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RELIABILITY ANALYSIS OF GEAR TRANSMISSION WITH CONSIDERING FAILURE CORRELATION

ANALIZA NIEZAWODNOŚCI PRZEKŁADNI Z UWZGLĘDNIENIEM KORELACJI USZKODZEŃ

Reliability analysis is of great importance in engineering practices. However, reliability analysis of mechanical system under considering correlation for multiple failure modes is very difficult. Gear is the key component in many mechanical transmission systems and therefore its reliability analysis is very important. Based on the standards of strength calculation of gears and stress-strength interference theory as well as copula theory, the reliability of gear transmission with three failure modes, including gear bending fatigue, gear flank contact fatigue and flank adhesion, is analyzed. The correlation of the three failure modes is studied and reliability of their correlation is also evaluated based on the selected copula functions. The proposed method can be used to facilitate the design, manufacturing, and maintenance planning of gears.

Keywords: *Reliability analysis; contact stress; bending stress; multiple failure modes; failure correlation.*

Analiza niezawodności ma ogromne znaczenie w praktyce inżynierskiej. Jednakże, analiza niezawodności układu mechanicznego z uwzględnieniem korelacji dla mnogich przyczyn uszkodzeń jest trudnym zadaniem. Koło zębate jest kluczowym elementem w wielu przekładniach mechanicznych i dlatego analiza jego niezawodności jest niezwykle ważna. W oparciu o normy obliczania wytrzymałości kół zębatych i teorię interferencji naprężeń i wytrzymałości, a także teorię kopuł, przeanalizowano niezawodność przekładni zębatej uwzględniając trzy przyczyny uszkodzeń: zmęczenie zginające koła zębatego, zmęczenie stykowe boku zęba i przyczepność boku. Prześledzono korelację trzech przyczyn uszkodzeń i oceniono niezawodność ich korelacji na podstawie wybranych funkcji kopuł. Proponowana metoda może być stosowana w celu ułatwienia projektowania, produkcji i planowania konserwacji przekładni.

Słowa kluczowe: *Analiza niezawodności; naprężenie stykowe; naprężenie zginające; mnogie przyczyny uszkodzeń; korelacja uszkodzeń.*

1. Introduction

Gear is an important component of mechanical transmission. Gear transmission has been recognized as one of the most important mechanical transmission forms because it has a series of advantages, such as broad power and speed ratio in scope, high transmission efficiency, compact structure and so on. In recent years, with rapid development of machine tools, aircrafts, and automobiles, gear transmission has become an extremely important form of mechanical transmission [1, 4, 6, 8].

Reliability of mechanical transmission relies on the most critical component, e.g. gear, of the system [10]. The failure of gears in mechanical transmission will lead to a poor performance, sometimes even serious accidents and subsequently great economic loss. Therefore, developing an effective and accurate fatigue reliability evaluation model for gear transmission has been a hot topic in the gear engineering community [2, 3, 11].

Failure modes of gears are complicated, such as teeth broken, teeth surface pitting, teeth wear, teeth bonding and teeth plastic deformations and so on, because in most cases, gears work under high speed, heavy load and strong impulse conditions. Therefore, multiple failure modes should be considered simultaneously for analyzing reliability of gears. In this paper, the gear transmission of heavy machine tools is analyzed. Tooth bending fatigue, gear contact fatigue and flank adhesion are major failure modes of heavy duty gears. The failures of weak points are dependent of each other, because all the roots (with maximum bending stress) and surfaces (with maximum contact stress) in a gear subject to the same environmental conditions. Thus, the failure dependence of a gear in the transmission system should be considered. Reliability models of gear transmission with common cause failures are developed without the assumption of failure independence [5, 6, 12, 14, 15].

The paper is organized as follows. In Section 2, the strength calculation standards for gears and the copula theory as well as stress-strength interference theory are briefly introduced. Reliability calculation models are developed in Section 3. The proposed method is validated by a gear transmission of heavy machine tools with three failure modes in Section 4. Conclusions are dawn in Section 5.

