JAISCR, 2021, Vol. 11, No. 2, pp. 85 – 97
10.2478/jaiscr-2021-0006

# TYPE-2 FUZZY LOGIC SYSTEMS IN APPLICATIONS: MANAGING DATA IN SELECTIVE CATALYTIC REDUCTION FOR AIR POLLUTION PREVENTION

Adam Niewiadomski, Marcin Kacprowicz

*Institute of Information Technology, Lodz University of Technology,*
*Wólczańska 215, 90-924 Łódź, Poland*

*E-mail: marcin.kacprowicz@mm.wloclawek.pl*

*Submitted: 26th March 2020; Accepted: 5th November 2020*

**Abstract**

The article presents our research on applications of fuzzy logic to reduce air pollution by DeNOx filters. The research aim is to manage data on Selective Catalytic Reduction (SCR) process responsible for reducing the emission of nitrogen oxide (NO) and nitrogen dioxide ($NO_2$). Dedicated traditional Fuzzy Logic Systems (FLS) and Type-2 Fuzzy Logic Systems (T2FLS) are proposed with the use of new methods for learning fuzzy rules and with new types of fuzzy implications (the so-called ”engineering implications”). The obtained results are consistent with the results provided by experts. The main advantage of this paper is that type-2 fuzzy logic systems with ”engineering implications” and new methods of learning fuzzy rules give results closer to expert expectations than those based on traditional fuzzy logic systems. According to the literature review, no T2FLS were applied to manage DeNOx filter prior to the research presented here.
**Keywords**: Selective Catalytic Reduction (SCR), fuzzy management of DeNOx filter, fuzzy logic systems, ”engineering” fuzzy implications, learning fuzzy rules.

## 1 Introduction

Attempts to create systems working similarly to (or even replacing) human-being in different activities are very common nowadays. Many approximations of human-being actions by machines and/or software can be found in different fields and realized using various methods. Expert systems are used for engineering tasks [1], in high voltage diagnostic systems [2] or as classification systems [3]. Fuzzy Logic Systems (FLS) are used as components of experts systems when expert knowledge cannot be expressed unambiguously, or is too difficult or too complex to be described in traditional and/or mathematical manners (e.g. quantitative terms). Examples for progress of fuzzy logic systems and type-2 fuzzy logic systems are given in numerous publications, e.g. [4, 5, 6, 7]. Quoted methods aim is to give models of uncertain data for systems that are supposed to give results as close to those given by a human-being as possible. Systems using fuzzy logic are very popular, mostly because data necessary for operations can be obtained from experts using natural language. This allows a much easier way to determine input data and does not require knowledge on fuzzy logic or information systems from experts in a field. Fuzzy logic systems are successfully used in many solutions, showing greater effectiveness than traditional (linear) models. Various examples describe the use of FLSs in nuclear power plants [8], crane control [9], elevator control [10], train control [11], or water quality control [12, 13], and many others.

The rest of the paper is organized as follows: Section 2 describes the specificity of the Selective Catalytic Reduction process and uncertainty of knowledge appearing in managing it. This description is based on our previous publications [14, 15, 16, 17], however, we provided it again to make the report on the newest results complete and to give the background for comparative analysis. Similarly, in Section 3, we comment on basics of designing traditional fuzzy logic systems to manage the SCR process and on their previously introduced modifications like using the so-called engineering implications [15] or learning rules algorithms [17]. In Section 4, Type-2 Fuzzy Logic Systems are designed and tested in managing the SCR process. Analogously to traditional FLSs in Section 3, also the presented T2FLSs are improved via using extended implications in the inference block, Section 4.2, and the fuzzy rules are being learned with newly proposed iterative algorithms, Section 4.3. The results showing better performance of Type-2 Fuzzy Logic Systems are collected and commented in Section 5 and the paper is concluded in Section 6.

## 2 Knowledge specification and data uncertainty in Selective Catalytic Reduction

One of the most efficient methods for reducing nitrogen oxides (NO, $NO_2$) in gases exhausted as by-products of combustion, is Selective Catalytic Reduction (SCR) [18]. The ammonia gas $NH_3$ is a reductor and injected to the reduction chamber, see Figure 1. The chemical model of this process is given by (1), (2)

$$4NO+4NH_3+O_2=4N_2+6H_2O, \quad (1)$$

$$6NO_2+8NH_3=7N_2+12H_2O. \quad (2)$$

The DeNOx filter performs the catalytic reduction and its main task is to reduce nitrogen oxides in chemical processes in which these oxides are harmful by-products. As for now, managing the parameters of this process must take place under human control due to the non-linearity of the process and many factors that affect the efficiency of the chemical reaction, so parameters of the DeNOx filter are controlled by human-being (of course, in large industrial installations). **The main goal of this paper is to propose fuzzy logic systems and type-2 fuzzy logic systems to support (or even replace) experts managing the DeNOx filter**, see Figure 2. Therefore, the attempt to develop fuzzy logic systems extended with new implications and new methods of learning fuzzy rules is supposed to increase the efficiency and reduce human expert effort, as solutions based on linear models do not give satisfactory results, especially, the results are not similar enough to human actions.

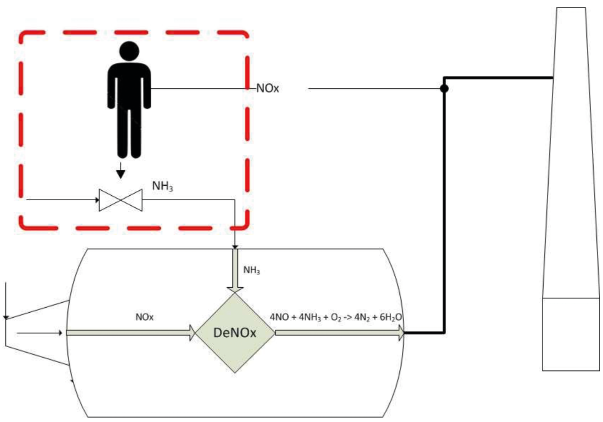


**Figure 1**. A schema of Selective Catalytic Reduction (SCR) performed by the DeNOx filter.

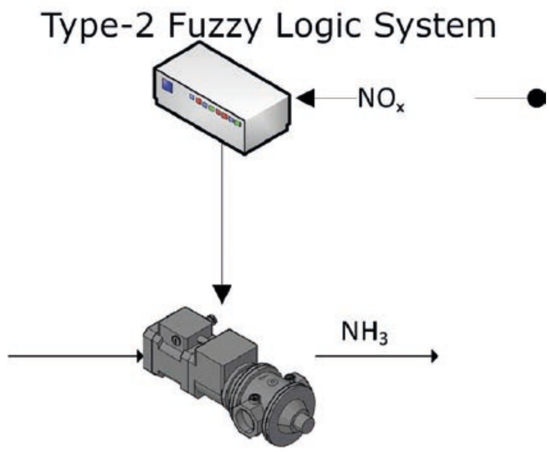


**Figure 2**. Selective catalytic reduction (SCR) – determining the settings of abrasion of the ammonia ($NH_3$) dosing valve into the reaction chamber

