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THE ROLE OF ECONOMIC UNCERTAINTY ON THE BLOCK ECONOMIC VALUE – A NEW VALUATION APPROACH

ROLA CZYNNIKA NIEPEWNOŚCI PRZY OBLICZANIU WSKAŹNIKA RENTOWNOŚCI – NOWE PODEJŚCIE

The block economic value (*BEV*) is one of the most important parameters in mine evaluation. This parameter can affect significant factors such as mining sequence, final pit limit and net present value. Nowadays, the aim of open pit mine planning is to define optimum pit limits and an optimum life of mine production scheduling that maximizes the pit value under some technical and operational constraints. Therefore, it is necessary to calculate the block economic value at the first stage of the mine planning process, correctly. Unrealistic block economic value estimation may cause the mining project managers to make the wrong decision and thus may impose inexpiable losses to the project. The effective parameters such as metal price, operating cost, grade and so forth are always assumed certain in the conventional methods of *BEV* calculation. While, obviously, these parameters have uncertain nature. Therefore, usually, the conventional methods results are far from reality. In order to solve this problem, a new technique is used base on an invented binomial tree which is developed in this research. This method can calculate the *BEV* and project *NPV* under economic uncertainty.

In this paper, the *BEV* and project *NPV* were initially determined using Whittle formula based on certain economic parameters and a multivariate binomial tree based on the economic uncertainties such as the metal price and cost uncertainties. Finally the results were compared. It is concluded that applying the metal price and cost uncertainties causes the calculated block economic value and net present value to be more realistic than certain conditions.

Keywords: metal price uncertainty, operating cost uncertainty, binomial tree, block economic value, net present value

Wskaźnik rentowności jest jednym z najważniejszych parametrów przy ocenie ekonomicznej kopalni. Parametr może warunkować kolejne czynniki, takie jak kolejność prowadzenia wybierania, określenie limitu wybierania oraz wartość bieżąca netto. W chwili obecnej celem planowania działalności kopalni odkrywkowej jest zdefiniowanie optymalnych granic odkrywki i optymalnego cyklu produkcyjnego

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zakładu, tak by maksymalnie zwiększyć wartość odkrywki z uwzględnieniem ograniczeń narzuconych przez uwarunkowania techniczne i ruchowe. Dlatego też konieczne jest określenie wskaźnika rentowności już w początkowych etapach planowania działalności kopalni. Nierealistyczne wyliczenia rentowności prowadzić mogą do podejmowania złych decyzji, co pociągnie za sobą straty przedsiębiorstwa. Niektóre parametry, takie jak cena metali, koszty działania, zawartość metalu w rudzie uważane są za parametry pewne w tradycyjnych metodach określania rentowności, podczas gdy w rzeczywistości parametry te zawierają pewien element niepewności. Dlatego też częstokroć wyniki uzyskane przy użyciu metod tradycyjnych znacznie odbiegają od rzeczywistości. W celu rozwiązania tego problemu, opracowano nową metodę wykorzystującą drzewa dwumianowe. Metoda ta umożliwia obliczania wskaźnika rentowności i wartości bieżącej netto w warunkach niepewności.

W pracy określono najpierw wskaźnik rentowności i prognozowaną wartość bieżącą netto w oparciu o wzór Whittle'a, bazujący na pewnych parametrach ekonomicznych oraz przy użyciu diagramu wielu zmiennych z uwzględnieniem niepewności warunków ekonomicznych, takich jak ceny metali czy koszty wydobycia. Porównano wyniki uzyskane w oparciu o obydwie metody. Stwierdzono, że przyjęcie cen metali i kosztów wydobycia jako wielkości niepewnych przy obliczaniu wskaźników rentowności i wartości bieżącej netto daje bardziej realistyczne wyniki niż gdy parametry te przyjmowane są jako pewne.

Słowa kluczowe: niepewność ceny metalu, niepewności związana z wysokością kosztów wydobycia, drzewo dwumianowe, wskaźnik rentowności, wartość bieżąca netto

1. Introduction

The aim of open pit mine planning is to define optimum pit limits and an optimum life of mine production scheduling that maximizes the pit value under some technical and operational constraints. The basic input to this process is a set of block values representing the net economic worth of each block. Based on the estimated block economic values, the optimizer selects the optimum destination of each block so as to maximize the overall pit value under some given technical constraints. A dollar value is usually assigned to each block by estimating the revenue of recoverable metal at a given fixed metal price and subtracting applicable mining, processing and other costs. Many researchers such as Ataee-pour (2005), Whittle (1988, 1999) and so forth have worked on the block economic value equations. The Whittle *BEV* equation is shown below:

$$BEV = T_o GRP - T_0 C_p - TC_m \tag{1}$$

where:

- BEV — is block value, \$,
- T_o — is tonne of ore in the block,
- G — is grade, unit/tonne,
- R — is recovery,
- P — is unit price, \$/unit,
- C_p — is processing cost, \$/tonne,
- T — is total amount of rock (ore and waste) in the block,
- C_m — is mining cost, \$/tonne.

Block value estimations using current, common conventional procedures are based on three main implied assumptions:

1. The ore grade or metal content of each block is known with certainty.
2. Economic variables such as metal prices and operating costs are known with certainty.
3. The economic value of all blocks is calculated at the present time like a static parameter and it is assumed that there are no possible future revisions.

The aim of this paper is to consider the effect of economic parameters uncertainties on the block value estimation. Thus, the grade uncertainty is not considered here, although this uncertainty has a significant effect on the mine planning and BEV (Parhizkar et al., 2011). The second implied assumption in conventional open pit mine planning is that economic variables such as metal prices and operating costs are fixed. In other words, economic variables do not change throughout the life of mine and are known with certainty. Obviously, considering the uncertain nature of the mentioned parameters, this assumption is far from realistic. Looking at the history of metal markets and cost charts, it is not difficult to conclude the probability of metal prices and operating costs remaining unchanged is null in both the short and long-term. Many researchers such as Brennan and Schwartz (1985), Trigeorgis (1993), Moyen et al. (1996), Kelly (1998), Moel and Tufano (2002), Monkhouse and Yeates (2005), Abdel Sabour and Poulin (2006), Samis et al. (2006), Jaszczuk and Kania (2008), Meagher et al. (2009), Akbari et al (2009) and Dehghani and Ataee-pour (2011) have worked on the market uncertainty. But they never show the effect of metal price uncertainty on the BEV. Figure 1 shows the possible copper price paths over a period of three years (2010-2012). Each path represents a possible scenario for the future copper prices. According to this figure, five possible paths are available for changing the metal price from 2010 to 2012. Moreover, the metal price in the year 2012 may include three particular values.

