TIME-FREQUENCY ANALYSIS OF TIME-VARIANT SYSTEMS

Kajetan DZIEDZIECH, Wiesław J. STASZEWSKI, Tadeusz UHL
Akademia Górniczo-Hutnicza im. Stanisława Staszica, Katedra Robotyki i Mechatroniki
Al. Mickiewicza 30, 30-059 Kraków, Fax : +48 12 634 3505, E-mail: dziedzie@agh.edu.pl, w.j.staszewski@agh.edu.pl, tuhl@agh.edu.pl

Summary

System identification is an important and often complex process in many areas of engineering. This process is not easy when parameters of the analysed system vary with time. In such cases classical methods fail to identify parameters properly. The work demonstrated in this paper deals with time-frequency representations for identification of natural frequencies of time-variant systems. The method involves the estimation of time-variant transfer functions. A "Crazy Climbers" algorithm - based on Monte Carlo simulations and Markov chains - is used to overcome difficulties associated with the method.

Keywords: system identification, time-frequency analysis, functional ridges, "Crazy Climbers"

CZASOWO-CZĘSTOTLIWOŚCIOWA ANALIZA SYSTEMÓW ZMIENNYCH W CZASIE

Streszczenie

Identyfikacja parametrów systemów mechanicznych jest bardzo ważnym i skomplikowanym procesem. Proces ten jest o wiele bardziej skomplikowany kiedy dotyczy systemów mechanicznych, których parametry zmieniają się w czasie. W takim przypadku klasyczne metody identyfikacji nie są w stanie poprawnie zidentyfikować tych parametrów. Artykuł zajmuje się wykorzystaniem reprezentacji czasowo-częstotliwościowych w celu identyfikacji częstotliwości drgań rezonansowych systemów o zmiennych w czasie parametrach. Jednym z kroków podczas estymacji funkcji przejścia jest dzielenie spektrum odpowiedzi przez spektrum wymuszenia, co często prowadzi do dzielenia przez wartości bliskie zera, a to prowadzi do nieskończonych (lub niezdefiniowanych) wartości. W celu ominięcia tego problemu zastosowano probabilistyczną metodę „CrazyClimbers”, opartą na symulacjach Monte Carlo oraz łańcuchach Markova.

Słowa kluczowe: identyfikacja systemów, analiza czasowo-częstotliwościowa, grzbiety funkcji, "CrazyClimbers"

1. INTORDUCTION

Estimation of instantaneous frequency of mono-component signals *x(t)* can be obtained from the instantaneous phase which is argument of complex function of Hilbert transform of given signal [1]. It is well known that that the Hilbert transform can be calculated from the Fourier transform by a proper manipulation of spectral components. When multi-component signals are used some additional signal processing is required to extract the instantaneous frequency. This results from the simple fact, that every multi-component signal can be decomposed into a series of mono-component signals, e.g. by means filtering or the Empirical Mode Decomposition (EMD). The latter leads to a number of the so-called Intrinsic Mode Functions (IMFs). Instantaneous frequencies can be extracted from each of these IMFs in same manner as in case of mono-component signals. The entire process is often called the Hilbert-Huang Transform (HHT) despite the fact that the EMD is s not a transform in a mathematical sense. Nevertheless the method is a powerful tool in signal processing and has been used to solve many engineering problems [2].

An alternative approach for estimation of instantaneous frequencies can be proposed when ridges of time-frequency representations are analysed. Intuitively, ridges are curves that concentrate signal's energy. Methods concerning identification of instantaneous frequency with the use of ridges are relatively well established in the case of mono-component signals. The simplest approach for ridge identification is based on calculation of local time-frequency maxima [3]. It is well known that local maxima (or in fact ridge values) correspond to instantaneous frequency of signal. This process can be written as

$$|\mathrm{TF}(t,\omega_t)| = |\max_\omega \mathrm{TF}(t,\omega)| \quad (1)$$

where *TF* denotes time-frequency transformation or distribution, t denotes time variable, ω is frequency variable and max_ω is maximal value along frequency variable. It is important to note that the above definition is limited to mono-component signals only. The ridge-based method can be also applied to multi-component signals when appropriate time-frequency filtering is used. However this is not an easy task particularly when noisy data - often associated with real measurements - are used. In such cases various optimization algorithms are required to obtain ridges.