2. Reliability calculation model

2.1. Stress-Strength Interference (SSI) theory

The stress–strength interference (SSI) model has been widely used for reliability analysis of mechanical components. Mathematically, the SSI theory presents the failure probability P_f of a mechanical system as the probability that the stress exceeds the strength. The reliability R is the probability that the stress is less than the allowable strength, denoted as [2, 6]:

$$R = P(\sigma_a > \sigma_B) \tag{1}$$

Reliability R of a component can be calculated, if the probability density functions of the allowable strength $f_a(\sigma_a)$ and the actual stress $f_B(\sigma_B)$ are known. The random variable U is a measurement for the distance between the actual stress and the allowable strength:

$$U = \sigma_a - \sigma_B \tag{2}$$

$P_R = P(U > 0)$ is the reliability; $P_F = P(U \le 0)$ is the failure probability.

If the random stress σ_B and allowable strength σ_a are normally distributed respectively with the mean values and standardized deviations $(\bar{\sigma}_B, S_B)$ $(\bar{\sigma}_a, S_a)$, the probability density function of a normal distributed stress can be determined as follows:

$$f_B(\sigma_B) = \frac{1}{S_B\sqrt{2\pi}} e^{-\left[\frac{(\sigma_B - \bar{\sigma}_B)^2}{2S_B^2}\right]} \tag{3}$$

Similarly, the probability density function of the allowable strength can be determined. The random variable U is, likewise, normally distributed. The unreliability for the interference area of the two distributions can be calculated using the following equation:

$$Z = \frac{\bar{\sigma}_a - \bar{\sigma}_B}{\sqrt{S_a^2 + S_B^2}} \tag{4}$$

Then, the reliability can be simply calculated as follows:

$$R = \Phi\left(\frac{\bar{\sigma}_a - \bar{\sigma}_B}{\sqrt{S_a^2 + S_B^2}}\right) \tag{5}$$

2.2. Reliability calculation based on gear bending fatigue

The bending stress of the gear root is the biggest stress in gear transmission process under the alternating bending stress, and the tooth is easy to produce fatigue crack and crack expansion under the alternating bending stress which will lead to tooth bending fatigue fracture. The bending stress can be calculated by [9, 15, 16]:

$$\sigma_F = Y_{Fa} Y_{Sa} Y_\varepsilon Y_\beta \frac{F_t}{bm_n} K_A K_V K_{F\beta} K_{Fa} \tag{6}$$

where m_n is the normal module, Y_{Fa} is the tooth form factor, Y_{Sa} is the bending stress concentration coefficient, Y_ε is the contact ratio factor, Y_β is the helix angle coefficient, F_t is the rated tangential tooth force at transverse pitch, b is the active face width, K_A is the work condition coefficient, K_V is the dynamic load coefficient, K_{Fa} is the load distribution coefficient, $K_{F\beta}$ is the longitudinal load distribution coefficient.

The tooth bending fatigue strength is defined as:

$$\sigma'_{Flim} = \sigma_{Flim} Y_{ST} Y_{NT} Y_{srelt} Y_{Rrelt} Y_X \tag{7}$$

where σ_{Flim} is the experimental gear bending fatigue strength, Y_{ST} is the experimental gear tooth stress concentration coefficient, Y_{NT} is the life coefficient, Y_{srelt} is the relative sensitive coefficient, Y_{Rrelt} is the relative surface condition coefficient, Y_X is the size coefficient.