The knowledge necessary to design fuzzy logic systems is obtained from experts being process engineers and specifying settings, in particular the degree of opening of the ammonia dispensing valve to the reaction chamber of the filter. As for now, they are the only sources of knowledge on that, for several reasons:

1. The SCR process depends not only on the concentration of NO and $NO_2$, but also on:
   (a) Working temperature of the system,
   (b) Atmospheric conditions (affecting emission and SCR process),
   (c) The type of platinum mesh used as a catalyst,
   (d) The age of the platinum mesh (its thickness proportional to its use) used as a catalyst.
2. No measurements or difficult measurements of properties that affect the process, e.g. type and thickness of the catalytic mesh, the working temperature of the process [18].
3. Expert knowledge used to manage the SCR process is difficult or impossible to express with traditional sets or parameters and their dependencies; the specificity of the process of selecting parameters in SCR allows, in practice, for linguistic descriptions only. The intuition and experience of an expert, playing important role in the process, is also difficult to be structured.
4. Logistic and/or economic conditions external to the SCR process, allowing, for example, periodic increase of concentration of nitrogen oxides released to the atmosphere at the price of purchasing additional gas emission permit, financial policy, changes in law on emission, etc.

The above-mentioned facts seem to be sufficient arguments to choose fuzzy logic system and type-2 fuzzy logic systems as tools to manage parameters of the SCR process in the DeNOx filter.

# 3 Managing the SCR process with fuzzy logic systems

This paper is focused on managing data used in the Selective Catalytic Reduction process using type-2 fuzzy logic systems (see Sections 4 and 5). We intend to is to show that a better solution to the problem can be proposed if T2FLS (based on type-2 fuzzy sets) are developed from the former (type-1) fuzzy logic systems, to control DeNOx filters better and more efficiently [14, 15, 16, 17].

Hence, at first, this Section resumes briefly our former works, in which traditional (Type-1) FLSs have been constructed to solve the problem. The research began with implementing of known architectures of fuzzy logic systems [19, 20, 21], to simulate the dispensing ammonia to the DeNOx reaction chamber. The results of this work are published in [14, 15, 16, 17]. Here, we recall them briefly to provide the reader with the background for analysis presented in Section 5.

## 3.1 Engineering implications in FLSs

By engineering implications we understand fuzzy implication-like operators that not necessarily complete all the axioms proposed for traditional fuzzy implications, see Table 2 and [22, 23, 24], however, they are intuitive and can be successfully applied as implication operators in fuzzy logic systems. Our previous results described in [15] point out that efficiency of designed systems may raise if using properly designed engineering implications, so we claim (and finally prove) that the supposed result — to calculate parameters as close to those proposed by human experts as possible — is achieved in type-2 fuzzy logic systems. The proposed implications are given by formulae (3) and (4). They are the so-called "engineering" implications, meaning that they work properly in fuzzy logic systems but they do not need to complete the axioms given for fuzzy implications to consider them as generalizations of the classic implication (see Tables 1, 2).

$$I_{K_1}(x,y) = \frac{\sqrt{xy}}{x+y-xy}, x,y \in (0,1], \quad (3)$$

$$I_{K_2}(x,y) = \frac{xy}{x+y}, \text{ for } x \neq 0 \vee y \neq 0, x,y \in [0,1]. \quad (4)$$

**Table 1**. Completing $T$-norms axioms by the proposed engineering fuzzy implications (3), (4).

| | Axiom of $T$-norms | $I_{K_1}$ | $I_{K_2}$ |
|---|---|---|---|
| 1 | commutativity | YES | YES |
| 2 | associativity | NO | NO |
| 3 | monotonicity | YES | YES |
| 4 | neutral element | NO | NO |

**Table 2**. Completing fuzzy implication axioms by engineering fuzzy implications (3), (4).

| | Axiom of fuzzy implications | $I_{K_1}$ | $I_{K_2}$ |
|---|---|---|---|
| 1 | $a \leq c \rightarrow I(a,b) \geq I(c,b)$ | NO | NO |
| 2 | $b \leq c \rightarrow I(a,b) \leq I(a,c)$ | NO | YES |
| 3 | $I(0,a) = 1$ | YES | YES |
| 4 | $I(a,1) = 1$ | NO | NO |
| 5 | $I(1,0) = 0$ | YES | YES |

Figures 3 and 4 show graphic representations of engineering implications (3), (4).

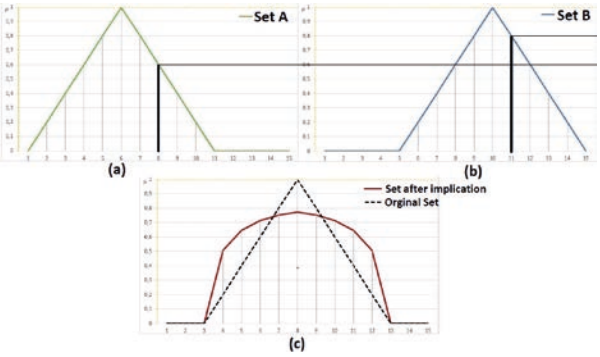


**Figure 3**. A result of implication $I_{K_1}$ (c) for sample fuzzy sets $A, B$ with triangular membership functions (a), (b)

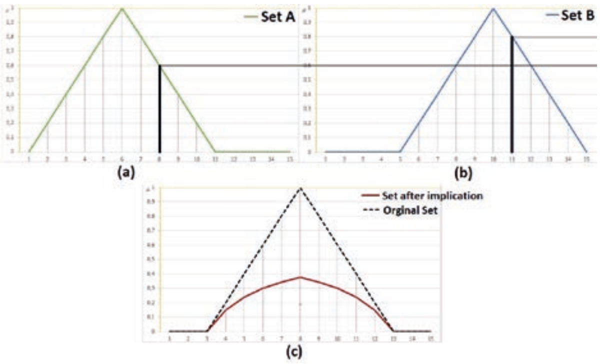


**Figure 4**. A result of implication $I_{K_2}$ (c) for sample fuzzy sets $A, B$ with triangular membership functions (a), (b)