Various approaches can be used for identification of time-variant systems. Parametric methods are often used for estimation of FRFs for time-variant systems. These methods are based on parametric models. The Auto-Regressive (AR), Auto-Regressive Moving-Average (ARMA), Auto-Regressive Moving-Average with eXogenous inputs (ARMAX) models and many other approaches can be used for LTI systems [4]. Time-variant systems are often analysed using similar but modified approaches. There exist a number of time-dependent parametric methods, starting from simple models such as the Time-dependent AR (TAR) and Time-dependent ARMA (TARMA) to more complex and computationally demanding models such as the Recursive Maximum Likelihood estimated TARMA (RML-TARMA), Smoothness Priors TARMA (SP-TARMA), Functional Series TARMA (FS-TARMA) and other variations of these models are discussed in [5]–[8].

The objective of the paper is to present alternative method to parametric methods. This method will be based on non-parametric transformations such as Short Time Fourier Transform (STFT), in combination with Crazy Climbers algorithm. However, other time-frequency or even time-scale representations can be also used, as shown in [9], [10].

The structure of the paper is as follows. For the sake of completeness Section 2 briefly describes input-output analysis of classical time-invariant systems. The Time-Variant Frequency Response Functions (TVFRF) is presented in Section 3. The concept of ridges is presented in Section 4. The optimization algorithm used for ridge extraction is shown in Section 5. Simulated examples and results are given in Section 6. Finally the paper is concluded in Section 7.

2. INPUT-OUTPUT ANALYSIS OF SYSTEMS

Different methods can be used for signals. The two most common approaches utilise time responses and power spectra. Analysis of time response leads to information on signal amplitude and localisation of events in time. The simplicity is the major advantage of this approach. Relatively very little signal post-processing is required to obtain basic information. In contrast, analysis in the frequency domain, based for example on power spectra, provides information about frequency content of the analysed signal. The well-known Fourier transform defined as:

$$X(\omega) = F[x(t)] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt \qquad (2)$$

can be used to obtain power spectra. In modal analysis the FRF defined as the frequency-domain ratio between the output (or response) $Y(\omega)$ and input (or excitation) $X(\omega)$, i.e.

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{F[y(t)]}{F[x(t)]} \qquad (3)$$

allows for modal parameters (natural frequency, damping and mode shapes) to be estimated. Once the FRF is given the response can be obtained in the time domain using the inverse Fourier transform as

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} Y(\omega)e^{j\omega t}d\omega \qquad (4)$$

Another way of representing dynamic response of mechanical systems to given excitations is the well-known impulse response function defined as

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(\omega)e^{j\omega t}d\omega \qquad (5)$$

It is clear that the response of the analysed system can be obtained from the impulse response function using the convolution theorem as

$$y(t) = \int_{-\infty}^{+\infty} h(s)\, x(t-s)ds \qquad (6)$$

The major disadvantage of the classical method described in this section is the fact that the Fourier Transform is capable to analyse properly only time-invariant signals. The application of this approach to time-variant systems may lead to incorrect FRFs and identified physical/modal parameters. This is the reason why other approaches are required.

3. TIME-VARIANT FREQUENCY RESPONSE FUNCTION

The classical FRF can be extended intuitively for time-variant systems to provide time-frequency localisation capability. When the analysis is limited to small periods of time that exhibit time-invariant behaviour, time-variations are negligibly small and the Time-Variant Frequency Response Function (TVFRF) can be defined as

$$H(t,\omega) = \frac{Y(t,\omega)}{X(t,\omega)} = \frac{TF[y(t)]}{TF[x(t)]} \qquad (7)$$

where TF denotes time-frequency transformation or distribution. More information about this ratio can be found in [9]–[11]. For simplicity, the work presented in this paper utilises the STFT to obtain input and output time-frequency spectra needed to calculate the TVFRF in Equation (7).