According to the stress-strength interference theory, the limit state function of bending fatigue is defined as:

$$f(\sigma) = \ln(\sigma'_{Flim}) - \ln(\sigma_F) \tag{8}$$

According to the Eq. (8), the mean of function can be calculated as:

$$\begin{aligned} E\left[f(\sigma)\right] &= E\left[\ln\left(\sigma'_{Flim}\right)\right] - E\left[\ln\left(\sigma_F\right)\right] \\ &= E\left[\ln\left(\frac{\sigma'_{Flim}}{\sigma_F}\right)\right] \end{aligned} \tag{9}$$

where:

$$E\left[\ln(\sigma'_{Flim})\right] = \ln\left[E(\sigma_{Flim}) E(\mathrm{Y}_{ST}) E(\mathrm{Y}_{NT}) E(\mathrm{Y}_{srelt}) E(\mathrm{Y}_{Rrelt}) E(\mathrm{Y}_X)\right] \tag{10}$$

$$E\left[\ln(\sigma_F)\right] = \ln\left[E(\mathrm{Y}_{Fa}) E(\mathrm{Y}_{Sa}) E(\mathrm{Y}_\varepsilon) E(\mathrm{Y}_\beta) \frac{E(F_t)}{E(b)E(m_n)} E(K_A) E(K_V) E(K_{F\beta}) E(K_{Fa})\right] \tag{11}$$

The variance of the function is as follow:

$$D\left[f(\sigma)\right] = D\left[\ln(\sigma'_{Flim})\right] + D\left[\ln(\sigma_F)\right] \tag{12}$$

Supposed the random variable x is normally distributed, the variance of function $y = \ln(x)$ is:

$$\sigma^2(y) = \left[\left.\frac{dy}{dx}\right|_{E(x)} \sigma(x)\right]^2 = \left[\frac{\sigma(x)}{E(x)}\right]^2 = C^2(x) \tag{13}$$

For multivariable function y composed of multiple independent random variables, the expression is:

$$y = ax_1^{m_1} x_2^{m_2} \ldots x_n^{m_n} = a\prod_{i=1}^{n} x_i^{m_i} \tag{14}$$

$x_1, x_2, \cdots, x_n$ is independent random variables.

According to Eqs. (13) and (14), the variable coefficient of y using the first order Taylor expansion can be expressed as:

$$C_y^2 = \sum_{i=1}^{n} m_i^2 C_{x_i}^2 \tag{15}$$

According to Eq. (12), we have:

$$D\left[f(\sigma)\right] = C^2(\sigma'_{Flim}) + C^2(\sigma_F) \tag{16}$$

According to Eqs. (6), (15), and (16), the variable coefficient of $\sigma'_{F\lim}$ and σ_F can be respectively expressed as:

$$C^2(\sigma_F) = C_{Y_{Fa}}^2 + C_{Y_{Sa}}^2 + C_{Y_e}^2 + C_{Y_b}^2 + C_{F_t}^2 + C_b^2 + C_{m_n}^2 + C_{K_A}^2 + C_{K_V}^2 + C_{K_{F\beta}}^2 + C_{K_{Fa}}^2 \tag{17}$$

$$C^2(\sigma'_{Flim}) = C_{\sigma_{Flim}}^2 + C_{Y_{ST}}^2 + C_{Y_{NT}}^2 + C_{Y_{srelt}}^2 + C_{Y_{Rrelt}}^2 + C_{Y_X}^2 \tag{18}$$

Substituting Eqs. (11) and (17) into the reliability formula, the reliability index for gear bending fatigue strength can be given by:

$$\beta_F = \frac{E(f_\sigma)}{\sigma(f_\sigma)} = \frac{E\left[\ln\left(\dfrac{\sigma'_{Flim}}{\sigma_F}\right)\right]}{\sqrt{C^2(\sigma'_{Flim}) + C^2(\sigma_F)}} \tag{19}$$

Reliability for gear bending fatigue strength is:

$$R = \Phi(\beta_F) \tag{20}$$

2.3. Reliability design based on contact fatigue

The fatigue life of gears has been studied over many years, and the gear contact fatigue performance is very important from the former studies. Gear tooth contact fatigue is a key characteristic of the gear and affected by design geometry, material, manufacturing methods and other variables. Surface contact fatigue is the common cause of gear failure. It results in damage to the contacting surfaces which can significantly reduce the load-carrying capacity of components, and may ultimately lead to the complete failure of a gear.

