Proceedings of the 2nd EAA International
Symposium on Hydroacoustics
24-27 May 1999, Gdańsk-Jurata
POLAND

Horizontal Refraction and Acoustic Signals Fluctuations Caused by Internal Waves Packets in Shallow Water [1]

B. Katsnelson, S. Pereselkov

Voronezh State University, Universitetskaya sq.1, Voronezh 394593, RUSSIA

e-mail: katz@mph.vsu.ru

The subject of the paper is to consider the propagation of low frequency sound field in shallow water in presence of soliton-like internal waves (SLIW). It is supposed that shallow water parameters are closed to ones of acoustic trace in Yellow Sea. We assume as well that SLIW spread across stationary acoustic trace. The problem of sound propagation is considered within the framework of 3-D model of the shallow water. The theory of "horizontal rays and vertical modes" is applied to take into account influence of horizontal refraction due to SLIW. As shown in the paper SLIW lead periodically to "focusing" and "defocusing" of the horizontal rays. The estimations of the sound fluctuations due to SLIW demonstrates that value of fluctuations can reach up to 10-20 dB (between moments of the "focusing" and "defocusing"). The results presented in the paper can be used as basis of remote sensing of the internal waves packets in shallow water.

1. Introduction

Soliton-like internal waves (SLIW) are rather widespread phenomenon in shelf zone of ocean. SLIW cause a significant vertical displacements of termocline layer in water column. The amplitudes of the vertical displacement are $\sim 5-10\,m$ on rather small distance in horizontal direction ($\sim 100-400\,m$). As result SLIW lead to sizeable variability of acoustic features of the shallow water environment in both time and space domains. So sound propagation in shallow water with SLIW is differed strongly from one in shallow water without SLIW.

At last time, SLIW influence on acoustic propagation in a shallow water has been researched intensively by both theoretical and experimental investigators [1-4].

In the Ref.[1] it is shown that resonant mode-coupling between the lower-order acoustic modes and higher-order modes caused by SLIW could be reason of significant loss of sound intensity on acoustic trace in Yellow Sea. Authors of Ref. [2] examined sound fluctuations induced by series of SLIW in shallow water trace on Washington Shelf. In the Ref. [3] it is obtained by numerical simulation that SLIW induced low-frequency sound fluctuations strongly depend upon source/receiver depths, transmission distance and SLIW propagation direction. Authors of Ref. [4] use the numerical simulation of sound propagation in presence of SLIW to interpret the experimental data which obtained in Japan Sea.

But it should be pointed that numerical simulations of sound propagation in Ref.[1-4] are carried out within framework of the 2-D model of shallow water environment. As known the 2-D treatment assumes that horizontal refraction and

[1] This work was supported by RFBR, grant 97-05-64878

azimutthal scattering of sound are negligible. But these acoustic effects can be sizable at some conditions (for example, if SLIW front and acoustic trace are parallel). So it is obvious that 3-D model of shallow water would be desirable to complete the researches of the SLIW influence on sound propagation.

In given work we consider propagation of low-frequency acoustic signals in shallow water in presence of SLIW which cross stationary acoustic track. This problem is considered sound within framework of 3-D model of the shallow water environment. To take into account influence of horizontal rays refraction due to SLIW we apply theory of "horizontal rays and vertical modes". We show that SLIW cause in a different time moments a "focusing" and "defocusing" of horizontal rays. One of paper goals is the estimation of the sound fluctuations on acoustics trace ($\sim 10\,km$) between time moments of the "focusing" and "defocusing"

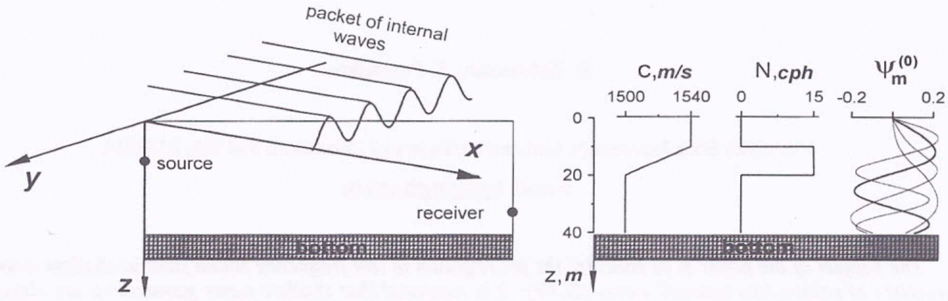

Fig. 1. Shallow water model.

