DETERMINATION OF VARIABLE PSEUDO-VISCOSITY COEFFICIENTS FOR OILS WITH THE RIVLIN-ERICKSEN PROPERTIES

Andrzej Miszczak

Gdynia Maritime University, Faculty of Marine Engineering
Morska Street 83, PL 81 225 Gdynia, Poland
tel.: +48 58 6901348, fax: +48 58 6901399
e-mail: miszczak@am.gdynia.pl

Abstract

Modern oils, that lubricate the sliding friction pairs, contain more and more additives. These additives change the oil properties to the non-Newtonian. Furthermore, the friction and wear products and also the products of combustion in internal combustion engines, result in a change of the oil properties, from Newtonian to the non-Newtonian. Frequently, researchers suggest to use so-called „smart fluids", for example, the ferro-oils as lubricants of sliding friction pairs exposed to strong magnetic fields or the absence of gravity. The ferro-oils are also characterized by the non-Newtonian properties. The viscosity characteristics of these oils are described rather well by the Rivlin-Ericksen constitutive equation.

The Rivlin-Ericksen constitutive equation contains the coefficients, which are difficult to estimate or determine experimentally. One of the possibilities to determine these coefficients is the method proposed by Prof. K. Wierzcholski [12]. This method requires the experimental results of dynamic viscosity changes as a function of shear rate. The measurements of dynamic viscosity should be conducted for the widest possible range of shear rates. Then, using the viscosity curves and nonlinear system of equations, the auxiliary coefficients are determined. These coefficients generate the equations that describe the variable pseudo-viscosity coefficients.

Not all oils have an intensive exponential character of viscosity changes in dependence on shear rate. In these cases it is possible to determine the constant coefficients of pseudo-viscosity [1, 3, 7, 8].

This paper presents the examples of determination of the variable pseudo-viscosity coefficients for the selected lubricating oils and ferro-oils.

Keywords: *pseudo-viscosity coefficients, apparent viscosity, Rivlin-Ericksen equation, viscosity curves*

1. Introduction

The non-Newtonian oils are liquids, where besides the classical viscosity changes, according to the pressure and temperature, the viscosity also depends on the shear rate. Most of the new or operated lubricating oils have the non-Newtonian properties. The factors affecting the change in the oil properties to non-Newtonian, are: organic substances getting into the oil during operation, dust, soot, the wear and combustion products of the combustion engine or magnetic particles, e.g. Fe_3O_4. These impurities and additives can increase or decrease the value of viscosity, relative to the basic value of the lubricant viscosity [8].

In the existing theoretical research on lubrication of sliding friction pairs with the non-Newtonian, viscoelastic oils, generally, the Rivlina-Ericksen model is used as constitutive relationship, which describe the correlation between the shear stress and shear rate. This model also formulate the relation between the apparent viscosity and shear rate, which is a distinguishing feature of all lubricating agents with the non-Newtonian properties. These relationships contain the α, β and γ pseudo-viscosity coefficients characterizing the type of lubricant. In the numerous previous studies, these coefficients were taken as constant values, independent of the shear rate [1, 2, 9].

The viscosity measurements of the lubricants at varying shear rates show that the adoption of the constitutive relationships described by the Rivlin-Ericksen formula causes, that they correspond with the experimentally measured values of viscosity only in the case of the assumption of variable pseudo-viscosity coefficients [6, 7]. The results of numerous experimental measurements of viscosity changes according to shear rate indicated, that the decrease in oil dynamic viscosity with increasing shear rate is exponential. That kind of changes in the viscosity values requires an adoption of the variable pseudo-viscosity coefficients in the model of Rivlin-Ericksen.

Relying on the method of K. Wierzcholski [12] for determining the variable pseudo-viscosity coefficients for the Rivlin-Ericksen constitutive relationship, it was decided to check on few sample oils, the efficiency and precision of determination of these coefficients with this method.

The aim of this study is to verify the accuracy of the K. Wierzcholski's method of determination of the variable pseudo-viscosity coefficients for the Rivlin-Ericksen model.

Furthermore, on the basis of the measured viscosity curves, the author determines the equation describing the viscosity changes as a function of shear rate by identifying the fitted trend line equation, and also the accuracy of that method will be specified.

In case of analysis of sliding friction pairs lubricated with oils with properties described by the Rivlin-Ericksen model, the type of lubricating oil pseudo-viscosity coefficients (constant or variable in dependence on the shear rate) can affect the value of the flow parameters and operating conditions of slide bearings.