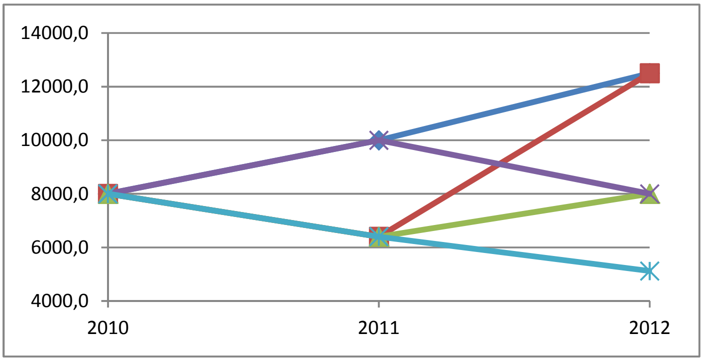

Fig. 1. A sample of possible copper price paths

There are many researchers such as Henry et al. (2004), Dimitrakopoulos and Abdel Sabour (2007) who focus on both geological and market uncertainty. But the operating costs are considered as a certain parameter in most of the previous research works. However, this parameter may be unpredictably changed by market variations, government policy changes, novel technology, management adjustments and so forth. Thereupon, in order to determine the real and correct block economic value, it is necessary to consider the operating costs uncertainty.

The third assumption indicates the block economic value is determined at the present time and has a static nature. Whiles, the block economic value is dynamic because the metal price and operating cost are dynamic over time. On the other hand, the block economic value has a close relationship with mining sequence determination. Therefore, value of each block must be determined according to their mining time and using a dynamic procedure.

In this paper, the *BEV* and project *NPV* were initially determined using Whittle formula based on certain economic parameters and a multivariate binomial tree based on the economic uncertainties such as the metal price and cost uncertainties. Finally the results were compared.

2. Multivariate binomial tree

The binomial model is a well-known alternative discrete time, which is developed by Cox et al. (1979). The method of binomial pricing trees is a flexible, powerful, and quite superb method. A binomial pricing tree is a structure that maps all possible trajectories of metal price through time as are allowed by the model. This structure consists of nodes and branches. Each node in a given layer, and therefore corresponds to a potential stock price at a particular point in time. Nodes are identified with traversal probabilities and option valuations, as well as with metal prices. Each branch or path in a binomial pricing tree represents a possible transition from one node to another node later in the tree and has a probability and a ratio associated with it. Branches to higher nodes reflect up probabilities (pr) and multipliers (u), while branches to lower nodes implement the down probabilities ($1 - pr$) and multipliers (d). A schematic binomial tree on the metal price at time zero (P_0) with three steps are shown in Figure 2. The up (u) and down (d) factors and the probability of occurrence were determined using the following formula:

$$u = e^{\sigma\sqrt{\delta t}} \tag{2}$$

$$d = e^{\sigma\sqrt{\delta t}} = \frac{1}{u} \tag{3}$$

$$pr = \frac{(1 + rf) - d}{u - d} \tag{4}$$

The basic inputs are the volatility of the metal price or operating cost (σ), the risk-free rate (rf), stepping time (δt).

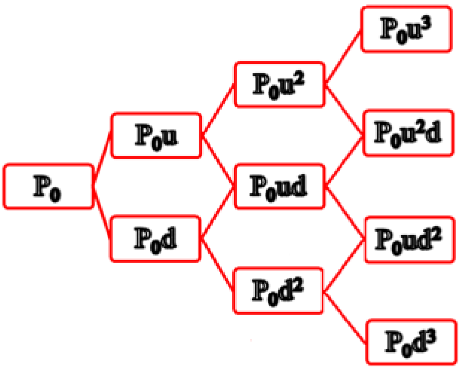

Fig. 2. Three time step binomial tree

As mentioned above, the binomial tree has a great ability to calculate and solve the problems with one uncertainty quantity. Due to the fact that the available uncertainties are more than one uncertainty in this research, i.e. price and cost uncertainties, the conventional binomial tree cannot model the uncertainties. Therefore, the researchers try to develop a suitable binomial tree which can survey two uncertainty sources simultaneously. For this reason, at first it is necessary to prepare a preliminary data analysis. Then the probability of occurrence of each uncertainty can be calculated considering the dependency between the main uncertainties to another. Figure 3 shows the developed binomial tree.

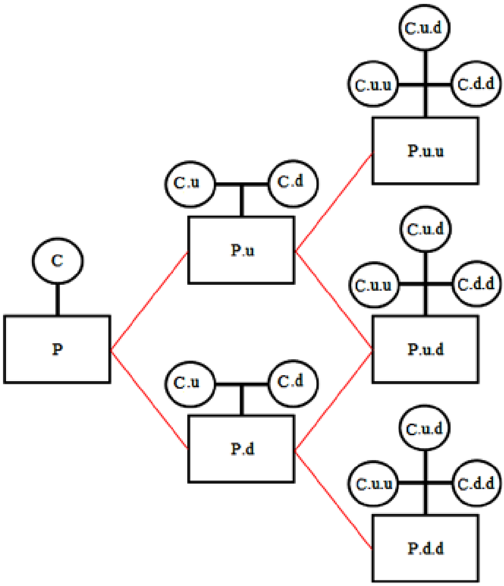

Fig. 3. The developed binomial tree

In figure 3, the squares and circles represent the main and secondary uncertainty quantities, respectively. For example, as the cost variation depends on price variation, metal price is the main uncertainty (P in square in Figure 3) and operating cost is the secondary uncertainty (C in circle in Figure 3). The metal price nodes are available in each period depending on increasing or decreasing the metal price. Each metal price node has a group of cost branches. This means the amount of cost can increase or decrease in each metal price node. In the developed binomial tree, the number of the metal price node and cost node in each cost branch is equal to the period number (n). But the entire number of cost nodes in each period is equal to the square of the period number (n^2). For example, in Figure 3, in the third period, the number of metal price nodes, cost branches and cost nodes are three, three and nine, respectively. In each metal price node, the amounts of corresponding costs are the same in cost branches. But the occurrence probability of each cost node is not the same. The occurrence probability is calculated by dividing the number of considered occurrence per total number of the occurrences. It is obvious the sum of the cost uncertainty probability is equal to one in each period.

3. Methodology

Equation 1 is used for determining the block economic value in the Whittle method. The requested parameters in this formula are metal price, operating cost, processing cost, block volume and total recovery. These parameters must be determined correctly. For this purpose, the engineers and planners use the current parameters for calculating the block economic value in the future. Finding the final pit limit is the main reason for this issue. After finding the value for each block, in order to maximize the net present value of extracting the blocks, the mining sequence is determined by attending to the technical constraint. The Roman (1974) presented method is used for finding the mining sequence.