Equation (7) is relatively simple often not easy to use in practice. For values of $TF[x(t)]$ that are close to zero, the $H(t,\omega)$tends to infinity. This makes it very difficult to interpret. That is why additional post-processing is required to avoid the above problem.

4. RIDGES OF FUNCTIONS

For the sake of completeness concept of canonical representation of real signals is recalled in this section for better understating related to functional ridges. It is well known that any real signal can be represented in terms of its instantaneous amplitude A and phaseφ

$$\mathrm{x(t)} = \mathrm{A(t)}\cos(\varphi(\mathrm{t})) \tag{8}$$

Instantaneous frequency can be easily derived from instantaneous phase:

$$\omega(\mathrm{t}) = \frac{1}{2\pi}\frac{\mathrm{d}\varphi(\mathrm{t})}{\mathrm{dt}} \tag{9}$$

More information on both instantaneous signal characteristics be found in [12], [13]. Various methods can be used to obtain the instantaneous amplitude and frequency of signals. The most commonly used algorithm is based on the Hilbert transform. These characteristics can be also obtained from the time-frequency and time-scale representation. The major areas where the signal energy is concentrated need to be localized. These areas form a smooth curve that joins local maxima of the transform. The curve is often called a ridge of the transform. Ridges can be then used to obtain the instantaneous frequency and amplitude. The former is obtained when ridge localization is performed. The process of combining single ridges into sequence is called chaining. The latter is achieved when the amplitude corresponding to ridge area is analyzed.

When Equation (7) is used the entire concept of ridges can be utilised in system analysis. Then instead of instantaneous frequency, natural frequency will be considered.

5. RIDGE EXTRACTION PROCEDURE

5.1 Crazy Climbers Algorithm

The time-variant FRF defined by Equation (7) does not involve any data averaging in the time domain. When additionally the data analysed are noisy and close vibration modes are involved, the process of ridge extraction and ridge chaining can lead to significant numerical errors and difficult interpretation. Various post-processing algorithms can be applied to avoid such difficulties. The so-called "Crazy Climbers" algorithm is one of the possible methods that can be used in practice. This method is based on the Monte Carlo Markov Chain (MCMC) simulations. The main idea of the method is to use the TVFRF to generate a random walk on the time-frequency plane, in such way that the random walker is attracted by the ridges of the hills. In addition, the random walk is done at a given "temperature" which changes with time. The temperature is gradually decreased in the process, as in the simulated annealing algorithm. However, contrary to the simulated annealing procedure, the motion of the walker is unrestricted in one direction and walker is never stuck on the ridge. Thanks to the temperature schedule, each climber is expected to spend most of his time walking along one or another ridge involved. Therefore there are a number of walkers instead of just one and the entire procedure is suitable for multi-degree-of-freedom (MDOF) systems. Thanks to these random walks, one can create the so-called occupation measures. These occupation measures are created for each point on the time-frequency plane. They represent numbers of time that certain points have been visited by walkers. The occupation measures are expected to have higher values near ridges as they are more attractive for climbers. The "Crazy Climbers" algorithm has a great advantage in terms of possible mode separation when closed vibration modes are involved. More detailed description of the entire algorithm can be found in [14], [15]. When occupation measures are created the results are chained to obtain skeletons. The chaining procedure consists of two major steps. Firstly, thresholding of occupation measures is performed. Values below the pre-defined fixed value τ are forced to zero, i.e.

$$\rho(\mathrm{t},\omega_\mathrm{n}) = \begin{cases}\rho(\mathrm{t},\omega_\mathrm{n}), \text{if } \rho(\mathrm{t},\omega_\mathrm{n}) \geq \tau \\ 0, \text{otherwise}\end{cases} \tag{10}$$

where ρ is the occupation measure obtained by means of the "Crazy Climbers" algorithm. The second step considers chaining the relevant ridges into a number of skeletons in such way that maximal points are connected together when moving along time direction. The entire procedure allows for ridge extraction that is essential for system identification.