### 3.2 Learning fuzzy rules in FLSs

The rule base is an essential component of each fuzzy system. Fuzzy rules are implications in terms of fuzzy logic: IF antecendent THEN succedent. Usually, rules express expert knowledge, dependencies, facts and/or observations that are too complex or too difficult to be expressed in traditional quantitative terms. However, it may happen that an expert is not able to propose the optimal set of rules for a given system, and, frequently, the rule base must be processed (e.g. reduced, learned or rebuilt). In this experiment, novel algorithms for learning fuzzy rules are proposed and applied, since those based on known literature methods, e.g. [25, 26], did not provide satisfying results. As the designed fuzzy systems collect data every 2s, additional conditions on computational complexity and time for the proposed learning rules algorithms must be assumed. As a result of the research, three new learning fuzzy rules (LFR) algorithms are proposed, working as follows: two inputs, $x_1 \in \mathcal{X}_1, x_2 \in \mathcal{X}_2$, and one output $y \in \mathcal{Y}$ for each rule are considered as elements of fuzzy sets in $\mathcal{X}_1, \mathcal{X}_2$, $\mathcal{Y}$, respectively. Using all input fuzzy sets, $nF_y$ IF-THEN rules are generated, where $n = \prod_{i=1}^{k} F_i$, $k$ is the number of input universes of discourse $\mathcal{X}_1, \mathcal{X}_2, \ldots, F_i$ is the number of fuzzy sets in a given $\mathcal{X}_i$, and $F_y$ is the number fuzzy sets in $\mathcal{Y}$. Each rule has one succedent $j$, where $j = 1, 2, \ldots, F_y$. The training data is represented by three sets of elements, $x_1, x_2$ and $y$. Learning fuzzy rules is based on the following steps:

1. All possible combinations of fuzzy rules from fuzzy sets of input and output data are created.
2. Input data is fuzzified and inference is performed.
3. Membership degree to each of the output fuzzy sets is evaluated.
4. The counter of each rule (of each succedent) is increased depending on satisfying certain conditions in a given algorithm:
   - Algorithm 1 increments succCount only for the rule with the largest membership degree of succedent.
   - Algorithm 2 increments the succCount for each rule with a non-zero membership degree of succedent.
   - In Algorithm 3, for each rule with a non-zero membership degree of succedent, the succCount is increased with this membership degree.
5. **For a given antecedent the rule with the largest counter is selected**.

**Algorithm 1** Learning fuzzy rules I ($LFR_I$)

**for all** succedents **do**
  **if** membership degree is the largest **then**
    succCount $\leftarrow$ succCount $+ 1$
  **end if**
**end for**

**Algorithm 2** Learning fuzzy rules II ($LFR_{II}$)

**for all** succedents **do**
  **if** membership degree $> 0$ **then**
    succCount $\leftarrow$ succCount $+ 1$
  **end if**
**end for**

**Algorithm 3** Learning fuzzy rules III ($LFR_{III}$)

**for all** succedents **do**
  **if** membership degree $> 0$ **then**
    succCount $\leftarrow$ succCount $+$ membership degree
  **end if**
**end for**

**Example.**
The rule in its basic form, i.e. before learning:
IF (NO IS Low) AND ($NO_2$ IS Higher Than Acceptable) THEN Valve opening angle IS High
Rules after learning Algorithm 1, before choosing the best rule:
IF (NO IS Low) AND ($NO_2$ IS Higher Than Acceptable) THEN Valve opening angle IS Low - succCount $= 0$
IF (NO IS Low) AND ($NO_2$ IS Higher Than Acceptable) THEN Valve opening angle IS Medium - succCount $= 0$
IF (NO IS Low) AND ($NO_2$ IS Higher Than Acceptable) THEN Valve opening angle IS High - succCount $= 56$
IF (NO IS Low) AND ($NO_2$ IS Higher Than Acceptable) THEN Valve opening angle IS Very High - succCount $= 96$
Hence, the best rule, chosen via succCount=96, is:
**IF (NO IS Low) AND ($NO_2$ IS Higher Than Acceptable) THEN Valve opening angle IS Very High**

The results obtained by the designed FLSs are presented in Table 4 and discussed in Section 5.

## 4 Type-2 fuzzy logic systems managing data in the SCR process

Promising results in managing parameters of the SCR process obtained in our previous experiments, cf. [15] and Section 3, based mostly on traditional and on interval-valued fuzzy sets (aka interval type-2 fuzzy sets), are the main premise to consider general type-2 fuzzy sets and type-2 fuzzy logic systems to be applied. This idea is mainly inspired by many successful applications of type-2 fuzzy logic systems [27, 28, 29, 30, 31, 32] supported via pioneer descriptions and explanations of building T2FLSs by Mendel et al. [22, 33, 34, 35, 36]. Our first attempt to apply higher order fuzzy logic systems in managing the SCR process is presented in [15], however from the current point of view, it was limited to interval type-2 fuzzy set only; here, we present T2FLSs based on general (mostly triangular type-2 fuzzy sets) and the results achieved are promising.

A general type-2 fuzzy set $\widetilde{A}$ in $\mathcal{X}$ is defined as a set of pairs: $\widetilde{A} = \{\langle x, \mu_{\widetilde{A}}(x)\rangle : x \in \mathcal{X}$ where $\mu_{\widetilde{A}}(x) : \mathcal{X} \to \mathcal{FS}([0,1])$ is a membership function of a type-2 fuzzy set and $\mathcal{FS}([0,1])$ is a set of all fuzzy sets in $[0,1]$. Type-2 fuzzy sets express membership degrees as fuzzy sets in $[0,1]$ so primary and secondary membership functions are distinguished. Especially, in secondary membership functions and secondary membership degrees, we see an opportunity to represent imprecise knowledge (given by experts) more adequately than in case of traditional fuzzy sets. In particular, we intend to represent levels of expertise of experts, the so-called "confidence levels", with secondary membership degrees to type-2 fuzzy sets on input and output of designed systems, see Figure 5 and 6.