After finding the mining sequence, the net present value for the mining limit will be calculated using equation 5. Having the project net present value, the managers can decide on the implementation or suspension of the project.

$$NPV = \sum_{n=1}^{N} \frac{BEV_n}{(1+i)^n} \tag{5}$$

where:

NPV — is the project net present value,
n — is the time period,
BEV — is the current block economic value,
i — is the discount rate.

The blocks net present value is calculated based on the current information, while these blocks will be extracted in future years. Considering the constant amount for metal price and operating cost is not correct because of their past variations. Consequently, the BEV calculation is not correct based on these constant parameters. For example, Figure 4 shows the copper price variation for the past ten years.

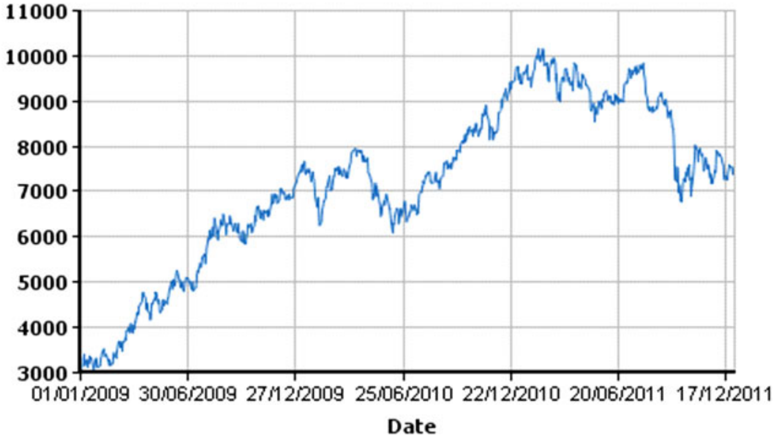

Fig. 4. Copper price chart from 2009-2011 (LME-2011)

In conventional method, if a pit was designed in June 2009, its blocks value and its net present value will be calculated based on the price of 5000 $ per tons, while the price in 2010 and 2011 changes to 6500 $ per tons and 9000 $ per tons, respectively. Thereupon, the results are very conservative and far from reality based on the constant price.

For eliminating the mentioned problems, a method must be applied which can calculate the metal price and operating cost uncertainties in future years. Then, this method determines the amounts of block economic value and project net present value, considering these economic uncertainties.

For this purpose, in the first step, the uncertainty of price and cost will be calculated using the developed binomial tree and then their effect will be determined on the block economic value and present net present value. In the proposed method, the probability of increasing or decreasing the amount of cost was determined in the case of increasing or decreasing the price using the price and cost historical data and a preliminary analysis. Also, the price and cost volatility was calculated using the historical data.

The amount of up and down for each uncertainty was calculated having the volatility and using equations 2 and 3. Therefore, the price and cost nodes were constructed in binomial tree. At the next step, the occurrence probability was determined using equation 4 for each metal price node. Finally the block economic value was calculated using the constructed binomial tree and the equation 6. For simplicity, it is assumed that the blocks are completely ore or waste.

$$BEV_n = T(G_n R_n P_{ni} - C_{nj}) \tag{6}$$

where

T — is block tonnage,
n — is the period number,
G — is the block grade,
R — is the recovery,
P_{ni} — is the i^{th} price node in the n^{th} period in metal price binomial tree,
C_{nj} — is the j^{th} cost node in the n^{th} period in total cost binomial tree.

If the value of each block was less than zero equation 6 will be changed as follows:

$$BEV_n = T.OC_{nj} \tag{7}$$

where: OC_{nj} is the j^{th} cost node in the n^{th} period in mining operating cost binomial tree

Therefore, there is more than one economic value for each block. This advantage helps the managers and planners to select a correct decision in the case of mutations. In the next step, the amount of probability of price-cost variation was multiplied by each block economic value. In the final step, discounted cash flow is calculated by assuming that the conventional mining sequence is correct, using equation 8.

$$DCF_n = BEV_n + \frac{pr.DCF_{n+1} + (1-pr).DCF_{n+1}}{(1+rf)} \tag{8}$$

where

pr — is price changes probability,
rf — is the risk free rate.

Considering the metal price and cost uncertainties and also offering various probable economic values for each block, it is obvious that the new method represents more real consequences than the conventional method. To clarify this point, a numerical example is represented in the next section.

4. Numerical example

In this section, the *BEV* and project *NPV* were determined using the Whittle equation and multivariate binomial tree method. For this purpose, two sections of copper and Iron grade block model were assumed.

4.1. Copper Mine

A section of copper grade block model was assumed (Fig. 5). Each block must be extracted in one year. The first block was extracted in 2010.

0.1	0.8	0.1	0.7	0.1
	0.7	0.9	0.8	
		0.8		

Fig. 5. Copper grade block model

The price and cost data are prepared from Grasberg copper mine. Grasberg mine complex is located in the rugged highlands of the Sudirman mountain range in the province of Irian Jaya, Indonesia. Grasberg contains the largest single gold reserve and one of the three largest open pit copper reserves of any mine in the world. Grasberg is a copper – gold porphyry deposit which is hosted by a magnetite – chalcopyrite and born skarn. The proven reserve of this deposit is 2.4 billion tons of ore grading 1.13% Cu, 1.06 g/t Au and 3.85 g/t Ag. The Grasberg deposit is mined by the open pit method. The process of metal price and cost changes in this mine from 1991 to 2010 are shown in Figure 6. Table 1 presents the supplementary information.

TABLE 1

Supplementary information

Item	Amount	Unit
Total recovery	82	%
Block dimension	10×10×5	m
Density	3	Ton/m^3
Cut off grade	0.3	%
Copper price in 2010	7457.9	\$/ton
Mining operating cost in 2010	1276.0	\$/ton
Processing cost in 2010	2649.6	\$/ton
Discount rate	7	%
Risk free rate	7	%

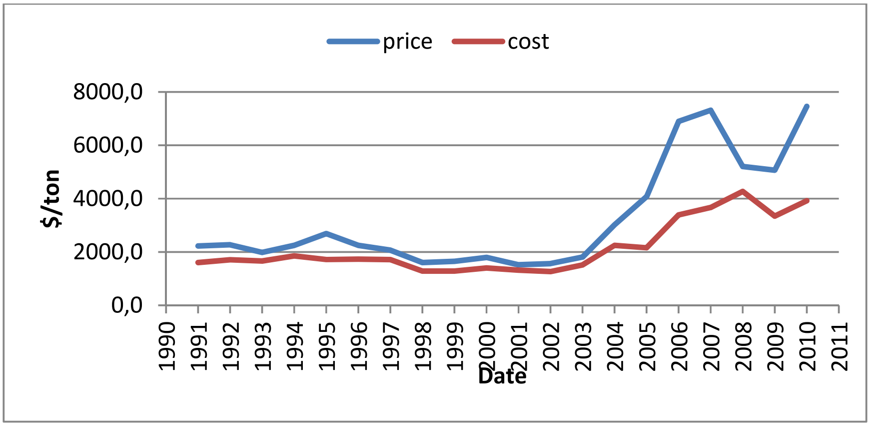

Fig. 6. The copper price and cost process in 1991-2010 (Infomine, 2011)

4.1.1. BEV and NPV calculation using the conventional method

Equation 1 is used for calculating the block economic value. For instance, the BEV is calculated in the first and second left side blocks.