2. Shallow Water Model.

Let us consider sound propagation within framework of the following 3-D model of shallow water (see Figure 1). We suppose that SLIW propagate in y-direction of Cartesian coordinate system, x-axis of the is directed parallel to wave front of SLIW, z-axis is directed downward.

We model the shallow water environment as water 3-D layer limited by free halfspace ($z<0$) and homogeneous absorbing bottom ($z>H$). The bottom density is ρ_b and sound speed is $c_b(1-i\alpha/2)$, where α is determined by bottom absorption. Unperturbed (without SLIW) sound speed profile $c(z)$ and buoyancy frequency $N(z)$ in water layer has rather thermocline shape corresponding to summer period.

The SLIW lead to sound speed profile perturbation $\delta c(y,z)$:

$$\delta c(y,z) = QN^2(z)\Phi(z)\zeta(y) \qquad (1)$$

where $Q = 2.4c^2/m$ - some constant, $\Phi(z)$ - the lowest mode of internal waves, corresponding to SLIW, $\zeta(y)$ - vertical displacements of thermocline layer due to SLIW.

According to experimental data Ref. [5-6] SLIW on ocean shelf are grouped in packets with general length about $\sim 3-5\,км$. Such packet contains about $6-8$ internal solitons. The width of each internal soliton varies in interval $200-400\,m$.

The front of SLIW in horizontal direction is approximately plane (radius of curvature is up to a few tens of km) and parallel to coast line. That is why we suppose that perturbation of sound speed $\delta c(y,z)$ in Eq. (1) does not depend upon x-coordinate (x-axis parallel to SLIW front).

Because of the IW-speed in y -direction is rather small ($0.5-1\,m/s$), the shift of packet during $10-20\,\sec$ (time of propagation of sound signal along acoustic trace) is about $10-20\,m$. This is much less than width of internal soliton. So we will solve the problem in frozen-time approximation. We will construct sound field of harmonic source (frequency is equal $f=\omega/2\pi$) which is placed at the point $\vec{r}_r=(x_r,y_r)$. Factor $\exp(-i\omega t)$ will be omitted in the following consideration, and t will be considered as a parameter.

3. Theoretical Background.

We will find sound field pressure at the point of receiver $\Psi(\vec{r},z)$ according to the theory of vertical modes and horizontal rays [7]. Within the framework of this theory sound field $\Psi(\vec{r},z)$ in receiver point has form

$$\Psi(\vec{r},z)=\sum_{m,l}A_{ml}(\vec{r})\psi_l(\vec{r};z)\exp[i\theta_{ml}(\vec{r})] \quad (2)$$

In Eq.(2) acoustic modes $\psi_l(\vec{r},z)$ satisfy to the following boundary-value problem:

$$\frac{\partial^2\psi_l(\vec{r};z)}{\partial z^2}+\left[k^2(\vec{r};z)-\xi_l^2(\vec{r})\right]\psi_l(\vec{r};z)=0,$$

$$\psi_l(\vec{r};z)|_{z=0}=0, \quad (3)$$

$$\left[\psi_l(\vec{r};z)+g(\xi_l(\vec{r}))\frac{\partial\psi_l(\vec{r};z)}{\partial z}\right]_{z=H}=0.$$

In the Eq. (3) function $g(\xi)$ is determined by bottom model. For example for liquid homogeneous absorbing bottom:

$$g(\xi_l)=\frac{\rho_b}{\rho(H)}\left(\xi^2{}_l-k^2n_b{}^2(1+i\alpha)\right)^{-1/2} \quad (4)$$

As it follows from Eq.(4) eigen value of the problem (3) are complex $\xi_l=q_l+i\frac{\gamma_l}{2}$. The imaginary part $\gamma_l/2$ of modal value ξ_l describes

The functions $\theta_{ml}(\vec{r})$ (eikonal) and $A_l(\vec{r})$ (amplitude) in Eq.(2) are determined by ray equations for horizontal plane ($\vec{r}=x,y$):

$$(\nabla_r\theta_{ml})^2=q_l^2, \quad (5)$$

$$2\nabla_rA_{ml}\nabla_r\theta_{ml}+A_{ml}\nabla_r^2\theta_{ml}+q_{ml}\gamma_{ml}A_{ml}=0. \quad (6)$$

where $\nabla_r=(\frac{\partial}{\partial x},\frac{\partial}{\partial y})$.