Gear contact stress is defined as [16, 17]:

$$\sigma_H = Z_H Z_E Z_e Z_B \sqrt{\frac{F_t}{bd_1}\frac{u+1}{u} K_A K_V K_{H\beta} K_{HA}} \tag{21}$$

where F_t is the rated tangential tooth force at transverse pitch, b is the active face width, K_A is the work condition coefficient, K_V is the dynamic load coefficient, $K_{H\beta}$ is the longitudinal load distribution coefficient, K_{HA} is the transverse load distribution coefficient, Z_H is the nodal field coefficient, Z_E is the elastic coefficient, Z_e is the contact ratio coefficient, Z_B is the spiral angle coefficient, d_1 is the pinion pitch diameter, u is the gear ratio.

Contact fatigue strength of tooth faces is defined as:

$$\sigma'_{Hlim} = \sigma_{Hlim} Z_N Z_R Z_V Z_L Z_W Z_X \tag{22}$$

where σ_{Hlim} is the experimental flank contact fatigue strength, Z_N is the life coefficient, Z_R is the tooth fineness coefficient, Z_V is the velocity coefficient, Z_L is the lubricant coefficient, Z_W is the work harden coefficient, Z_X is the size coefficient.

According to stress-strength interference theory, the limit state function is defined as:

$$g(\sigma) = \ln(\sigma'_{Hlim}) - \ln(\sigma_H) \tag{23}$$

According to Eq. (23), the mean of the function is calculated as:

$$\begin{aligned} E\left[g(\sigma)\right] &= E\left[\ln(\sigma'_{Hlim})\right] - E\left[\ln(\sigma_H)\right] \\ &= E\left[\frac{\ln(\sigma'_{Hlim})}{\ln(\sigma_H)}\right] \end{aligned} \tag{24}$$

where:

$$E\left[\ln(\sigma'_{Hlim})\right] = \ln\left[E(\sigma_{Hlim})E(Z_N)E(Z_R)E(Z_V)E(Z_L)E(Z_W)E(Z_X)\right] \tag{25}$$

$$E\left[\ln(\sigma_{Hlim})\right] = \ln\left[\sqrt{\frac{u\pm1}{u}} E(Z_H)E(Z_E)E(Z_e)E(Z_B)\sqrt{\frac{E(F_t)}{E(bd_1)E(d_1)} E(K_A)E(K_V)E(K_{H\beta})E(K_{HA})}\right] \tag{26}$$

The variances of the function is:

$$\begin{aligned} D\left[g(\sigma)\right] &= D\left[\ln(\sigma'_{Hlim})\right] + D\left[\ln(\sigma_H)\right] \\ &= C^2\left[\ln(\sigma'_{Hlim})\right] + C^2\left[\ln(\sigma_H)\right] \end{aligned} \tag{27}$$

The variable coefficients of σ'_{Hlim} and σ_H are respectively given by:

$$C^2(\sigma'_{Hlim}) = C_{\sigma_{Hlim}}^2 + C_{Z_N}^2 + C_{Z_R}^2 + C_{Z_V}^2 + C_{Z_L}^2 + C_{Z_W}^2 + C_{Z_X}^2 \tag{28}$$

$$C^2(\sigma_{Hlim}) = C_{Z_H}^2 + C_{Z_E}^2 + C_{Z_e}^2 + C_{Z_B}^2 + \frac{1}{4}\left(C_{F_t}^2 + C_{K_A}^2 + C_{K_V}^2 + C_{K_{H\beta}}^2 + C_{K_{HA}}^2\right) \tag{29}$$

Substituting Eqs. (26) and (27) into the reliability formula, the reliability index for gear contact fatigue strength can be calculated as:

$$\beta_H = \frac{E(g_H)}{\sigma(g_H)} = \frac{E\left[\dfrac{\ln(\sigma'_{Hlim})}{\ln(\sigma_H)}\right]}{\sqrt{C^2\left[\ln(\sigma'_{Hlim})\right] + C^2\left[\ln(\sigma_H)\right]}} \tag{30}$$