- The copper grade is 0.1 in the first left side block. This block is a waste block because its grade is less than cut off grade (0.1 < 0.3). so the *BEV* is calculated using equation 7:

$$BEV = -TC_m = -(1500 \times 1276.0) = -1.9\ MUSD$$

- The copper grade is 0.8 in the second left side block. This block is an ore block because its grade is greater than cut off grade (0.8 > 0.3). so the *BEV* is calculated using equation 1:

$$BEV = T_o GRP - T_0 C_p - TC_m =$$
$$= 1500(0.8 \times 7457.9 \times 0.82 - 1276.0 - 2649.6) = 1.5\ MUSD$$

Figure 7 shows the economic value for each block.

–1.9	1.5	–1.9	0.5	–1.9
	0.5	2.4	1.5	
		1.5		

Fig. 7. Block economic value (M$)

The mining sequence was determined based on the Roman method. Figure 8 shows the mining sequence.

7	1	2	3	5
	8	4	6	
		9		

Fig. 8. Mining sequence

The project net present value was calculated considering the block economic value, the mining sequence and using equation 5, as follow:

$$NPV = \sum_{n=1}^{N} \frac{BEV_n}{(1+i)^n} = \frac{1.5}{1.07^1} + \frac{-1.9}{1.07^2} + \ldots + \frac{1.5}{1.07^9} = 1.55\,\text{M\$}$$

The project *NPV* is 1.55 M$ using the Whittle equation and situation of certain metal price and operating cost.

4.1.2. *BEV* and *NPV* calculation using the new method:

In this section, the *BEV* and project *NPV* were calculated using a set of real price and cost data of the Grasberg copper mine. The volatility, probability and amounts of up and down were determined as shown in Table 2 using the historical data and equations 2-4.

TABLE 2

Price and cost volatility

	Price	Mining cost	Total cost
Volatility	24.1%	20.1%	18.0%
Up	1.273	1.223	1.198
Down	0.785	0.818	0.835
Probability	0.584		

The binomial trees of copper prices, total cost and mining operating cost are illustrated for 10 years in Figures 9-11. For instance, to calculate the upside node copper price in 2011, the copper price in 2010 should be multiplied by the upside factor (7457.9 × 1.273 = 9494.5) and for downside node, the copper price in 2010 should be multiplied by the downside factor (7457.9 × 0.785 = 5858.2). Consequently, to work out the range of copper prices up to 2018, a similar approach has been utilized.

Table 3 shows the block specification in each year. The probability of occurrence of each block economic value was determined using the preliminary analysis on the historical price and cost data as shown in Figure 12. For example, there are four nodes in year 2011. These nodes illustrate the probability of four situations, i.e. price and cost go up, price goes up but cost goes down, price goes down but cost goes up and price and cost go down, respectively.

TABLE 3

Block specification

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018
Grade (%)	0.8	0.1	0.7	0.9	0.1	0.8	0.1	0.7	0.8
Tonnage (ton)	1500	1500	1500	1500	1500	1500	1500	1500	1500
Recovery (%)	82	82	82	82	82	82	82	82	82

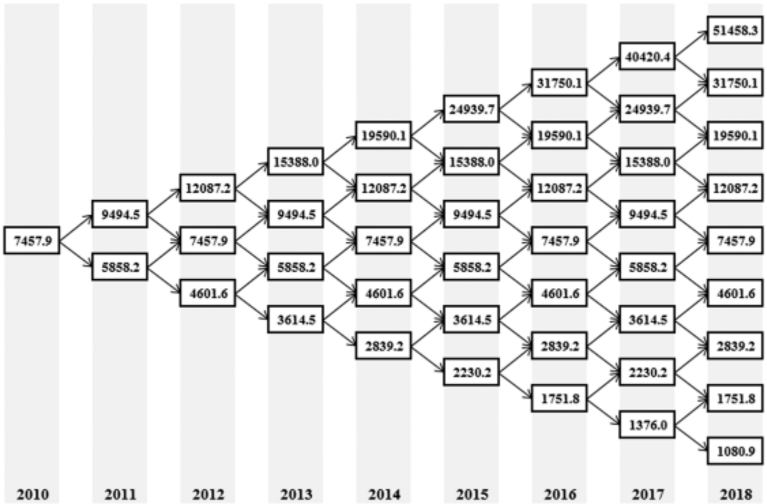

Fig. 9. The metal price binomial tree ($/ton)

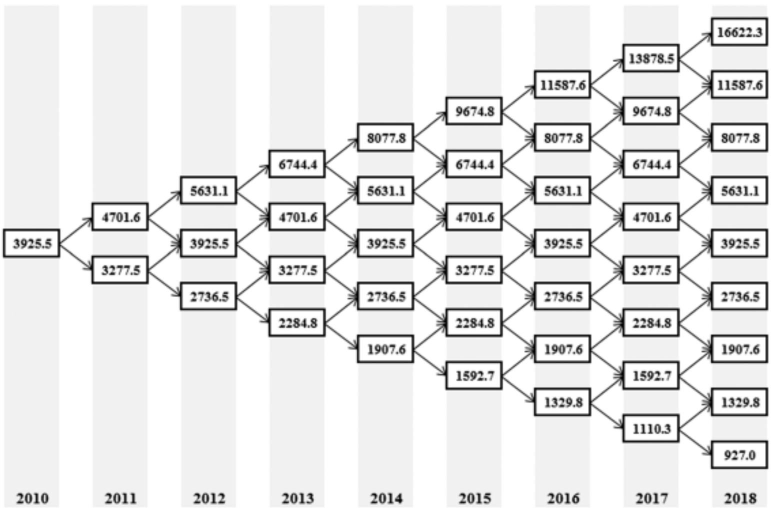

Fig. 10. The total cost binomial tree ($/ton)