5.2 Ridge Optimization

Extracted and appropriately chained ridges are often non-smooth and have no physical meaning when natural frequencies are analysed. This is due to noise and/or numerical errors. Ridge optimization needs to be performed to obtain the best ridge solution. In order to achieve this, an assumption concerning the expected skeleton have to be made and some additional physical information have to be taken into account. The first step considered arises from the previously made definition of ridges. Ridges can be found from the areas of concentration of energy, i.e.

$$\varepsilon_1(\omega_n(t)) = -\int H(t, \omega_n(t))dt \qquad (11)$$

where the integral in the above equation defines the sum of TVFRF values along which the skeleton passes. One should note that this integral is multiplied by minus one. This is required by the optimization algorithm used; optimization concerns minimization of penalty function.

Since this method concerns real mechanical systems, smoothness of the ridge is expected. Mathematically this requires minimizing the second penalty function given in the form

$$\varepsilon_2(\omega_n(t)) = \lambda \int |\omega_n'(t)|^2 dt + \mu \int |\omega_n''(t)|^2 dt \qquad (12)$$

where λ and μ are arbitrary parameters defining weights of the relevant integrals. Minimization of these integrals leads to more smooth results. The penalty functions - given by Equations (11) and (12) - can be combined to define one penalty function, i.e.

$$\varepsilon(\omega_n(t)) = \varepsilon_1(\omega_n(t)) + \varepsilon_2(\omega_n(t)) \qquad (13)$$

that was used in the entire optimization process to extract smooth ridges. The optimization process can utilise genetic algorithms, as demonstrated in [16].

6. NUMERICAL SIMULATIONS

This section demonstrates the capability of the system identification procedure. The focus is on the "Crazy Climbers". Numerical simulations are used to define a simple MDOF system. The 2-DOF damped mass-spring lumped parameter system, illustrated in Figure 1, is simulated using the MATLAB/Simulink computation platform. The sampling frequency was equal to 1 kHz in these numerical simulations.

The system analysed is heavily damped and involves a time-variant mass element; the mass decreased with time. As a result, one of the natural frequencies was expected to vary in time.

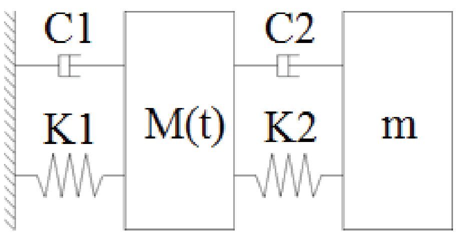

Figure 1. 2-DOF system with time-variant mass

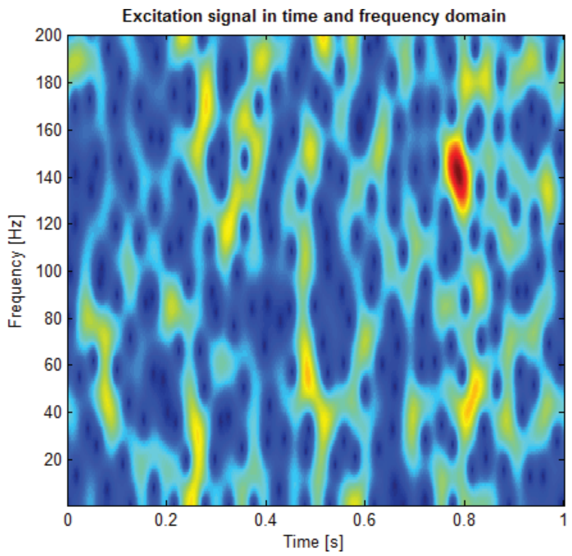

Figure 2. Time-frequency distribution for the Gaussian white noise excitation signal

The Gaussian white noise was used as an input signal to provide a broadband excitation. Figure 2 gives the time-frequency distribution for the excitation signal.