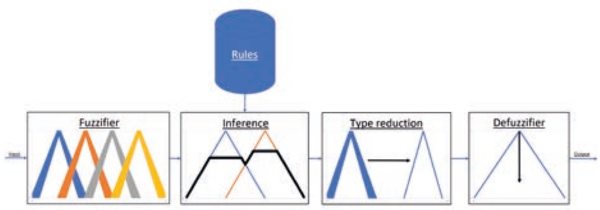


**Figure 5**. The general schema of a type-2 fuzzy logic system

### 4.1 Representing expert knowledge in type-2 fuzzy logic system

Usually, knowledge acquired from an expert may contain errors, subjectivity, or another drawbacks affecting performance of projected systems. Because of that, frequently, knowledge and initial data for a system are collected from more than one expert, to eliminate subjective assessments, different experiences, etc. Obviously, representing these data, usually different from different experts, is a crucial point in designing systems. Moreover, aggregating data may and should take into account different levels of expertise of experts providing their knowledge.

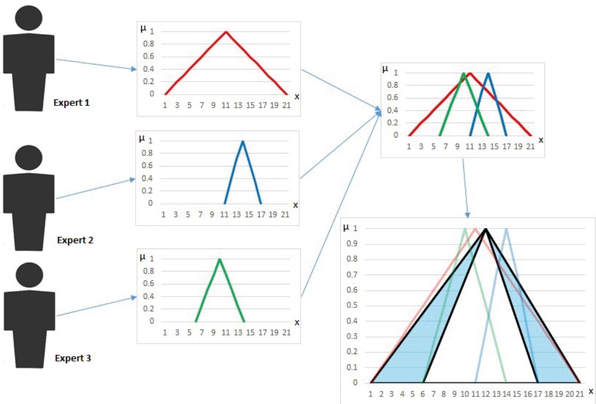


**Figure 6**. Constructing type-2 fuzzy sets based on knowledge of a few experts

Type-2 fuzzy logic systems, unlike traditional FLSs (in which information from many sources must be averaged before applying, e.g. in a rule base), make it possible to represent and/or aggregate experts proposals without loss of important (though seeming secondary) information that may positively affect final results (system outputs). Usually, in type-2 fuzzy logic systems, aggregating data from different experts is done with primary and secondary membership degrees taken into account, e.g. as values and their weights. A symbolic schema of such an aggregation is given in Figure 6. Secondary membership functions of type-2 fuzzy sets on inputs and outputs make it possible to express confidence levels of experts, or, to be more precise, "level of confidence to expert knowledge" with real numbers in $[0,1]$. In practice, it is made by taking into account the linguistic values proposed by experts as separate primary membership functions and assigning to each of them secondary membership degrees. For the systems designed in this experiment, the level of confidence is related to his/her seniority/experience. We use the following weights to describe experts' seniority: Expert 1: $w_1 = 3$, Expert 2: $w_2 = 20$, Expert 3: $w_3 = 13$, related to years they work for the company. Primary and secondary membership functions are assumed to be triangular. The expert knowledge on the SCR process is represented in terms of fuzzy sets and aggregated as follows:

1. Experts propose membership functions for the imprecise values considered in the system, i.e. for concentration of NO, $NO_2$. The labels are: (a) Low, (b) Medium, (c) High, (d) Higher than acceptable.

2. The proposals for each label are taken as primary membership functions of a type-2 fuzzy set representing that label and their membership degrees – as primary membership degrees.

3. Vertex of triangular membership functions for the resulting type-2 fuzzy set is evaluated (5)

$$x = \frac{\sum_{i=1}^{n} x_i w_i}{\sum_{i=1}^{n} w_i}, \tag{5}$$

   where $x$ is the coordinate of the vertex of primary membership function after aggregation, $x_i$ is the coordinate of the top of the fuzzy set of the $i$-th expert, and $w_i$ is the weight assigned to the $i$-th expert.

4. Secondary membership degrees of a resulting type-2 fuzzy set are evaluated as normalized experts' weights: $w_1 = 0.15$, $w_2 = 1.0$, $w_3 = 0.65$

Analogously to aggregating input data from experts, the output data must also be aggregated. In this case, for each sample of entry data on concentration of nitrogen oxides, the proper opening of ammonia dosing valve is proposed by each of experts. Next, the data are aggregated via (6)

$$y = \frac{\sum_{i=1}^{n} y_i w_i}{\sum_{i=1}^{n} w_i}, \tag{6}$$

where $y \in [0,100]\%$, and $w_i$ is the weight assigned to the $i$-th expert. Sample values of input and output and corresponding results of aggregation via (6) are collected in Table 3.

**Table 3**. Sample values of concentration of NO, $NO_2$ (col. 1., 2.), the ammonia valve opening given by experts (col. 3.-5.) and aggregated via (6) (col. 6.)

| Concentrat-ion NO [$mg/m^3$] | Concentrat-ion $NO_2$ [$mg/m^3$] | Expert 1 [% opening valve $NH_3$] | Expert 2 [% opening valve $NH_3$] | Expert 3 [% opening valve $NH_3$] | Aggregated data [% opening valve $NH_3$] |
|---|---|---|---|---|---|
| 57 | 128 | 30 | 33 | 32 | 32 |
| 35 | 86 | 27 | 30 | 31 | 30 |
| 287 | 13 | 52 | 57 | 56 | 56 |
| 176 | 110 | 49 | 55 | 55 | 55 |
| 398 | 80 | 100 | 100 | 100 | 100 |
| 62 | 250 | 93 | 59 | 57 | 61 |

## 4.2 Engineering type-2 fuzzy implications

In Section 3.1, the so-called engineereing applications are propsed for traditional FLSs. Now, we enhance them to be useful implication operators for inference using type-2 fuzzy rules. Let $\widetilde{A}, \widetilde{B}$ – type-2 fuzzy sets in $\mathcal{X}, \mathcal{Y}$, respectively

$$\mu_{\widetilde{A}}(x) = \int_{u \in J_x} \frac{f_x(u)}{u}, \quad \mu_{\widetilde{B}}(y) = \int_{v \in J_y} \frac{g_y(v)}{v}, \tag{7}$$

where $J_x, J_y \subseteq (0,1]$ are sets of all primary membership degrees to $\widetilde{A}$, $\widetilde{B}$, respectively. The general form of a type-2 fuzzy implication is

$$\mu_{\widetilde{A} \to \widetilde{B}}(x,y) = \int_{u \in J_x} \int_{v \in J_y} \frac{I_1(f_x(u), g_y(v))}{I_2(u,v)}, \tag{8}$$

where $I_1, I_2$ are fuzzy implications, e.g. min. Using engineering fuzzy implications (3) or (4) for $I_2$ and min for $I_1$, we obtain the following engineering type-2 fuzzy implications

$$\widetilde{I}_{K_1} : \mu_{\widetilde{A} \to \widetilde{B}}(x,y) = \int_{u \in J_x} \int_{v \in J_y} \frac{\min\{f_x(u), g_y(v)\}}{\left(\frac{\sqrt{uv}}{u+v-uv}\right)}, \tag{9}$$

$$\widetilde{I}_{K_2} : \mu_{\widetilde{A} \to \widetilde{B}}(x,y) = \int_{u \in J_x} \int_{v \in J_y} \frac{\min\{f_x(u), g_y(v)\}}{\left(\frac{uv}{u+v}\right)}. \tag{10}$$