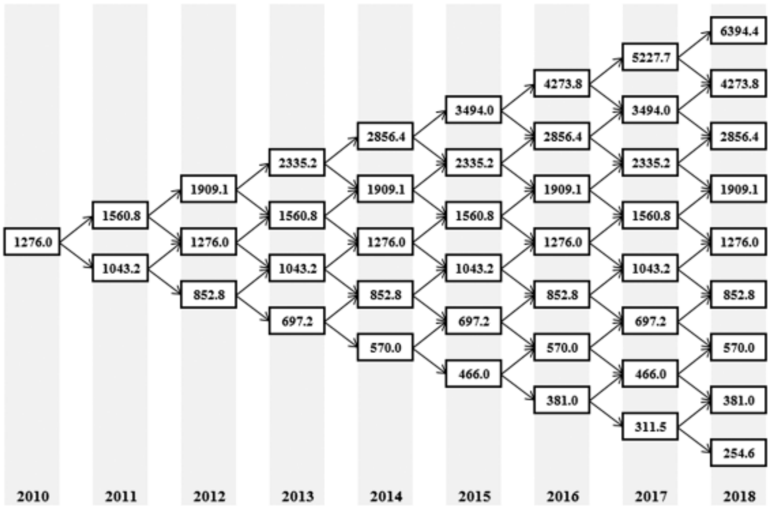

Fig. 11. The mining operating cost binomial tree ($/ton)

The amount of block economic value which is calculated using equations 6 and 7 is shown in Figure 13. For example, the first node in 2012 is calculated as follow:

$$BEV_n = T(G_n R_n P_i - C_j) =$$
$$= 1500(0.7 \times 0.82 \times 12078.2 - 5631.1) = 1.96\ MUSD$$

2010	2011	2012	2013	2014	2015	2016	2017	2018
1.00	0.47	0.17	0.00	0.00	0.00	0.00	0.00	0.00
	0.16	0.22	0.24	0.13	0.07	0.00	0.00	0.00
	0.11	0.00	0.00	0.06	0.07	0.07	0.00	0.00
	0.26	0.06	0.00	0.00	0.00	0.00	0.00	0.00
		0.33	0.06	0.00	0.00	0.00	0.00	0.00
		0.06	0.18	0.00	0.00	0.00	0.00	0.00
		0.00	0.18	0.06	0.00	0.00	0.00	0.00
		0.11	0.00	0.25	0.07	0.00	0.00	0.00
		0.06	0.00	0.00	0.13	0.07	0.00	0.00
			0.00	0.00	0.07	0.07	0.00	0.00
			0.12	0.00	0.00	0.14	0.08	0.00
			0.18	0.13	0.00	0.00	0.23	0.00
			0.00	0.25	0.00	0.00	0.00	0.17
			0.00	0.00	0.07	0.00	0.00	0.00
			0.06	0.00	0.13	0.00	0.00	0.00
			0.00	0.00	0.07	0.00	0.00	0.00
				0.00	0.00	0.14	0.00	0.00
				0.06	0.00	0.07	0.00	0.00
				0.06	0.00	0.00	0.15	0.00
				0.00	0.00	0.00	0.08	0.00
				0.00	0.07	0.00	0.00	0.08
				0.00	0.27	0.00	0.00	0.17
				0.00	0.00	0.00	0.00	0.08
				0.00	0.00	0.00	0.00	0.00
				0.00	0.00	0.21	0.00	0.00
					0.00	0.07	0.00	0.00
					0.00	0.00	0.00	0.00
					0.00	0.00	0.08	0.00
					0.00	0.00	0.08	0.00

0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00
0.00	0.07	0.00	0.08
0.00	0.07	0.00	0.00
0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00
	0.00	0.31	0.00
	0.00	0.00	0.00
	0.00	0.00	0.00
	0.00	0.00	0.00
	0.00	0.00	0.33
	0.00	0.00	0.08
	0.00	0.00	0.00
	0.00	0.00	0.00
	0.00	0.00	0.00
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Fig. 12. Block economic value probability

2010	2011	2012	2013	2014	2015	2016	2017	2018
1.45	-2.34	1.96	6.92	-4.28	10.03	-6.41	13.98	25.70
	-1.56	4.52	9.98	-2.86	14.42	-4.28	20.29	33.25
	-2.34	6.30	12.12	-1.91	17.49	-2.86	24.69	38.52
	-1.56	-2.86	13.61	-1.28	19.62	-1.91	27.75	42.19
		0.53	0.39	-0.85	21.11	-1.28	29.89	44.75
		2.32	3.46	-4.28	22.15	1.04	31.37	46.53
		-2.86	5.59	-2.86	0.63	1.91	32.41	47.77
		-1.91	7.08	-1.91	5.03	-6.41	33.14	48.64
		-1.28	-3.50	-1.28	8.09	-4.28	0.66	49.24
			-2.34	-0.85	10.23	-2.86	6.96	6.31
			1.57	-4.28	11.71	-1.91	11.36	13.86
			3.06	-2.86	12.75	-1.28	14.42	19.13
			-3.50	-1.91	-5.24	-0.85	16.56	22.80

-2.34	-1.28	-3.50	0.41	18.05	25.35
-1.56	-0.85	2.29	-6.41	19.08	27.14
0.57	-4.28	4.43	-4.28	19.81	28.38
	-2.86	5.92	-2.86	-7.84	29.25
	-1.91	6.95	-1.91	-5.24	29.85
	-1.28	-5.24	-1.28	3.13	-9.59
	-0.85	-3.50	-0.85	6.20	1.90
	-4.28	-2.34	-0.57	8.33	7.16
	-2.86	0.85	-6.41	9.82	10.83
	-1.91	2.34	-4.28	10.86	13.39
	-1.28	3.38	-2.86	11.58	15.17
	-0.85	-5.24	-1.91	-7.84	16.42
		-3.50	-1.28	-5.24	17.28
		-2.34	-0.85	-3.50	17.89
		-1.56	-0.57	1.12	-9.59
		0.13	-6.41	3.26	-6.41
		1.17	-4.28	4.75	-4.28
		-5.24	-2.86	5.79	3.45
		-3.50	-1.91	6.51	6.01
		-2.34	-1.28	-7.84	7.79
		-1.56	-0.85	-5.24	9.03
		-1.05	-0.57	-3.50	9.90
		-0.70	-6.41	-2.34	10.50
			-4.28	0.13	-9.59
			-2.86	1.62	-6.41
			-1.91	2.65	-4.28
			-1.28	3.38	-2.86
			-0.85	-7.84	1.45
			-0.57	-5.24	3.23
			-6.41	-3.50	4.48
			-4.28	-2.34	5.34
			-2.86	-1.56	5.95
			-1.91	-1.05	-9.59
			-1.28	0.72	-6.41
			-0.85	1.45	-4.28
			-0.57	-7.84	-2.86
				-5.24	-1.91
				-3.50	0.42
				-2.34	1.67
				-1.56	2.53
				-1.05	3.14
				-0.70	-9.59
				0.25	-6.41
				-7.84	-4.28
				-5.24	-2.86
				-3.50	-1.91
				-2.34	-1.28
				-1.56	-0.85
				-1.05	0.80
				-0.70	1.40
				-0.47	-9.59
					-6.41
					-4.28
					-2.86
					-1.91
					-1.28
					-0.85
					-0.57
					-0.38
					-9.59
					-6.41
					-4.28
					-2.86
					-1.91
					-1.28
					-0.85
					-0.57
					-0.38