Reliability for gear contact fatigue strength is:

$$R = \Phi(\beta_H) \tag{31}$$

2.4. Reliability design based on flank adhesion

Generally, we should consider not only contact fatigue strength and bending fatigue strength, but also the scuffing failure during the design of a high speed heavy gear. Flank adhesion damage occurs on gear teeth if they are operated with an inadequate lubricant film between the teeth. High surface temperatures then arise from the frictional heating, local welding and surface dragging as well as scoring therefore tend to occur. Because flank adhesion failure usually occurs in the sudden onset of high-speed heavy conditions, thereby limiting load capacity and the service life [3, 5, 6, 17].

According to GB/Z 6413, tooth of gear integral temperature θ_S is:

$$\theta_S=\theta_M+1.5\times\left\{0.12\frac{F_t}{b}K_A K_B K_{B\beta}K_{B\gamma}\left(\frac{1}{\upsilon_\Sigma \eta_{ail}}\right)^{0.25}\left(\frac{R_{a1}+R_{a2}}{2Q_{redc}}\right)^{0.25}X_M X_{BE}X_{\alpha\beta}\frac{K_{B\gamma}^{0.75}\upsilon^{0.5}}{|a|^{0.25}}\frac{1}{X_Q X_{Ca}}X_\varepsilon\right\} \quad (32)$$

where θ_M is the body temperature, $K_{B\gamma}$ is the twist coefficient of abrasion, X_M is the coefficient of thermal expansion, X_{BE} is the addendum coefficient, $X_{\alpha\beta}$ is the coefficient of pressure angle, X_Q is the contact ratio, X_{Ca} is the addendum modification coefficient, X_ε is the scuffing calculate contact ratio factor, R_{a1}, R_{a2} are the arithmetic average roughness values.

The scuffing temperature limit is defined as:

$$\theta_B=\theta_M+1.5\theta_{fla\,\mathrm{int}} \quad (33)$$

where $\theta_{fla\,\mathrm{int}}$ is flash temperature.

According to the stress-strength interference theory, the limit state function is defined as:

$$g_S=\theta_B-\theta_S \quad (34)$$

According to Eq. (23), the mean and variable of the function are as follows:

$$E(g_S)=E(\theta_B)-E(\theta_S) \quad (35)$$

$$\begin{aligned}D(\theta_{\mathrm{int}})&=D(\theta_M)+1.5^2 D(\theta_{fla\,\mathrm{int}})\\&=C_{\theta_M}^2 E^2(\theta_M)+1.5^2 C(\theta_{fla\,\mathrm{int}})E^2(\theta_{fla\,\mathrm{int}})\end{aligned} \quad (36)$$

where:

$$C_{\theta_{fla\,\mathrm{int}}}=\sqrt{C_{F_t}^2+C_{K_A}^2+C_{K_B}^2+C_{K_{B\beta}}^2+C_{K_{B\gamma}}^2+C_{X_M}^2+C_{X_{BE}}^2+C_{X_{\alpha\beta}}^2+C_{X_\varepsilon}^2} \quad (37)$$

The reliability index for gear flank adhesion can be calculated as:

$$\begin{aligned}\beta_\theta&=\frac{E(g_S)}{\sigma(g_S)}=\frac{E(g_S)}{\sqrt{D(g_S)}}\\&=\frac{E(\theta_{\mathrm{int}\,s})-E(\theta_{\mathrm{int}})}{\sqrt{D(\theta_{\mathrm{int}\,s})+D(\theta_{\mathrm{int}})}}\\&=\frac{E(\theta_{\mathrm{int}\,s})-E(\theta_{\mathrm{int}})}{\sqrt{C_{\theta_{\mathrm{int}\,s}}^2 E^2(\theta_{\mathrm{int}\,s})+C_{\theta_M}^2 E^2(\theta_M)+1.5^2 C_{\theta_{fla\,\mathrm{int}}}^2 E^2(\theta_{fla\,\mathrm{int}})}}\end{aligned} \quad (38)$$

Reliability of the flank adhesion is:

$$R=\Phi(\beta_\theta) \quad (39)$$

3. Reliability analysis of gears considering failure correlation

Generally, when component has multiple failure modes, the occurrence of any kind of failure mode will lead to component failure [12]. As a result, reliability of component with multiple failure modes can be regarded as a series system, as shown in Fig.1.