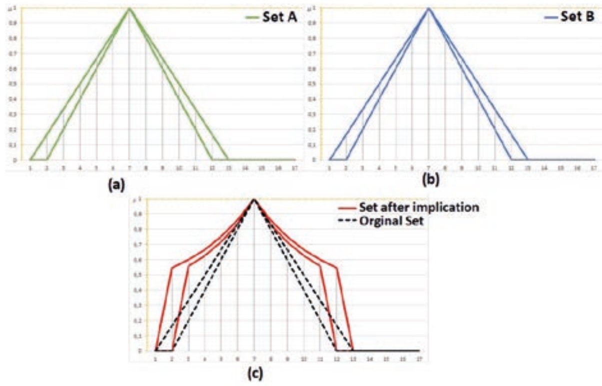


**Figure 7**. A result of type-2 fuzzy implication $\widetilde{I}_{K_1}$ (c) for sample type-2 fuzzy sets $\widetilde{A}, \widetilde{B}$ with triangular lower and upper membership functions (a), (b)

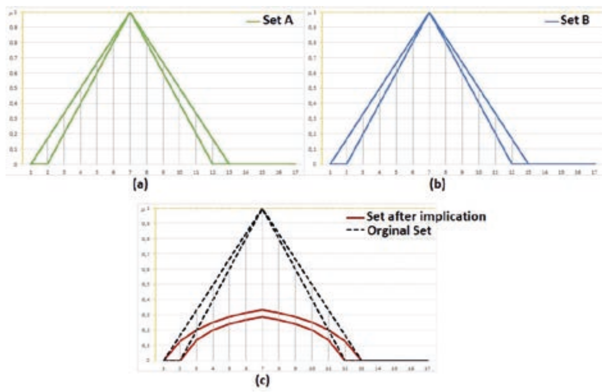


**Figure 8**. A result of type-2 fuzzy implication $\widetilde{I}_{K_2}$ (c) for sample type-2 fuzzy sets $\widetilde{A}, \widetilde{B}$ with triangular lower and upper membership functions (a), (b)

The proposed engineering implications are used in the inference block of the designed type-2 fuzzy logic system, the schema is illustrated in Figure 5. Figures 7 and 8 illustrate sample type-2 fuzzy sets as inputs and outputs of implications (9) and (10), respectively.

### 4.3 Algorithms for learning type-2 fuzzy rules

Fuzzy rules for Type-2 Fuzzy Logic Systems are usually determined on the base of expert knowledge and their experience. This does not guarantee optimal solutions, and this is the main reason for enhancing the proposed system with learning fuzzy rules algorithms newly designed here with respect to T2FLS requirements: learning type-2 fuzzy rules algorithms, LT2FR. Besides, they extend Algorithms 1.-3 shown for traditional FLSs in Section 3.2.

The results obtained with type-2 fuzzy logic systems with learning rules algorithms 4.-6. are presented in Table 5 and commented in Section 5.2.

**Algorithm 4** Learning type-2 fuzzy rules I ($LT2FR_I$)

**for all** succedents **do**
  **if** defuzzified primary membership degree is the largest **then**
    succCount $\leftarrow$ succCount $+ 1$
  **end if**
**end for**

**Algorithm 5** Learning type-2 fuzzy rules II ($LT2FR_{II}$)

**for all** succedents **do**
  **if** defuzzified primary membership degree $> 0$ **then**
    succCount $\leftarrow$ succCount $+ 1$
  **end if**
**end for**

**Algorithm 6** Learning type-2 fuzzy rules III ($LT2FR_{III}$)

**for all** succedents **do**
  **if** defuzzified primary membership degree $> 0$ **then**
    succCount $\leftarrow$ succCount $+$ defuzzified primary membership degree
  **end if**
**end for**

### 4.4 Type-reduction and defuzzification

The type-reduction is the process of representing a type-2 fuzzy set by an adequate type-1 fuzzy set. In this research, we use the Centroid Method: via the Extension Principle, the centroid of a type-2 fuzzy set $\widetilde{B}$ in a finite $\mathcal{Y}=\{y_1, y_2, \dots, y_M\}$, $M \in \mathbb{M}$, $\mu_{\widetilde{B}}(y_i) = \int_{u \in J_{y_i}} f_{y_i}(u)/u$ where $J_{y_i} \subseteq [0,1]$, $i = 1, 2, \dots, M$, is a set of all primary membership of $y_i$ to $\widetilde{B}$, is given as

$$C(\widetilde{B}) = \int_{u_1 \in J_{y_1}} \dots \int_{u_M \in J_{y_M}} \frac{f_{y_1}(u_1) * \dots * f_{y_M}(u_M)}{\left( \frac{\sum_{i=1}^{M} y_i u_i}{\sum_{i=1}^{M} u_i} \right)}, \quad (11)$$

where $u_i \in J_{y_i}$, and $*$ is a $T$-norm.

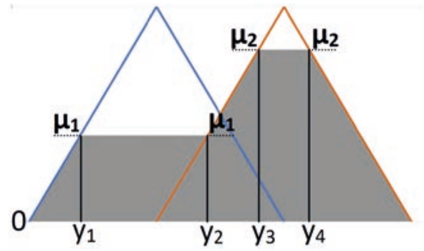


**Figure 9**. Graphical demonstration of the height method for defuzzification; $\mu_i$ is an activation value of a given fuzzy rule, and $y_i$'s are representative values for each of fuzzy sets in output.

The defuzzification for all simulations is done using the Height Method (12), that uses heights of each input fuzzy sets that create output fuzzy sets as antecedents of fuzzy rules (after type-reduction). The heights of fuzzy sets are $\mu_{Ci*}$ (taken as weights) and $y_i$ are representative points, see [37].

$$y^* = \frac{\sum_{i=1}^{M} y_i \mu_{Ci*}}{\sum_{i=1}^{M} \mu_{Ci*}}, \quad (12)$$

where $y^*$ is the value of the fuzzy output, $\mu_{Ci*}$ is the value of the activation of $i$-th fuzzy rule, $y_i$ is the element of $\mathcal{Y}$ representative for $i$-th output fuzzy set and $M$ is the number off all fuzzy sets affecting the output, compare Figure 9.