Fig. 13. Block economic value binomial tree (M$)

The final block economic value for each block was determined by multiplying the *BEV* probability (Fig. 12) in the *BEV* amount (Fig. 13) and dividing on the sum of the probability nodes. Figure 14 shows the final *BEV* binomial tree. For example, first node of 2011 is calculated as bellow:

$$BEV_{1,2011} = \frac{(-2.34\times 0.47)+(-1.56\times 0.16)}{(0.47+0.16)} = -1.36\,\text{M\$}$$

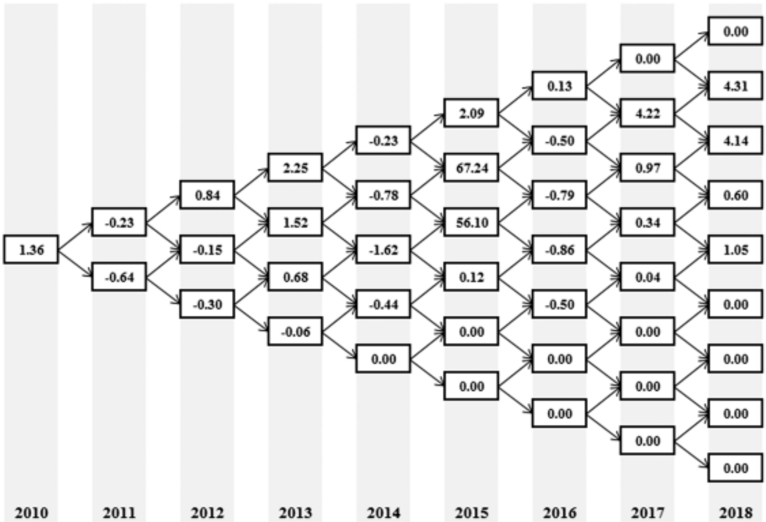

Fig. 14. The multiple *BEV* probability in *BEV* amount (M$)

Using the binomial tree method, the *DCF* can be calculated for each node from *BEV*. *BEV* in the last year of the project (in this example, year 2018) shows the value of discounted cash flow of the mine for the last year. This means that the last column in Figure 15 is the same as the last column in Figure 14. Equation 8 is used to calculate the remaining years. For example to calculate *DCF* in 2010, the formula should be:

$$DCF_n = BEV_n + \frac{pr.DCF_{n+1} + (1-pr).DCF_{n+1}}{(1+rf)}$$

$$= 1.45 + \frac{0.58\times 2.58 + 0.42\times 0.46)}{1.07} = 3.03\text{ M\$}$$

The project net present value using the multivariate binomial tree method is 3.03 M$.

4.2. Iron Mine

A section of iron grade block model was assumed (Fig. 16). Each block must be extracted in one year. The first block was extracted in 2010.

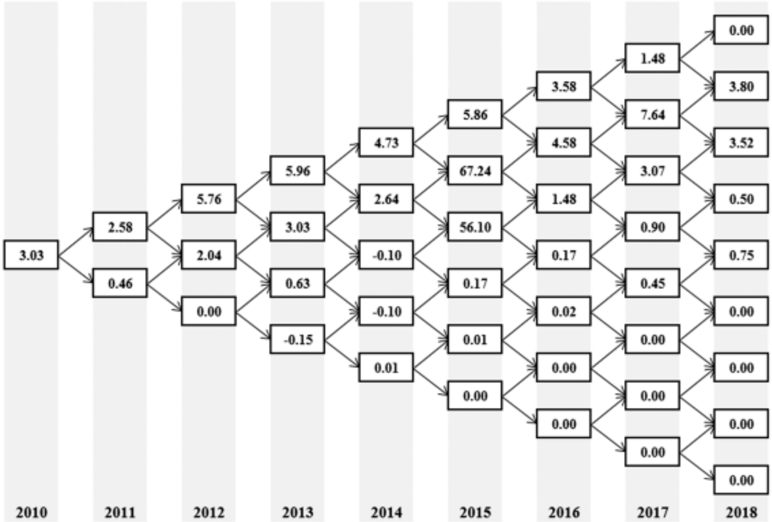

Fig. 15. The net present value binomial tree (M$)

45	35	20	20	20
	50	55	55	
		60		

Fig. 16. Iron grade block model

Shahrak Mine is located in the west of Iran. Conventional open pit mining methods are utilized at the Shahrak Mine. The ore-body is relatively flat. Table 4 presents the supplementary information.

TABLE 4

Supplementary information

Item	Amount	Unit
Total recovery	80	%
Block dimension	10×10×5	m
Density	3	Ton/m^3
Cut off grade	30	%
Iron price in 2010	58	$/ton
Mining operating cost in 2010	6.5	$/ton
Processing cost in 2010	11.2	$/ton
Discount rate	7	%
Risk free rate	7	%

4.2.1. *BEV* and *NPV* calculation using the conventional method

Equation 1 is used for calculating the block economic value. Figure 17 shows the economic value for each block.

4.77	-9.75	-9.75	-9.75	-9.75
	8.25	11.73	11.73	
		15.21		

Fig 17. Block economic value (T$)

The mining sequence was determined based on the Roman method. Figure 18 shows the mining sequence.

1	2	3	5	7
	4	6	8	
		9		

Fig. 18. Mining sequence

The project net present value was calculated considering the block economic value, the mining sequence and using equation 5, as follow:

$$NPV = \sum_{n=1}^{N} \frac{BEV_n}{(1+i)^n} = \frac{4.77}{1.07^1} + \frac{-9.75}{1.07^2} + \ldots + \frac{15.21}{1.07^9} = 5.35 \text{ T\$}$$

The project **NPV** is 5.35 T$ using the Whittle equation and situation of certain metal price and operating cost.

4.2.2. *BEV* and *NPV* calculation using the new method

In this section, the *BEV* and project *NPV* were calculated using a set of real price and cost data of the Shahrak iron mine. The volatility, probability and amounts of up and down were determined as shown in Table 5 using the historical data and equations 2-4.

TABLE 5

Price and cost volatility

	Price	Mining cost	Total cost
Volatility	7.9%	5.5%	6.3%
Up	1.083	1.056	1.065
Down	0.924	0.947	0.939
Probability	0.921		

The binomial trees of iron prices, total cost and mining operating cost are illustrated for 10 years in Figures 19-21.

Table 6 shows the block specification in each year. The probability of occurrence of each block economic value was determined using the preliminary analysis on the historical price and cost data as shown in Figure 22.