Firstly, the results obtained from numerical simulations were subjected to the classical analysis. The FRF for the system analysed is shown in Figure 3.

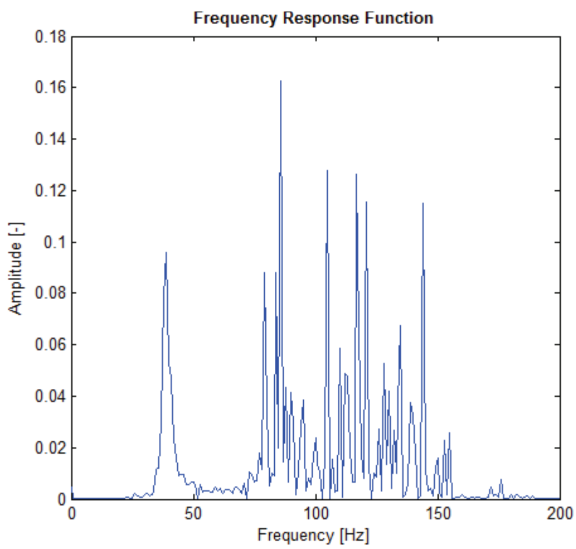

Figure 3. Classical FRF obtained with Fourier transform

The first natural frequency of 40 Hz is clearly exhibited by the FRF function. The second natural frequency is not so obvious due to the time-variant behaviour of the system. Clearly, the classical FRF analysis is not suitable for time-variant systems since natural frequencies are not properly identified.

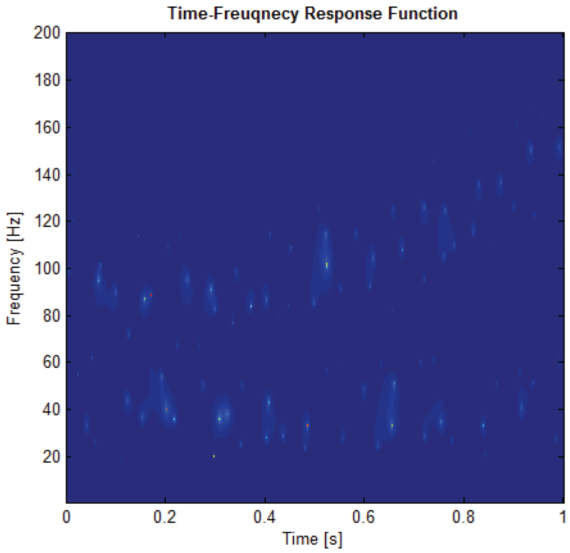

Figure 4. Time-variant FRF for the system shown in Figure 1

The excitation and response data were used to calculate the TVFRF from Equation (7). The result - presented in Figure 4 - is not easy for interpretation. That is why additional post-processing was performed. The "Crazy Climbers:" algorithm - described in Section 5.1 - was used to extract TVFRF ridges. The results of this extraction are shown in Figure 5. Here two vibration modes - represented by two white curves indicating varying natural frequencies - can be clearly identified. The natural frequency of the second mode involved increases with time, as expected. However, both natural frequencies extracted are very bumpy, due to the calculation procedure involved.

The optimization procedure - described in Section 5.2 - was performed to obtain smooth curves. The results are shown in Figure 5 as two red curves imposed on the white noisy characteristics.