## 5 Comparative study and discussion

The elaborated fuzzy logic systems and type-2 fuzzy logic systems are now tested via the simulation analysis and comparison. Each system produce sets of output values and these sets are compared to expert opinions. The general criterion is: the more similar proposals computed by a system to expert knowledge, the better the system.

### 5.1 Data sets and methods of comparison

All dataset used in the experiment are taken from the production installation logs of one of the biggest industrial locations in Poland, see [38]. The number of samples in each dataset is $100,000$ and it corresponds to about 56 hours of continuous work of the installation, since samples are read from sensors every 2s; this time interval is related to capabilities of the solenoid valves used to dispense ammonia, the inertia of the system and some legal regulations on emission of nitrogen oxides (the maximum permitted value is 400mg/m$^3$). Moreover, different datasets are selected from periods with different weather parameters. Finally, 6 data sets, $|X_1| = |X_2| = |X_3| = 10000$ and $|X_4| = |X_5| = |X_6| = 100000$ samples each, $X_i = \{x_1, x_2, \ldots, x_n\}$, where $x_j = (\bar{x}_1, \bar{x}_2) \in \mathcal{X}_{\text{NO}} \times \mathcal{X}_{\text{NO}_2}$, $i = 1, 2, \ldots, 6$, $j = 1, 2, \ldots, n$, $n = 10000$ for $i = 1, 2, 3$ or $n = 100000$ for $i = 4, 5, 6$. $x_j$ is pair of values of concentration of NO and $NO_2$ (read from sensors of the DeNOx system) expressed with integers in $[0, 400]$ mg/m$^3$.

The results calculated by each of tested variant of a designed fuzzy logic system are compared with the data proposed by experts. For each set of samples two vectors are created: $E$ – vector containing the output values proposed by the experts, $C$ – vector containing the output values computed by a fuzzy logic system. Both vectors of the same length, and they are compared using three different methods: minimum-maximum (min-max), Pearson Correlation Coefficient (PCC), and Mean Absolute Percent Error (MAPE).

$$\text{min-max}(E, C) = \frac{\sum_{i=1}^{n} \min\{e_i, c_i\}}{\sum_{i=1}^{n} \max\{e_i, c_i\}}, \qquad (13)$$

where $E = \{e_1, e_2, ..., e_n\}$, $C = \{c_1, c_2, ..., c_n\}$, $c_i$ is the value calculated by the fuzzy system, and $e_i$ is the value forecasted by the human expert, for the same index $i$. The values of the min-max$(E, C) \in [0, 1]$ method show the similarity of vectors $E$ and $C$. The maximum value of the min-max$(E, C)$ method is 1 – meaning the vectors are identical.

The next similarity measure is Pearson Correlation Coefficient, *PCC*

$$PCC(E, C) = \frac{\sum_{i=1}^{n} (e_i - \bar{e})(c_i - \bar{c})}{\sqrt{\sum_{i=1}^{n} (e_i - \bar{e})^2} \sqrt{\sum_{i=1}^{n} (c_i - \bar{c})^2}}, \qquad (14)$$

where $PCC(E, C) \in [-1, 1]$, $\bar{e} = \frac{1}{n} \sum_{i=1}^{n} e_i$, $\bar{c} = \frac{1}{n} \sum_{i=1}^{n} c_i$. Value $-1$ means total negative correlation between $E$ and $C$, 0 means no correlation, and 1 means total positive correlation.

The third measure is Mean Absolute Percentage Error (MAPE)

$$MAPE(E, C) = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{c_i - e_i}{c_i} \right|, \qquad (15)$$

and its value is expressed in $[\%]$ – the smaller percentage, the larger similarity.

### 5.2 Results and discussion

The simulation analysis of the most efficient type-1 fuzzy logic systems is now reminded, cf. [15, 16]. Input and output data are represented by traditional fuzzy sets with triangular membership functions. For each of two inputs and for the output, 4 fuzzy sets representing linguistic labels are defined. The results for traditional FLSs with different implications and learning fuzzy rules algorithms (see Section 3) are collected in Table 4. Row. 1 contains results produced by a traditioinal fuzzy logic system with $T$-norms as implication operators. It is clearly visible that using new engineering implications and/or new methods of learning fuzzy rules affects the final results, see rows 2., 3. (for engineering applications) and rows 4.-6. (for learning fuzzy rules algorithms), as similarity increases and MAPE decreases meaningfully. The results for the system described in row 7. are the most similar to expert expectations (FLS with engineering implication (3) and learning fuzzy rules via Algorithm 1).

Nevertheless, the main goal of the research is to design an test type-2 fuzzy logic systems. Details of their structure are given in Section 4. The created type-2 fuzzy logic systems are simulated in a few different variants. Type-2 fuzzy sets with triangular primary membership functions are used to represent input and output data. The basic variant of the system, based on triangular norms as implication operators, is then enhanced with engineering implications (9), (10) and/or learning type-2 fuzzy rules methods (Algorithms 4.-6.). Analogously to the values obtained via FLSs (see Table 4), the results of experiments are recorded as vectors $E$ and $C$ separately for each tested system, and compared to each other using similarity measures (13)-(15), see Table 5. The results of simulation for 7 chosen variants of type-2 fuzzy logic systems are collected in rows 3.-9. Besides, they are compared to previously applied traditional fuzzy logic systems:

**Table 4**. Values of min-max, PCC and MAPE for output given by T1FLSs and by experts

| | min-max | **PCC** | **MAPE** | **Description** |
|---|---|---|---|---|
| 1. | 0.925 | 0.910 | 19.35% | Traditional FLS based on $T$-norm min |
| 2. | 0.930 | 0.912 | 10.05% | FLS with engineering implication (3) |
| 3. | 0.930 | 0.912 | 10.18% | FLS with engineering implication (4) |
| 4. | 0.960 | 0.959 | 9.83% | FLS with learning fuzzy rules: Algorithm 1 |
| 5. | 0.951 | 0.945 | 9.95% | FLS with learning fuzzy rules: Algorithm 2 |
| 6. | 0.955 | 0.955 | 9.87% | FLS with learning fuzzy rules: Algorithm 3 |
| **7.** | **0.962** | **0.962** | **9.64%** | **FLS with engineering implication (3) and learning fuzzy rules via Algorithm 1** |

typical one based on $T$-norms (row 1.) and the best one with new methods applied (row 2.). The most efficient type-2 fuzzy logic system is based on the proposed engineering type-2 implication (9) and learning type-2 fuzzy rules Algorithm 4. It is visible that the use of type-2 fuzzy logic systems with the proposed algorithms for learning type-2 fuzzy rules applied improve visibly the results in comparison to T2FLSs without learning rules (rows 4., 5.), and, moreover, to previously applied traditional fuzzy logic systems (rows 1., 2.).

