining operation indicates the following:

Table 2 – The evaluation results of calculation accuracy for vertical displacement by the bimodal method.

No.	Mine	a	$tg\beta_1$	$tg\beta_2$	p, m	w^{max}_{obsw}, mm	w^{max}_{cal}, mm	ν_{avg}, mm	ν_0, mm	σ, mm	O, %	O_B, %	O_E, % Trough side	
1	Pokój	0.95	2.0	0.7	30	612	608	1.0	12.1	15.1	2.5	0.6	E −0.5	W 0.3
2	Pokój	0.91	3.0	1.8	45	708	720	1.8	8.0	10.6	1.5	−1.7	E 0.6	W 1.0
3	Piekary	1.04	1.9	0.8	N – 30 S – 135	1591	1560	4.7	41.9	56.1	3.5	1.9	N 0.5	S 2.2
4	Wujek Śląsk	0.89	1.8	0.6	70	651	644	5.3	23.0	29.2	4.5	1.1	W 12.4	E 6.9
5	Kazimierz-Juliusz	0.76	4.1	1.1	N – 85 S – 25	2223	2163	−21.2	60.7	89.9	4.0	2.7	N 0.5	S 1.0
6	Kazimierz-Juliusz	0.97	4.5	1.2	N – 40 S – 40	2591	2532	17.1	38.6	45.5	1.8	2.3	N 0.8	S 1.6
7	Bobrek-Centrum	0.86	4.4	1.9	N – 80 S – 140	1441	1407	−6.0	53.6	66.4	4.6	2.4	N −0.8	S 1.7
8	Wujek	0.12	1.9	0.9	0	166	159	−1.1	8.3	10.9	6.6	4.2	N 0.6	S 7.2

1. A greater level of conformity may be obtained for the observed and calculated profile of the subsidence trough by determining the parameters of the Knothe–Budryk theory with two radii r. The bimodal method enables to improve description of subsidence up to 46%. However, if the asymmetry of the subsidence trough profile is high, the difference between the results obtained by the bimodal and the classic methods is minor.
2. The values of the determined exploitation coefficient a in the bimodal method is greater than values calculated by the classic method considering two parameters $tg\beta_1$ and $tg\beta_2$. An increase of the coefficient values (even up to 0.2) is related to calculations which include surface mining influences over an extracted panel and over a coal seam.
3. The $tg\beta_1$ parameter describing surface mining influences over an extracted panel ranges from 1.8 to 4.5. The values may exceed 2.0 when the level of rock mass disturbance is high and extracted panels are large.
4. The $tg\beta_2$ parameter describing surface mining influences over a coal seam ranges from 0.6 to 1.9. After analysing the data, it is possible to conclude that the parameter is lower than 1.0 in the case of mining operation in the undisturbed rock mass or in the case of its resumption after a long break. However the parameter value increases in the case of mining operation in disturbed rock mass.
5. Calculated parameters of the Knothe–Budryk theory by the bimodal method for acquired examples of the mining operations may be defined as reliable. If the values of $tg\beta_1$ and $tg\beta_2$ are averaged, the value is similar to $tg\beta$ calculated by the classic method. Greater data set will allow to verify the above presented conclusion and to specify relationship between the parameters.

Table 3 – The evaluation results of calculation accuracy for vertical displacement by the classic method.

No.	Mine	a	$tg\beta$	p, m	w^{max}_{obsw}, mm	w^{max}_{cal}, mm	ν_{avg}, mm	ν_0, mm	σ, mm	O, %	O_B, %	O_E, % Trough side	
1	Pokój	0,73	1.6	30	612	582	10.8	21.2	24.4	4.0	4.9	E 2.6	W 4.4
2	Pokój	0.88	2.4	45	708	721	2.5	10.4	12.4	1.7	−1.8	E 0.6	W 1.1
3	Piekary	0.84	1.6	N – 30 S – 135	1591	1590	27.9	58.6	67.1	4.2	0.1	N 2.6	S 2.3
4	Wujek	0.70	1.5	70	651	683	−8.2	41.3	48.1	7.4	−4.8	W 16.6	E 10.7
5	Kazimierz-Juliusz	0.70	2.0	N – 85 S – 25	2223	2208	17.9	83.1	99.8	4.5	0.7	N 0.5	S 1.4
6	Kazimierz-Juliusz	0.89	2.2	N – 40 S – 40	2591	2636	41.3	71.9	84.3	3.3	−1.7	N 0.8	S 1.7
7	Bobrek-Centrum	0.79	3.1	N – 80 S – 140	1441	1418	6.4	53.3	69.4	4.8	1.6	N 0.5	S 2.4
8	Wujek	0.11	1.4	0	166	152	2.6	8.6	11.1	6.7	8.4	N 1.2	S 9.0