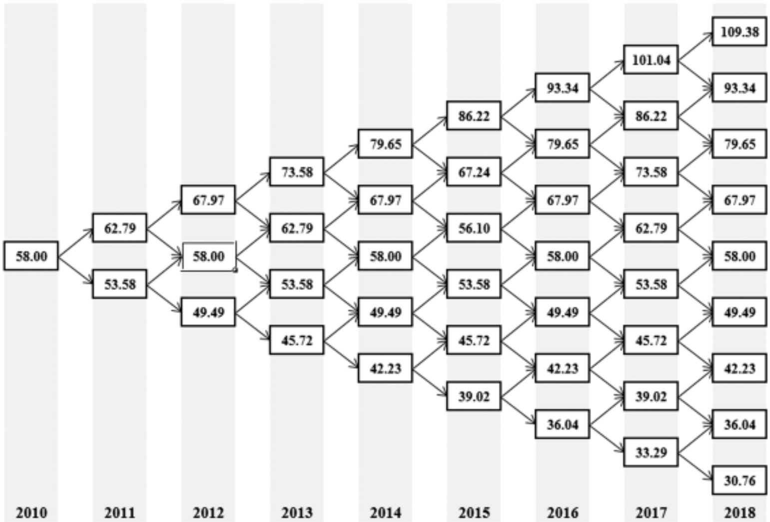

Fig. 19. The metal price binomial tree ($/ton)

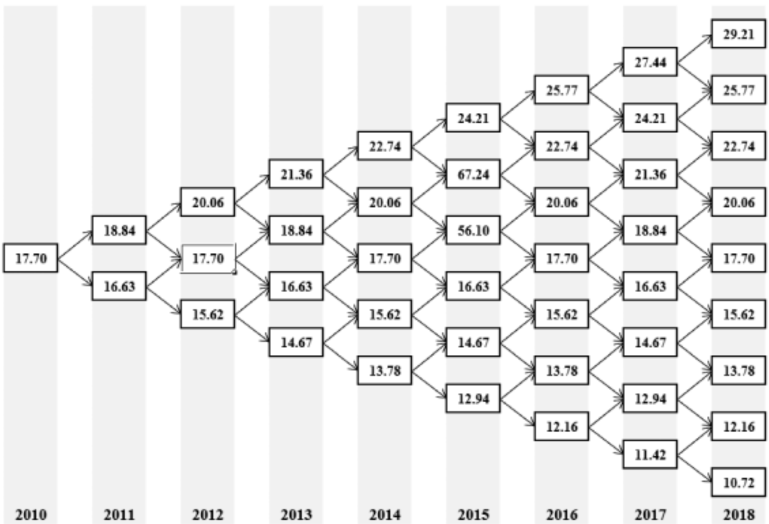

Fig. 20. The total cost binomial tree ($/ton)

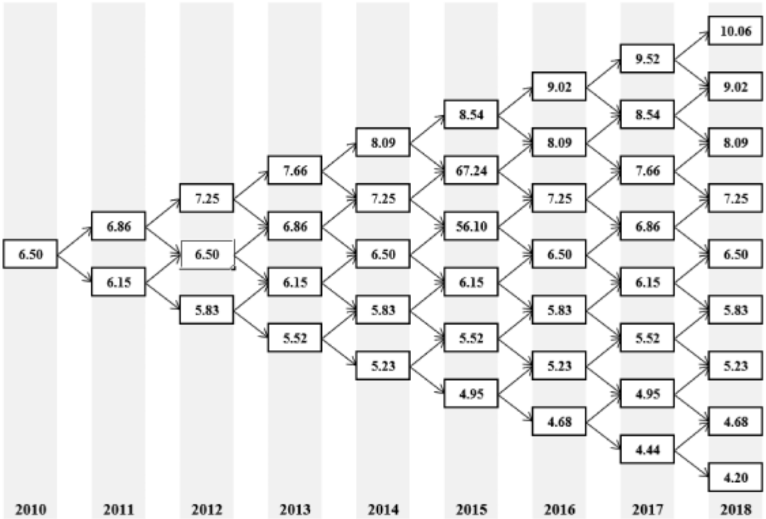

Fig. 21. The mining operating cost binomial tree ($/ton)

TABLE 6

Block specification

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018
Grade (%)	0.8	0.1	0.7	0.9	0.1	0.8	0.1	0.7	0.8
Tonnage (ton)	1500	1500	1500	1500	1500	1500	1500	1500	1500
Recovery (%)	82	82	82	82	82	82	82	82	82

The amount of block economic value which is calculated using equations 6 and 7 is shown in Figure 23.

2010	2011	2012	2013	2014	2015	2016	2017	2018
1.000	0.50	0.33	0.25	0.14	0.00	0.00	0.00	0.00
	0.20	0.11	0.25	0.14	0.17	0.00	0.00	0.00
	0.10	0.11	0.00	0.00	0.00	0.00	0.00	0.00
	0.20	0.00	0.00	0.00	0.17	0.00	0.00	0.00
		0.22	0.00	0.14	0.00	0.00	0.00	0.00
		0.00	0.13	0.00	0.00	0.20	0.00	0.00
		0.00	0.00	0.14	0.00	0.00	0.00	0.00
		0.11	0.00	0.00	0.17	0.00	0.00	0.00
		0.11	0.00	0.00	0.00	0.20	0.00	0.00
			0.00	0.00	0.00	0.00	0.00	0.00
			0.25	0.00	0.00	0.00	0.00	0.00
			0.00	0.00	0.00	0.00	0.25	0.00
			0.00	0.14	0.00	0.00	0.00	0.00
			0.00	0.00	0.00	0.00	0.00	0.00
			0.13	0.00	0.17	0.00	0.00	0.00
			0.00	0.00	0.00	0.00	0.00	0.00
				0.00	0.00	0.20	0.00	0.00
				0.14	0.00	0.00	0.00	0.00
				0.14	0.00	0.00	0.25	0.00
				0.00	0.00	0.00	0.00	0.00

0.00	0.00	0.00	0.00	0.00
0.00	0.33	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.33
0.00	0.00	0.40	0.00	0.00
	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00
	0.00	0.00	0.50	0.00
	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.78
	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00
		0.00	0.00	0.00
		0.00	0.00	0.00
		0.00	0.00	0.00
		0.00	0.00	0.00
		0.00	0.00	0.00
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				0.00