The similarity results presented in Table 5 can also be translated into absolute numbers expressing amount of nitrogen oxides, the emission of which is reduced thanks to applying type-2 fuzzy logic systems with new methods instead of traditional fuzzy logic. Although the results listed in Table 5 differ only slightly, one must take into account that these differences are related to huge amounts of NO and $NO_2$ emitted to the atmosphere. The year production of 8 100Mg is declared in [38], hence the accuracy of ammonia dosing (via the min-max method) improved with respect to expert expectations from 0.925 (by traditional FLS, row 1.) to 0.970 (by T2FLS in row 9.), means that the difference is 0.045, which is the equivalent of 365Mg less nitrogen oxides emitted to the atmosphere per year. Or in other words, it is $\sim$ 30Mg of nitrogen oxides per month, which is the equivalent of $\sim$ 80mln $m^3$ more air totally free of nitrogen oxides per month.

## 6 Conclusions

The new solutions in the field presented in this article and, especially, the results of analysis confirm that it is promising to develop applications of type-2 fuzzy logic that enable efficient management of gas emission in the filtration system DeNOx. The most important is that the type-2 fuzzy logic systems, especially these using the proposed engineering fuzzy implications and learning fuzzy rules algorithms (see Section 4) allow us to increase consistency of obtained results with experts' expectations, in comparison to results of type-1 fuzzy logic systems applied previously [14, 15, 16, 17]. This is because possible defining ”confidence levels” for experts as secondary membership degrees in terms of type-2 fuzzy sets. It should also be emphasized that the implementation of the proposed solutions meets the requirements of real-time systems and it can be applied continuously to analyse measurements taken every 2s. The average time necessary to calculate the degree of valve opening by the fuzzy logic system is 2ms and the same result is achieved for type-2 fuzzy logic systems.

All the research presented allow us to claim that the DeNOx filter system can be efficiently supported by the proposed fuzzy logic systems as well as by the proposed type-2 fuzzy logic systems. The important conclusion is that T2FLSs do it more precisely than traditional FLSs, mostly because of more thorough representation of imprecise information, that leads to better compatibility of results with expectations of experts. In future, further research on applications of higher order fuzzy logic systems in the computer management of industrial gases emission are worth continuing and may point at new opportunities and challenges in this field.

## References

[1] M. Agarwal and S. Goel, Expert system and it's requirement engineering process, in Recent Advances and Innovations in Engineering (ICRAIE), 2014, pp. 1–4, May 2014.

**Table 5**. Values of min-max, PCC and MAPE for output given by T1FLSs, T2FLSs and by experts

| | min-max | **PCC** | **MAPE** | **Description** |
|---|---|---|---|---|
| 1. | 0.925 | 0.910 | 19.35% | Traditional FLS based on $T$-norm min |
| 2. | 0.962 | 0.962 | 9.64% | FLS with engineering implication (3) and learning fuzzy rules Algorithm 1 |
| 3. | 0.936 | 0.935 | 16.38% | T2FLS based on $T$-norm min |
| 4. | 0.949 | 0.953 | 9.34% | T2FLS with engineering T2 implication (9) |
| 5. | 0.948 | 0.949 | 9.44% | T2FLS with engineering T2 implication (10) |
| 6. | 0.957 | 0.960 | 8.49% | T2FLS with learning T2 fuzzy rules Algorithm 4 |
| 7. | 0.954 | 0.955 | 9.16% | T2FLS with learning T2 fuzzy rules Algorithm 5 |
| 8. | 0.955 | 0.958 | 8.56% | T2FLS with learning T2 fuzzy rules Algorithm 6 |
| **9.** | **0.970** | **0.970** | **8.01%** | **T2FLS with engineering T2 implication (9) and learning T2 fuzzy rules Algorithm 4** |


[2] K. Zalis, Application of expert systems in diagnostics of high voltage insulating systems, in Solid Dielectrics, 2004. ICSD 2004. Proceedings of the 2004 IEEE International Conference on, vol. 2, pp. 691–694 Vol.2, July 2004.

[3] K. Bilal and S. Mohsin, Mu/hadith: A Cloud Based Distributed Expert System for Classification of Ahadith, in Frontiers of Information Technology (FIT), 2012 10th International Conference on, pp. 73–78, Dec 2012.

[4] J. T. Starczewski, General type-2 FLS with uncertainty generated by fuzzy rough sets, in Fuzzy Systems (FUZZ), 2010 IEEE International Conference on, pp. 1–6, July 2010.

[5] R. K. Nowicki, B. A. Nowak, J. T. Starczewski, and K. Cpałka, The learning of neuro-fuzzy approximator with fuzzy rough sets in case of missing features, in Neural Networks (IJCNN), 2014 International Joint Conference on, pp. 3759–3766, July 2014.

[6] J. T. Starczewski, A Triangular Type-2 Fuzzy Logic System, in Fuzzy Systems, 2006 IEEE International Conference on, pp. 1460–1467, 2006.

[7] N. C. Long and P. Meesad, Meta-heuristic algorithms applied to the optimization of type-1 and type 2 TSK fuzzy logic systems for sea water level prediction, in Computational Intelligence Applications (IWCIA), 2013 IEEE Sixth International Workshop on, pp. 69–74, July 2013.

[8] M. Kinoshita, T. Fukuzaki, T. Satoh, and M. Miyake, An automatic operation method for control rods in BWR plants, Meeting on In-Core Instrumentation and Reactor Core Assessment, Cadarache, France, 1988.

[9] J. A. Bernard, Use of rule-based system for process control, IEEE Contr. Sys. Mag, pp. 3–13, 1988.

[10] F. Fujitec, FLEX-8800 series elevator group control system, Fujitec Co., Ltd., Osaka, Japan, 1988.

[11] S. Yasunobu, S. Miyamoto, and H. Ihara, Fuzzy control for automatic train operation system, IFORS Int. Congress on Control in Transportation Systems, Baden-Baden, 1983.