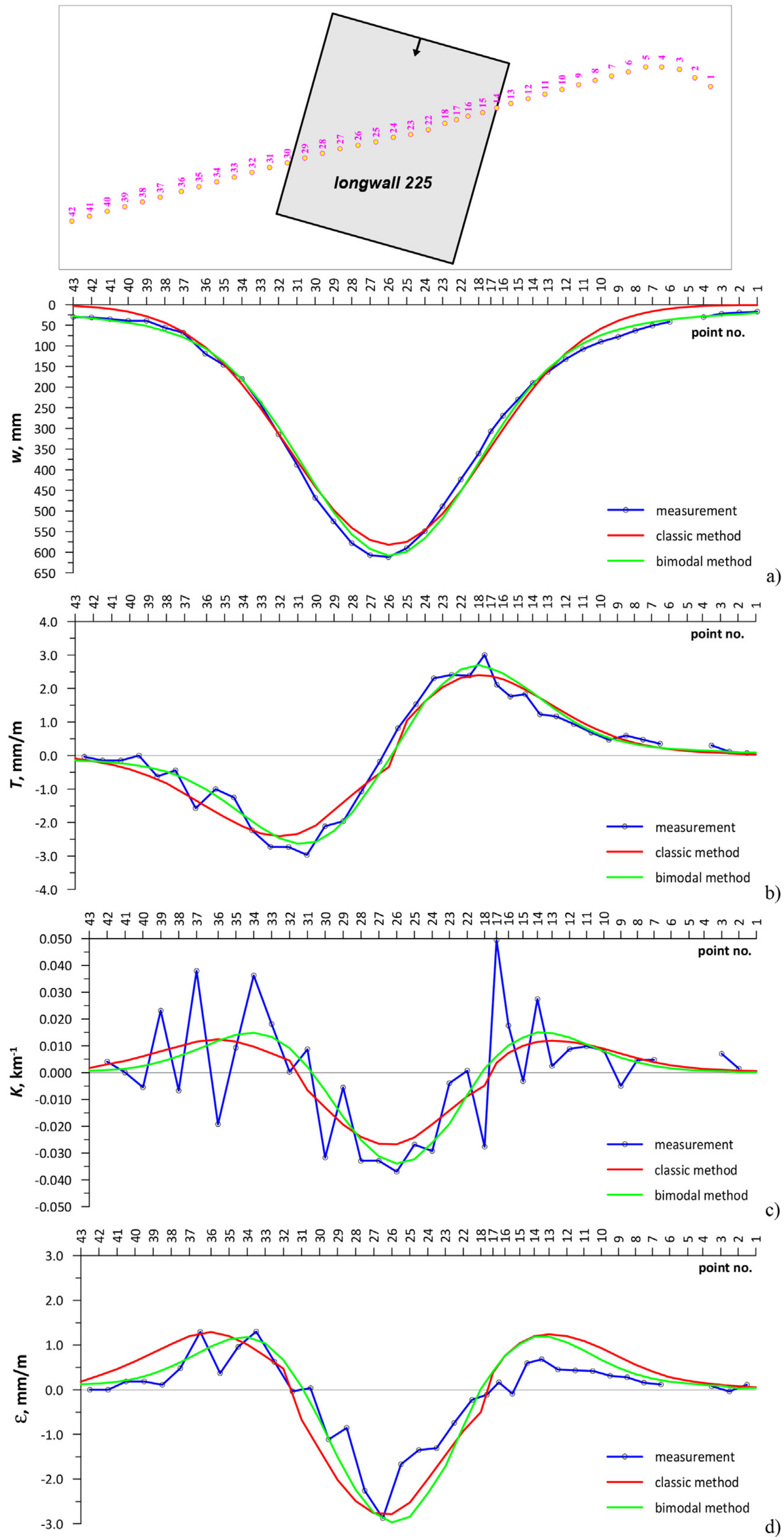

Fig. 1 – The outline of the longwall 225 in the bottom layer of the seam 502 and charts of deformation indicators along the measurement line: a) subsidence, b) tilts, c) curvatures, d) horizontal deformations.