It was generally considered that the parts and failure modes of mechanical system are mutually independent. Therefore reliability of a series system is:

$$R_S(t)=R_1R_2\cdots R_n=\prod_{i=1}^{n}R_i(t) \quad (40)$$

where $R_S(t)$ is reliability of the system, R_i is reliability of the ith failure mode.

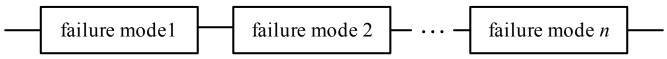

Fig.1. The series system

In practices, for the most engineering systems, their parts work in the same random load environment, and thus their failures are not mutually independent. Correlation is an inherent specialty of complicated mechanical systems, which is one of the greatest issues affecting and restricting mechanical reliability research [2, 6, 7, 10]. If the dependence of system failures is ignored, analysis of system reliability often leads to an excessive error. When we consider the correlation of mechanical components with multiple failure modes, reliability can be shown as:

$$\begin{aligned}R(t)&=P\{\min(T_1,T_2,\cdots,T_n)>t\}=P(T_1>t,T_2>t,\cdots,T_n>t)\\&=1-\sum_{i=1}^{n}P(T_i\le t)+\sum_{1\le i<j\le n}^{n}P(T_i\le t,T_j\le t)+\cdots+(-1)^k\times\\&\quad\sum_{1\le i_1<i_2<\cdots i_k\le n}^{n}P(T_{i_1}\le t,T_{i_2}\le t,\cdots,T_{i_k}\le t)+\cdots+(-1)^k P(T_{i_1}\le t,T_{i_2}\le t,\cdots,T_{i_k}\le t)\\&=1-\sum_{i=1}^{n}F_i(t)+(-1)^k\times\sum_{1\le i_1<i_2<\cdots<i_k\le n}^{n}C(F_{i_1}(t),F_{i_2}(t),\cdots,F_{i_n}(t))\end{aligned} \quad (41)$$

where $F_i(t)$ is a failure probability. $C(F_{i_1}(t),F_{i_2}(t),\cdots,F_{i_n}(t))$ is a copula function. As a useful tool to establish a joint distribution function from its marginal distributions, copula functions are often adopted to study correlation problems. Copulas provide a way of specifying joint distributions if only the marginal distributions are known. In terms of reliability problem with multiple failure modes, we can obtain a multivariate distribution for modeling joint behavior of failure modes using the marginal distributions of each failure mode and the copula function [7, 9, 16, 17].

Let $F_X(x)$ and $F_Y(y)$ denote the marginal distribution functions of variables X and Y, respectively. The joint distribution function $F_{X,Y}(x,y)$ can be expressed as:

$$F_{X,Y}(x,y)=C[F_X(x),F_Y(y)] \quad (42)$$

where $C(u,v)$ is the copula function.

If $F_X(x)$ and $F_Y(y)$ are continuous functions, $C(u,v)$ is unique. Since $F_X(x)$ and $F_X(x)$ are univariate functions and $C(u,v)$ is a copula function, then $F_{X,Y}(x,y)$ is a bivariate joint distribution function with marginal $F_X(x)$ and $F_Y(y)$.

Generally, the Archimedean copula functions are often adopted to build the joint distribution function. An N-dimensional Archimedean copula is given by:

$$C(u_1,u_2,\cdots,u_N)=\varphi(\varphi^{-1}(u_1),\varphi^{-1}(u_2),\cdots,\varphi^{-1}(u_N)) \quad (43)$$

where φ is the generator.