One can see, that real part $q_l(\vec{r})$ of modal value ξ_l. plays the role of refractive index for horizontal rays, which correspond to mode number l.

In our shallow water model the space dependence of modal value $q_l(\vec{r})$ is due to SLIW deforming the sound speed profile. SLIW cause a rather small corrections for sound speed profile. So space dependence of $q_l(\vec{r})$ can be presented as:

$$q_l(\vec{r})=q_l^0+\delta q_l(\vec{r}). \quad (7)$$

Value $\delta q_l(\vec{r})$ in Eq.(7) can be found by using the perturbation theory [8]:

$$\delta q_l(\vec{r})=\frac{1}{2q_l^0}\int_0^H\left[\psi_l^0(z)\right]^2\delta k^2(\vec{r},z)dz \quad (8)$$

Here $k^2(\vec{r},z)=k_0^2(z)+\delta k^2(\vec{r},z)$.

The value $\delta k^2(\vec{r},z)$ can be expressed through SLIW characteristics: $\zeta(\vec{r})$ - vertical displacement of thermocline:

$$\delta k^2(\vec{r},z)=2k_0^2QN^2(z)\Phi(z)\zeta(\vec{r}) \quad (9)$$

So the Eq.(8) and Eq.(9) allows to write the perturbation $\delta q_l(\vec{r})$ of modal value $q_l(\vec{r})$ by the following way:

$$\delta q_l(\vec{r})=\left\{\frac{Qk_0^2}{q_l^0}\int_0^H\left[\psi_l^0(z)\right]^2N^2(z)\Phi(z)dz\right\}\zeta(\vec{r}) \quad (10)$$

3. Analysis and Calculations.

Due to remarkable anisotropy in horizontal plane behavior SLIW can lead to substantial horizontal refraction and in turn to sizable fluctuations of sound field propagating approximately along wave front of SLIW.

According to 3-D model the acoustic features of shallow water environment depend upon y-coordinate only. So Eq. (10) leads to oscillation of δq_l as function of y:

$$\delta q_l(y)=\left\{\frac{Qk_0^2}{q_l^0}\int_0^H\left[\psi_l^0(z)\right]^2N^2(z)\Phi(z)dz\right\}\zeta(y) \quad (11)$$

On Fig.2 we present the results of numerical simulation of sound propagation within frame of the shallow water described above.

For these calculations we suppose that packet of SLIW has a sinusoidal form in both space and time:

$$\zeta(\vec{r},z)=\zeta(y)=\zeta_0\cos\{2\pi(y-v_st)/\Lambda\} \quad (12)$$

Here Λ is determined by width of internal soliton, v_s speed of SLIW propagation, ζ_0 - amplitude of vertical displacements of thermocline.

We assume that acoustic trace takes place along x-axis. So in each time moment the vertical displacements of thermocline are same on whole of acoustic trace.

To calculate trajectories of horizontal rays we use a numerical solution of Eq.(5). On the figures a), b), c) trajectories of horizontal rays closed to acoustic trace are shown for three different time moments t.

The figure a) corresponds to case when SLIW are absent. It means that $\zeta(y)=0$ and in turn $\delta q_l(y)=0$. In this case trajectories of horizontal rays are radial lines.

The figure b) corresponds to the situation, when source and receiver are placed at the maximum of vertical displacements. In this situation the horizontal waveguide is formed along acoustic