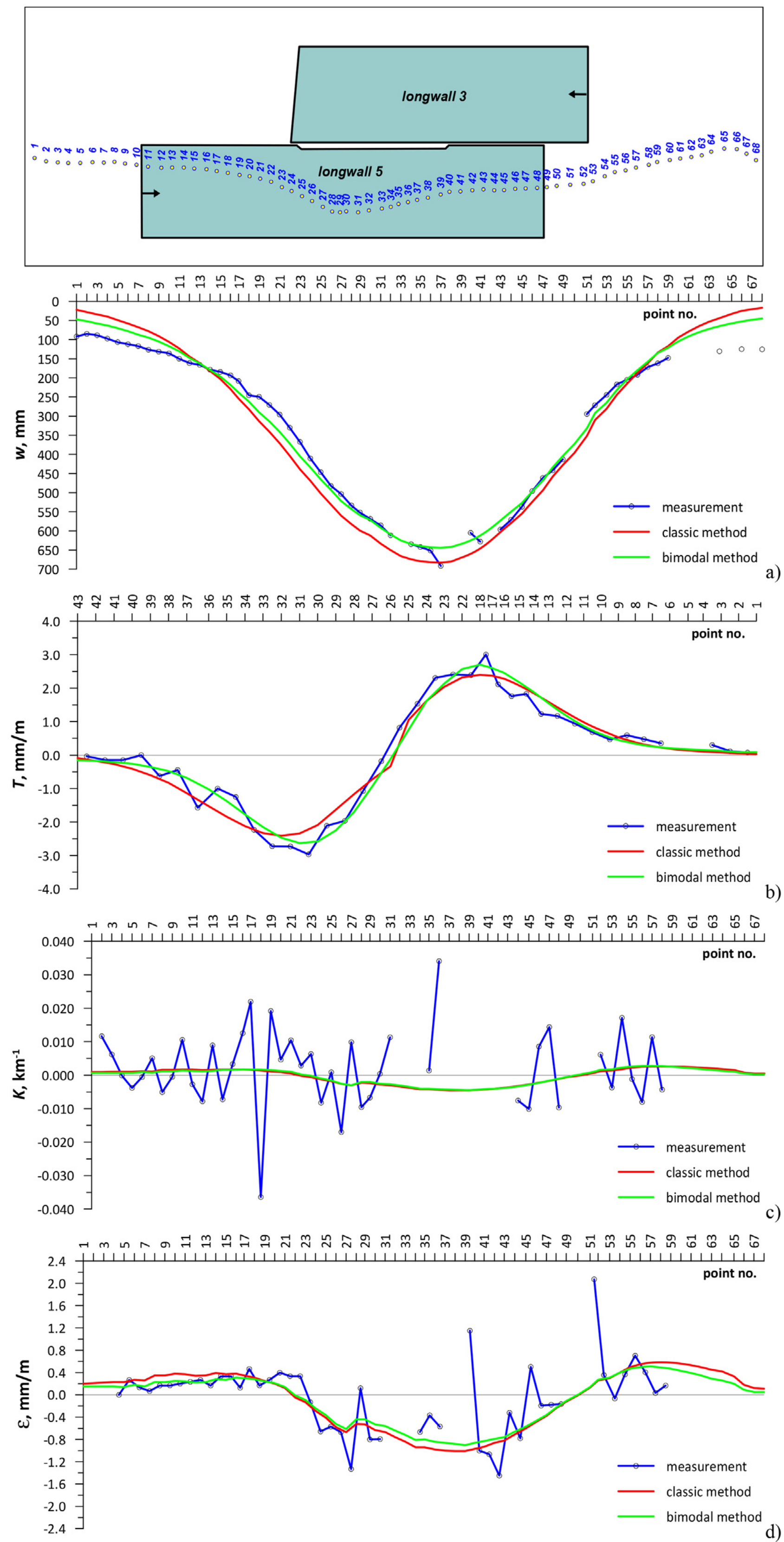

Fig. 2 – The outline of longwalls 3 and 5 in the seam 409 and charts of deformation indicators along the measurement line: a) subsidence, b) tilts, c) curvatures, d) horizontal deformations.