One of the important natural properties of the Archimedean copulas can be represented by the following expression,

$$C(u_1,u_2,u_3)=C[C(u_1,u_2),u_3] \quad (44)$$

$$C(u_1,u_2,u_3,u_4)=C[C(u_1,u_2,u_3),u_4] \quad (45)$$

$$C(u_1,u_2,\cdots,u_{N-1},u_N)=C\left[C(u_1,u_2,\cdots,u_{N-1}),u_N\right] \quad (46)$$

Eqs. (44-46) show that any N-dimensional Archimedean copula could be deduced by a two-dimension copula. In terms of mechanical parts, the failure modes are generally positive correlated, and the joint distribution function could be built by the Gumbel copula function. The expression of the Gumbel copula is as follows [10, 11, 14]:

$$C_G(u,v;\theta)=\exp\{-[(-\ln u)^{\frac{1}{\theta}}+(-\ln v)^{\frac{1}{\theta}}]^{\theta}\} \qquad \theta\in(0,1) \quad (47)$$

4. Numerical example

In this section, we use the proposed method to calculate reliability of a gear transmission for a heavy machine tool. The material of gear is 18Cr2Ni4WA. In accordance to the standard regulations or looking up in figures [13, 15, 16, 17], we get the mean values of each parameter of gear pairs, and the standard deviation of each parameter based on the aforementioned principles. The variable coefficients are shown in Table 1.

According to Table 1, the reliability index β_i, reliability R_i of each failure mode for gear are obtained, shown in Table 2.

According to Table 2, the major failure modes for gear are sorted as bending fatigue failure, flank adhesion failure and contact fatigue failure. Gear bending fatigue is a major failure mode. The more operating torque increases, the more gear bending fatigue strength will be. Therefore, gear tooth bending fatigue is a key characteristic of the gear and affected by geometry, material, manufacturing methods and other variables.

Using Eq. (35), according to the assumption of mutually independent, the reliability of the driving gear and driven gear respectively are:

$$R_1=R_{F_1}R_{H_1}R_{\theta_1}=0.9659$$

$$R_2=R_{F_2}R_{H_2}R_{\theta_2}=0.9748$$

Reliability of gear pair is $R=R_1R_2$=0.94159.

Table 1. The variable coefficient of gear

variable	variable coefficient	variable	variable coefficient
F_t	C_{F_t}=659.66	X_Q	C_{X_Q}=0.03
Y_{Fa1}	$C_{Y_{Fa1}}$=0.778	X_{BE}	$C_{X_{BE}}$=0.03
Y_{Fa2}	$C_{Y_{Fa2}}$=0.0703	Z_ε	C_{Z_ε}=0.045
Y_{Sa1}	$C_{Y_{Sa1}}$=0.0577	Z_β	C_{Z_β}=0.0478
Y_{Sa2}	$C_{Y_{Sa2}}$=0.0706	Z_F	C_{Z_F}=0.02
Y_ε	C_{Y_ε}=0.0357	Z_N	C_{Z_N}=0.03
Y_β	C_{Y_β}=0.004	Z_R	C_{Z_R}=0.036
Y_{ST}	$C_{Y_{ST}}$=0.0693	Z_V	C_{Z_V}=0.033
Y_{NT}	$C_{Y_{NT}}$=0.033	Z_L	C_{Z_L}=0.033
$Y_{\sigma relt1}$	$C_{Y_{\sigma relt1}}$=0.033	Z_W	C_{Z_W}=0.037
$Y_{\sigma relt2}$	$C_{Y_{\sigma relt2}}$=0.033	Z_E	C_{Z_E}=0.033
Y_{Rrelt1}	$C_{Y_{Rrelt1}}$=0.0351	Z_X	C_{Z_X}=0.033
Y_{Rrelt2}	C_{Rrelt2}=0.0351	Z_H	C_{Z_H}=0.116
Y_X	C_{Y_X}=0.0451	θ_M	C_{θ_M}=0.03
K_A	C_{K_A}=0.033	θ_{ints}	$C_{\theta_{ints}}$=0.03
K_V	C_{K_V}=0.033	$\sigma_{H\lim}$	$C_{\sigma_{H\lim}}$=0.06
$K_{H\beta}$	$C_{K_{H\beta}}$=0.055	$X_{\alpha\beta}$	$C_{X_{\alpha\beta}}$=0.032
$K_{H\alpha}$	$C_{K_{H\alpha}}$=0.0382	X_M	C_{X_M}=0.027
$K_{F\alpha}$	$C_{K_{F\alpha}}$=0.0382	X_{Ca}	$C_{X_{Ca}}$=0.03