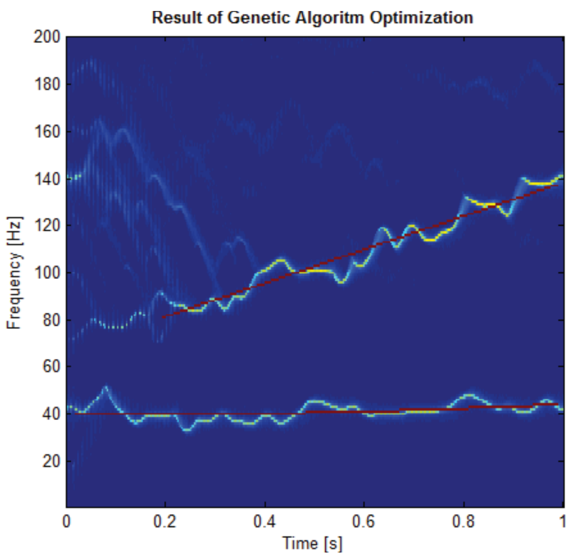

Figure 5. Ridges for the time-variant FRF shown in Figure 4

This clearly shows that the optimization procedure made the relevant ridges smoother and much easier for interpretation. The first natural frequency was estimated between 39 and 41 Hz, whereas the second natural frequency was found to vary between 80 to 140. The results in Figure 5 can be compared with Figure 6 where the so-called "frozen" spectra were used to obtain the natural frequencies of the analysed system. The dynamic behaviour of the system was frozen in time and the classical FRF was obtained. The results in Figure 6 shows that the time-variant behaviour exhibited by the TVFRF - shown in Figure 5 - is revealed properly. Thus the method can be used for identification of time-variant systems.

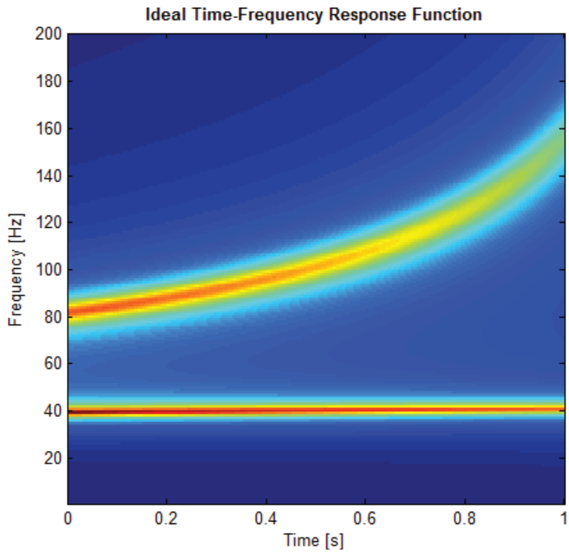

Figure 6. TVFRF obtained with frozen dynamical properties

7. CONCLUSIONS

The TVFRF based on the STFT was used for identification of time-variant systems. The "Crazy Climbers" algorithm together with the skeleton optimization schemes was applied to reveal varying natural frequencies of the simple 2-DOF time-variant system. The results show that the method can be used to reveal time-variant behaviour of the system and to extract its varying natural frequencies. It is important to note that the identification performance of the method have been tested using only one simple simulated example and one identified modal parameter. Future work should involve identification of the remaining modal parameters, i.e. mode shapes and damping. Also, more complex simulated and experimental systems should be investigated.

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Kajetan DZIEDZIECH is a PhD student in the Department of Robotics and Mechatronics at the AGH University of Science and Technology in Krakow, Poland. His research includes various aspects of structural dynamics, modal analysis and signal processing

Wieslaw J. STASZEWSKI is a Professor of Mechanical Engineering at the AGH University of Science and Technology in Krakow, Poland. He contributed to over 300 research publications in the field of structural dynamics, signal processing, structural health monitoring, smart structures and materials.

Tadeusz UHL is a Professor of Mechanical Engineering and Head of the Department of Robotics and Mechatronics at the AGH University of Science and Technology in Krakow, Poland. He is an author of over 600 publications in the field of mechatronics, structural health monitoring and structural dynamics.