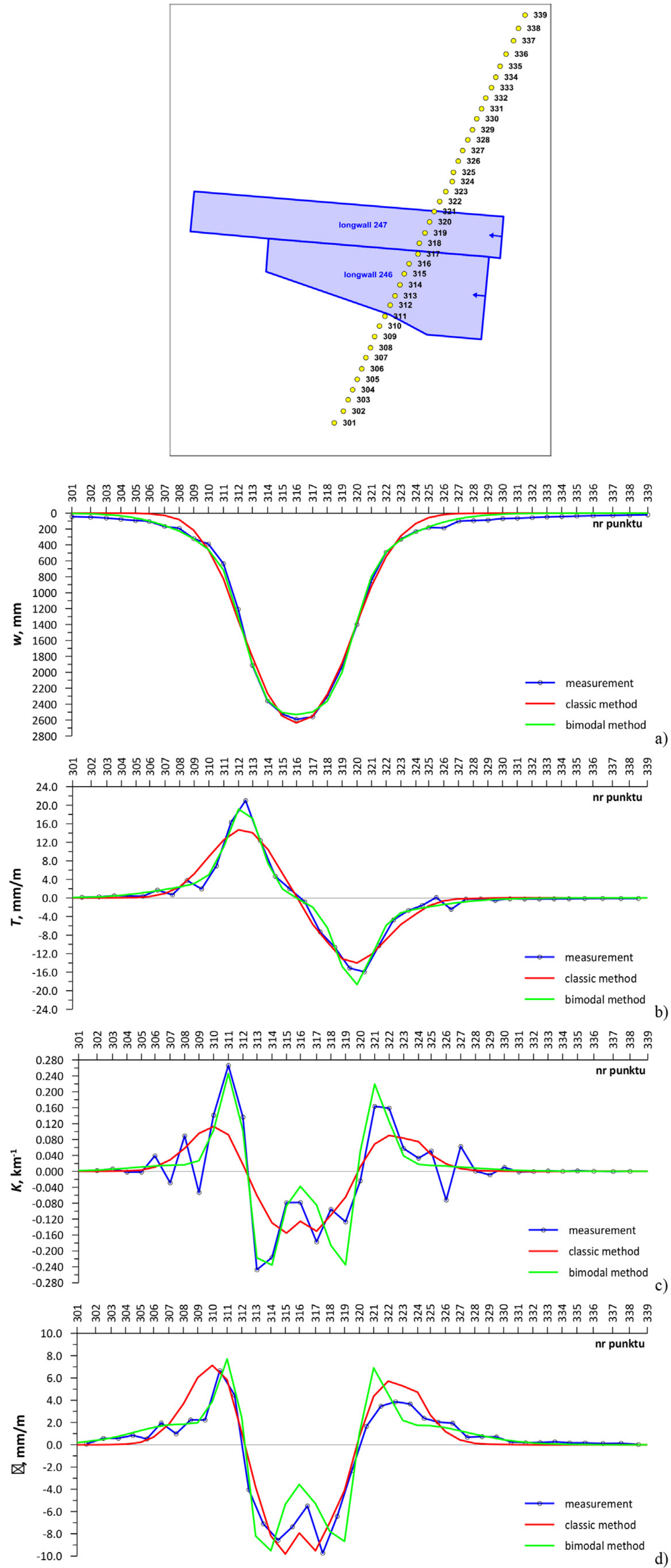

Fig. 3 – The outline of longwalls 246 and 247 in the IVth layer of the seam 510 and charts of deformation indicators along the measurement line: a) subsidence, b) tilts, c) curvatures, d) horizontal deformations.

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