[12] O. Itoh, K. Gotoh, T. Nakayama, and S. Takamizawa, Application of fuzzy control to activated sludge process, in Proc. 2nd IFSA Congress, Tokyo, Japan, pp. 282–285, July, 1987.

[13] O. Yagishita, O. Itoh, and M. Sugeno, Application of fuzzy reasoning to the water purification process, in Industrial Applications of Fuuy Control, M. Sugeno, Ed. Amsterdam: North-Holland, pp. 19–40, 1985.

[14] M. Kacprowicz and A. Niewiadomski, On Dedicated Fuzzy Logic Systems for Emission Control of Industrial Gases, in Trends in Logic XIII (A. Indrzejczak, J. Kaczmarek, and M. Zawidzki, eds.), pp. 113–130, 2014.

[15] A. Niewiadomski and M. Kacprowicz, Higher order fuzzy logic in controlling selective catalytic reduction systems, Bulletin of the Polish Academy of Sciences Technical Sciences, vol. 62, no. 4, pp. 743–750, 2014.

[16] M. Kacprowicz and A. Niewiadomski, Managing Data on Air Pollution Using Fuzzy Controller, in Computer Methods in Practice (A. Cader, M. Yatsymirskyy, and K. Przybyszewski, eds.), pp. 46–57, Exit Publishing House, Warsaw, Poland, 2012.

[17] K. Renkas, A. Niewiadomski, and M. Kacprowicz, Learning Rules for Hierarchical Fuzzy Logic Systems with Selective Fuzzy Controller Activation, Lecture Notes in Computer Science - Springer, vol. 9119, pp. 260–270, 2015.

[18] S. Prabhakar, M. Karthikeyan, K. Annamalai, and V. N. Banugopan, Control of emission characteristics by using Selective Catalytic Reduction (SCR) in D.I. diesel engine, IEEE Conference Publications, 2010.

[19] R. R. Yager and D. P. Filev, Fundamentals of modeling and fuzzy control (in Polish: Podstawy modelowania i sterowania rozmytego). WNT, Warsaw, 1995.

[20] J. Jantzen, Foundations of Fuzzy Control. John Wiley & Sons Ltd., England, 2007.

[21] D. Majumder and K. Dwijesh, Fuzzy logic and its application in technology and management. Narosa, 2007.

[22] J. M. Mendel, Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions. Prentice Hall, 2001.

[23] J. C. Fodor, Contrapositive symmetry of fuzzy implications, Fuzzy Sets and Systems, (1995).

[24] J. C. Fodor, On fuzzy implication, Fuzzy Sets and Systems, vol. 42, pp. 293–300, 1991.

[25] O. Cordon, F. Herrera, and P. Villar, Generating the knowledge base of a fuzzy rule-based system by the genetic learning of the data base, Fuzzy Systems, IEEE Transactions on, vol. 9, no. 4, pp. 667–674, 2001.

[26] R. Hammell and T. Sudkamp, Learning Fuzzy Rules From Data, in The Application of Information Technologies (Computer Science) to Mission Systems, pp. 1–10, 1998.

[27] K. Mittal, A. Jain, K. S. Vaisla, O. Castillo, and J. Kacprzyk, A comprehensive review on type-2 fuzzy logic applications: Past, present and future, Engineering Applications of Artificial Intelligence, vol. 95, 2020.

[28] S. Türk, M. Deveci, E. Özcan, F. Canıtez, and R. John, Interval type-2 fuzzy sets improved by Simulated Annealing for locating the electric charging stations, Information Sciences, vol. 547, pp. 641–666, 2021.

[29] E. Ontiveros, P. Melin, and O. Castillo, Comparative study of interval Type-2 and general Type-2 fuzzy systems in medical diagnosis, Information Sciences, vol. 525, pp. 37–53, 2020.

[30] R. Jafelice and W. Lodwick, Interval analysis of the HIV dynamics model solution using type-2 fuzzy sets, Mathematics and Computers in Simulation, vol. 180, pp. 306–327, 2021.

[31] Z. Ashraf, M. L. Roy, P. K. Muhuri, and Q. Danish Lohani, Interval type-2 fuzzy logic system based similarity evaluation for image steganography, Heliyon, vol. 6, no. 5, p. e03771, 2020.

[32] E. Ontiveros-Robles and P. Melin, A hybrid design of shadowed type-2 fuzzy inference systems applied in diagnosis problems, Engineering Applications of Artificial Intelligence, vol. 86, pp. 43–55, 2019.

[33] J. McCulloch and C. Wagner, On the choice of similarity measures for type-2 fuzzy sets, Information Sciences, vol. 510, pp. 135–154, 2020.

[34] Q. Liang and J. M. Mendel, Interval Type-2 Fuzzy Logic Systems: Theory and Design, IEEE Transactions on Fuzzy Systems, vol. 8, pp. 535–550, (2000).

[35] L. Wang and J. M. Mendel, Generating Fuzzy Rules by Learning from Examples, IEEE Transactions on Fuzzy Systems, vol. 22, pp. 1414–1427, 1992.

[36] D. Wu and J. M. Mendel, Recommendations on designing practical interval type-2 fuzzy systems, CoRR, vol. abs/1907.01697, 2019.

[37] H. H. Li and M. M. Gupta, Fuzzy Logic and Intelligent Systems. Springer, 2007.

[38] PKN ORLEN, Annual Report 2012 PKN Orlen, tech. rep., PKN ORLEN, 2012.

**Adam Niewiadomski** received M.Sc. degree in computer sciencefrom Lodz University of Technology, Lodz, Poland, in 1998, and the next two grades, Ph.D. and D.Sc. in computer science – from Polish Academy of Science, Warsaw, Poland, in 2001 and 2009, respectively. He was nominated professor in 2020 in the domain of engineering and technical sciences. Currently, he is a professor with Institute of Information Technology, Lodz University of Technology, Lodz, Poland. He published two monographies and, as author or co-author, over 100 papers. He promoted 5 Ph.Ds. His research interests include fuzzy and expert systems, soft computing methods, representations of uncertain information, automated theorem proving, and e-learning.

**Marcin Kacprowicz** after received M.Sc. degree in computer science at Lodz University of Technology, Lodz, Poland, in 2006 he started working in industry. He received Ph.D. in 2016 also at the Lodz University of Technology, Lodz, Poland. Currently is an assistant professor at the Institute of Information Technology, Lodz University of Technology, Lodz, Poland, and connects his connects with work for industry. The scope of research interests includes the widely understood artificial intelligence, in particular the application of fuzzy logic.