Fig. 22. Block economic value probability

2010	2011	2012	2013	2014	2015	2016	2017	2018
4.77	-10.30	-10.87	12.11	-12.13	20.59	-13.53	25.53	34.94
	1.43	-9.75	15.88	-10.87	24.87	-12.13	30.37	40.09
	-10.30	-8.74	19.21	-9.75	28.64	-10.87	34.65	44.65
	-9.23	-10.87	22.14	-8.74	31.97	-9.75	38.42	48.66
		-9.75	5.63	-7.84	34.90	-8.74	41.75	52.20
		-8.74	9.41	-12.13	37.50	1.73	44.69	55.33
		-10.87	12.73	-10.87	12.25	4.17	47.28	58.09
		-9.75	15.67	-9.75	16.52	-13.53	49.56	60.52
		-8.74	0.11	-8.74	20.30	-12.13	15.75	62.67
			3.88	-7.84	23.62	-10.87	20.59	23.39
			7.21	-12.13	26.56	-9.75	24.87	28.54
			10.14	-10.87	29.15	-8.74	28.64	33.10
			-11.48	-9.75	5.13	-7.84	31.97	37.11
			-10.30	-8.74	9.40	0.88	34.90	40.65
			2.49	-7.84	13.17	-13.53	37.50	43.78
			5.43	-12.13	16.50	-12.13	39.78	46.54
				-10.87	19.44	-10.87	7.40	48.97
				-9.75	22.03	-9.75	12.25	51.12
				-8.74	-12.81	-8.74	16.52	13.53
				-7.84	3.32	-7.84	20.30	18.69
				-12.13	7.10	-7.03	23.62	23.24
				-10.87	10.42	-13.53	26.56	27.25
				-9.75	13.36	-12.13	29.15	30.80
				-8.74	15.95	-10.87	31.43	33.92
				-7.84	-12.81	-9.75	0.28	36.68
					-11.48	-8.74	5.13	39.11
					1.91	-7.84	9.40	41.26
					5.24	-7.03	13.17	5.12
					8.17	-13.53	16.50	10.28
					10.76	-12.13	19.44	14.83
					-12.81	-10.87	22.03	18.84
					-11.48	-9.75	24.31	22.39
					-10.30	-8.74	-14.29	25.51
					0.81	-7.84	-12.81	28.27
					3.75	-7.03	3.32	30.70
					6.34	-13.53	7.10	32.85
						-12.13	10.42	-15.09
						-10.87	13.36	3.10
						-9.75	15.95	7.65
						-8.74	18.24	11.67
						-7.84	-14.29	15.21
						-7.03	-12.81	18.34
						-13.53	-11.48	21.09
						-12.13	1.91	23.53
						-10.87	5.24	25.67
						-9.75	8.17	-15.09
						-8.74	10.76	-13.53
						-7.84	13.05	1.53
						-7.03	-14.29	5.54
							-12.81	9.09
							-11.48	12.21
							-10.30	14.97
							0.81	17.40
							3.75	19.55
							6.34	-15.09
							8.62	-13.53
							-14.29	-12.13
							-12.81	0.32
							-11.48	3.86
							-10.30	6.98
							-9.23	9.74
							-8.28	12.18
							2.56	14.32
							4.85	-15.09
								-13.53
								-12.13
								-10.87
								-9.75
								2.53

5.28
7.72
9.86
-15.09
-13.53
-12.13
-10.87
-9.75
-8.74
1.48
3.91
6.06

Fig. 23. Block economic value binomial tree (T$)

The final block economic value for each block was determined by multiplying the *BEV* probability (Fig. 22) in the *BEV* amount (Fig. 23) and dividing on the sum of the probability nodes. Figure 24 shows the final *BEV* binomial tree.

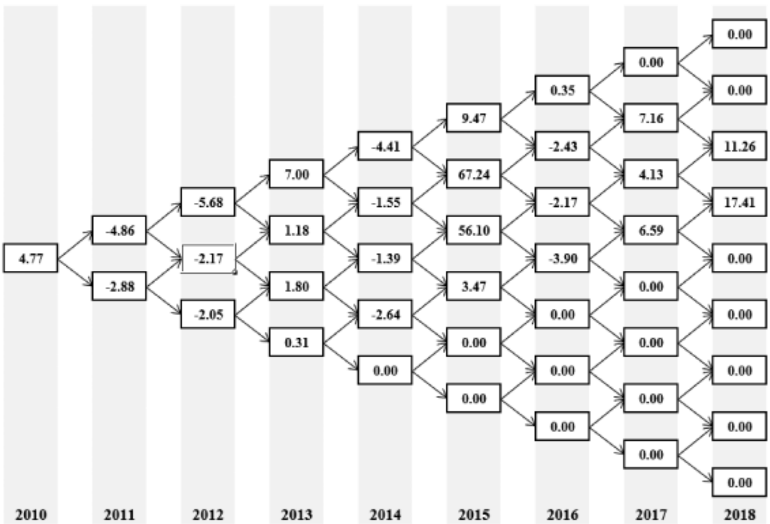

Fig. 24. The multiple *BEV* probability in *BEV* amount (T$)

Using the binomial tree method, the *DCF* can be calculated for each node from *BEV*. *BEV* in the last year of the project (in this example, year 2018) shows the value of discounted cash flow of the mine for the last year. This means that the last column in Figure 25 is the same as the last column in Figure 24. Equation 8 is used to calculate the remaining years.

The project net present value using the multivariate binomial tree method is 5.60 T$.

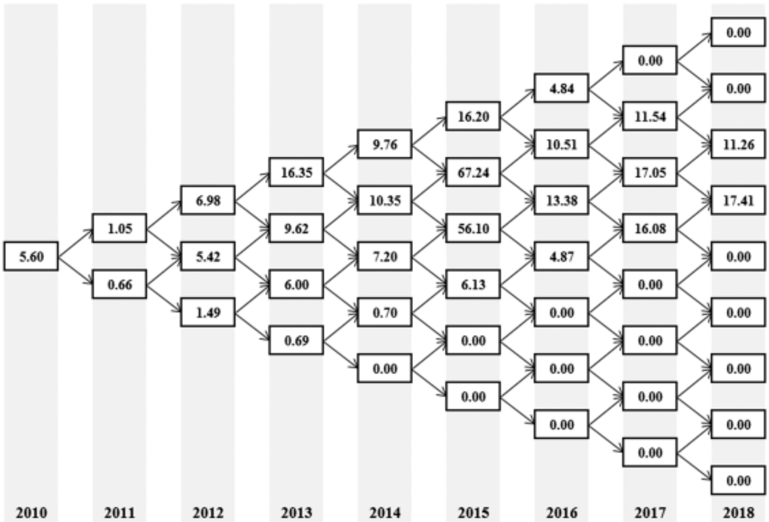

Fig. 25. The net present value binomial tree (T$)

5. Conclusion

In this research work a new binomial tree was developed, which can extensively be used in the problems with two uncertainties. For the first time, the role of cost uncertainty on the block economic value and net present value calculation process was represented. This paper shows that the cost uncertainty has a very important role in economic problems. The suggestive method may offer more than one economic value for each block depending on the price and cost variations. The amounts of block economic value and net present value of two real copper and iron mines were calculated and compared using two methods. Applying the metal price and cost uncertainties cause the block economic value and net present value to be calculated more realistically than certain conditions. For more investigation it is suggested that the BEV be calculated under geologic and economic uncertainties.

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