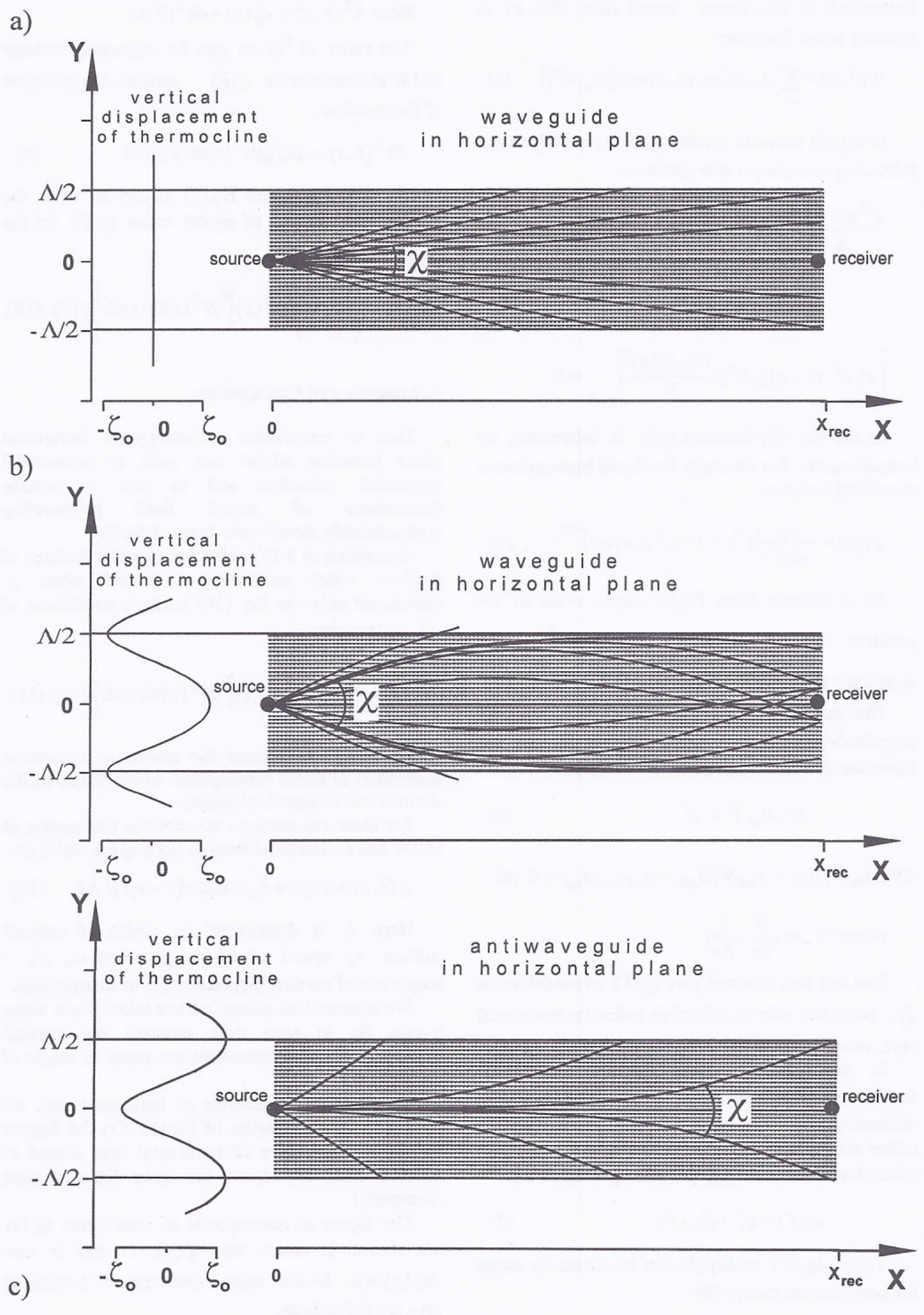

Fig.3 a) Cylindrical Spreading, b) Waveguide in horizontal plane, c) Antiwaveguide in horizontal plane.

trace. Part of horizontal rays is channelled by this waveguide in horizontal plane. This is case of the focusing of horizontal rays.

The figure c) corresponds to moment when source and receiver are placed at the minimum of vertical displacements. In contrast to case above in this moment the horizontal antiwaveguide is formed along acoustic trace. As result the horizontal rays are go away from acoustic trace more intensely than at cylindrical spreading. This is case of the defocusing of horizontal rays.

It means that generally speaking we can have different situations for horizontal rays in dependence on position of source with respect to wave front of internal soliton. More exactly because internal soliton moves at different moments of time source will be placed (in horizontal plane) as at the point of maximum of $\delta q_l(y)$ so at the point of minimum (and at other points). For the sound field at the point of observation (in accordance with this) we will have both the focused and defocused horizontal rays.

Lets us consider sound intensity of mode averaged on width of internal soliton. We suppose that source in horizontal plane is undirected and I_0 is intensity which radiated by source. We will discuss three cases corresponding to figures a), b), c).