Table 2. The index reliability and the reliability for gear

		β_i	$R_i=\Phi(\beta_i)$
gear bending fatigue failure	driving gear	2.02	$R_{F_1}=0.9783$
	driven gear	2.15	$R_{F_2}=0.9838$
gear contact fatigue failure	driving gear	3.5	$R_{H_1}=0.9935$
	driven gear	2.8	$R_{H_2}=0.9974$
gear flank adhesion failure	driving gear	2.5	$R_{\theta_1}=0.9938$
	driven gear	3.2	$R_{\theta_2}=0.993$

The results obtained by Monte Carlo simulation are

$R_{1MCS} = 0.9878$, $R_{2MCS} = 0.9762$, $R_{MCS} = 0.9754$.

where R_{1MCS} is reliability of a driving gear, R_{2MCS} is reliability of a driving gear, R_{MCS} is the reliability of gear pair. All the results are calculated using Monte Carlo simulation.

The relative error is:

$$\varepsilon = \left| R - R_{MCS} \right| / R_{MCS} = 3.4\%$$

According to the properties of the Gumbel copula $C(P_i, P_h)$ $(1 \le i, h \le 3)$, the $C(P_i, P_h, P_t)$ $(1 \le i < h < t \le 3)$ can be obtained:

$$C(P_{1F}, P_{1H}) = 0.2412$$
$$C(P_{1F}, P_{1\theta}) = 0.1262$$
$$C(P_{1H}, P_{1\theta}) = 0.0978$$

$$C(P_{2F}, P_{2H}) = 0.1348$$
$$C(P_{2F}, P_{2\theta}) = 0.0723$$
$$C(P_{2H}, P_{2\theta}) = 0.1527$$

$$C(P_{1H}, P_{1F}, P_{1\theta}) = C(C(P_{1H}, P_{1F}), P_{1\theta}) = 0.045$$

$$C(P_{2H}, P_{2F}, P_{2\theta}) = C(C(P_{2H}, P_{2F}), P_{2\theta}) = 0.032$$

$$C(P_1, P_2) = 0.1264$$

According to Copula theory, reliability of the driving gear and driven gear can be respectively given by:

$$R_1' = 0.9867$$

$$R_2' = 0.9884$$

Reliability of the gear pair is $R' = 0.9851$. The relative error is $\varepsilon = \left| R' - R_{MCS} \right| / R_{MCS} = 0.94\%$.

From aforementioned results, we know that the relative error for mutually independent of failure modes is greater than considers failure correlation. Since this paper only considers three main failure modes, so the difference of relative error is not obvious. When we consider multiple failure modes, the proposed method is superior to traditional methods without considering correlations.

5. Conclusions

This paper has established reliability model with three major failure modes: tooth bending fatigue, gear contact fatigue and gear scuffing failure. From the reliability calculation model, it is concluded that the primary failure mode of gear is the tooth surface contact fatigue failure and secondary failure mode is the gear scuffing failure. Based the copula theory, a reliability calculation method of the gear under considering correlation for multiple failure modes are developed. A comparative analysis has shown that the accuracy and practicality of the proposed model is higher than the model without consider failure correlation. However, correlations widely exist in practical engineering. Therefore, this method provides an effective and reliable approach to assess reliability of engineering systems.

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