Case I. Cylindrical Propagation. In this situation (see Fig. 2a) horizontal rays for which $|y| \le \Lambda/2$ at point of receiver ($x = x_{rec}$) have angles of sliding satisfing to:

$$|\chi| \le \frac{\Lambda}{2x_{rec}} \quad (13)$$

The total intensity (averaged on interval Λ) of these horizontal rays is determined by the following expression:

$$\bar{I}(x_{rec}) = \frac{1}{\Lambda}\frac{I_0}{2\pi}\frac{\Lambda}{x_{rec}} = \frac{I_0}{2\pi}\frac{1}{x_{rec}} \quad (14)$$

This intensity of acoustic mode decreases with range in according to known law $1/x_{rec}$.

Case II. Waveguide in Horizontal Plane. In this situation (see Fig. 2b)we assume that:

$$\frac{q_l(y)}{q_l^{\max}} \approx 1 - e_l|y| \quad (15)$$

where

$$e_l = \frac{\zeta_0 Q k_0^2}{\Lambda\left(q_l^0\right)^2}\int_0^H \left[\psi_l^0\right]^2 N^2(z)\Phi(z)dz \quad (16)$$

For this case horizontal rays with angles of sliding

$$|\chi| \le \sqrt{e_l \Lambda} \quad (17)$$

are channelled by horizontal waveguide. So whole intensity channelled by waveguide

$$\bar{I}(x_{rec}) = \frac{I_0}{\pi}\sqrt{e_l\Lambda} \quad (18)$$

One can see that averaged intensity does not depend upon range between source and receiver. (That is because we not take into account bottom absorption in this expression)

Case III. Antiwaveguide in Horizontal Plane. For this situation (see Fig. 2c) we suppose that:

$$\frac{q_l(y)}{q_l^{\max}} \approx 1 + e_l z \quad (18)$$

The horizontal ray passages through point with coordinate: $x = x_{rec}$ and $|y| \le \Lambda/2$ if angles of sliding satisfy to

$$|\chi| \le \frac{\Lambda}{x_{rec}} - 2e_l x_{rec}, \quad (19)$$

The total intensity (averaged on interval Λ) of these horizontal rays is determined by the following expression:

$$\bar{I}(x_{rec}) = \frac{I_0}{2\pi}\frac{1-(e_l/2\Lambda)x_{rec}^2}{x_{rec}} \quad (20)$$

The Eq.(20) is true for $x_{rec} \le \sqrt{\frac{2\Lambda}{e_l}}$.

One can see that averaged intensity of acoustic mode decreases with more strongly than in case of cylindrical spreading. In this situation the modal intensity is negligible on ranges $x_{rec} \approx \sqrt{\frac{2\Lambda}{e}}$.

4. Conclusion.

For typical parameters of internal soliton in shallow water the estimations of intensity (case II and case III) show that value of intensity fluctuations can reach up to $10-20\,\text{dB}$ (between moments of the focusing and defocusing).

Reference

1. Ji-xun Zhou, Xue-zhen Zhang : Resonant interaction of sound wave with internal solitons in the coastal zone *J. Acoust. Soc. Amer.* V.90(4), pp. 2042-2054, (1991).
2. Rubenstein D., Brill M.N. Acoustic variability due to internal waves and surface waves in shallow water // Ocean Variability and Acoustics Propagation ed. by J.Plotter and A.Warn-Varnas. Dordtecht: Kluwer Academic,. pp. 215-228 (1991)

3. Ji-Xun Zhou, Xue-Zhen Zhang, P.H. Rogers "Modal Characteristics of Acoustic Signal Fluctuations Induced by Shallow Water" IEEE Proc. of Conference "Oceans'96", Fort Lauderdale, Florida USA, V.I, pp.1-8. (1996)
4. S.V. Borisov, R.A. Korotchenko, A.N. Rutenko, M.U. Trofimov "Example of numerical simulation of internal waves influence on sound propagation in shallow water" // Acoustical Physics. V.42. №5. pp.702-705, (1996).
5. Konyaev K.V., Sabinin K.D. Waves inside ocean. St-Peterburg: Gidrometeoizdat, 1992, p.271.
6. Serebryanyi A.N. Manifestation of solitons features in internal waves on shelf. Izv. of AN "Physics of atmosphere and ocean" V..29 (2), pp.244-252, (1993).
7. Keller J.B. and Papadakis J.S (ed) , *Wave Propagation and Underwater Acoustics*. Springer-Verlag, Berlin-Heidelberg-New-York, 1979 , 227 pp.,
8. Katsnelson B.G. , Petnikov V.G. Shallow Water Acoustics - Moskow, Nauka, 1997, p.191