2. The method of determining the variable pseudo-viscosity coefficients

The method of determining the variable pseudo-viscosity coefficients for the Rivlin-Ericksen constitutive relationship is based on a modification of the known physical correlation between the stress tensor and tensors of deformation rate, in which the requested pseudoviscosity coefficients can be found, i.e.: [7-12]:

$$\mathbf{S} = -\,p\mathbf{I} + \eta_o\cdot\mathbf{A}_1 + \alpha\cdot(\mathbf{A}_1)^2 + \beta\cdot\mathbf{A}_2 + \gamma\cdot(\mathbf{A}_1)^2\,\mathbf{A}_2, \tag{1}$$

where:

$\mathbf{A}_1$, $\mathbf{A}_2$ – the defined tensors of deformation rate of dimension [s^{-1}], [s^{-2}],
α – the first experimental pseudo-viscosity coefficient of oil in [Pas^2],
β – the second experimental pseudo-viscosity coefficient of oil in [Pas^2],
γ – the third experimental pseudo-viscosity coefficient of oil in [Pas^4],
η_o – the dynamic viscosity of oil for low values of shear rate in [Pas].

The replacement of tensors $\mathbf{A}_1$, $\mathbf{A}_2$ by their dimension as a function of shear rate, i.e. dim$\mathbf{A}_1$=$\boldsymbol{\Theta}_1$=Θ and dim$\mathbf{A}_2$=$\boldsymbol{\Theta}_2$=Θ^2 in Eq. (1) have negligible impact on the change of stress in the case of thin boundary layers. Hence, the constitutive relationship (1) can be written in the following approximate form [12]:

$$\mathbf{S} \cong -\,p\mathbf{I} + \mathbf{A_1}\left[\eta_o + \alpha\,\boldsymbol{\Theta}_1 + \beta\frac{\boldsymbol{\Theta}_2}{\boldsymbol{\Theta}_1} + \gamma\,\boldsymbol{\Theta}_1\boldsymbol{\Theta}_2\right] \text{ lub } \mathbf{S} \cong -\,p\mathbf{I} + \mathbf{A_1}\eta_p\,, \tag{2}$$

where the apparent viscosity has the following form [12]:

$$\eta_p \equiv \eta_o + \alpha(\Theta)\cdot\Theta + \beta(\Theta)\cdot\Theta + \gamma(\Theta)\cdot\Theta^3\,. \tag{3}$$

The pseudo-viscosity coefficients α, β, γ in Eq. (3) may have constant or variable values . If the adoption of constant pseudo-viscosity coefficients does not give correct results or the results of an analytical solution describing the viscosity changes as a function of the shear rate differ significantly from the values obtained experimentally, the variable pseudo-viscosity coefficients should be applied, i.e.: $\alpha(\Theta)$, $\beta(\Theta)$, $\gamma(\Theta)$.

The unknown variable pseudo-viscosity coefficients generally should be adopted in the following class of the mathematical functions [12]:

$$\alpha(\Theta) = -\frac{\alpha_0}{1 + A_1\Theta + A_2\Theta^2}, \quad \beta(\Theta) = -\frac{\beta_0}{1 + B_1\Theta + B_2\Theta^2}, \quad \gamma(\Theta) = -\frac{\gamma_0}{1 + C_1\Theta + C_2\Theta^2}, \tag{4}$$

where:

α_0 – a positive characteristic value of the first experimental pseudo-viscosity coefficient of the investigated oil in [Pas^2],

β_0 – a positive characteristic value of the second experimental pseudo-viscosity coefficient of the investigated oil in [Pas^2],

γ_0 – a positive characteristic value of the third experimental pseudo-viscosity coefficient of the investigated oil in [Pas^4],

A_1 – the experimental coefficient independent of Θ expressed in [s],

A_2 – the experimental coefficient independent of Θ expressed in [s^2],

B_1 – the experimental coefficient independent of Θ expressed in [s],

B_2 – the experimental coefficient independent of Θ expressed in [s^2],

C_1 – the experimental coefficient independent of Θ expressed in [s],

C_2 – the experimental coefficient independent of Θ expressed in [s^2].

By substituting the functions (4) to relation (3), we get [12]:

$$\eta_p \equiv \eta_o - \frac{\alpha_0\Theta}{1+A_1\Theta+A_2\Theta^2} - \frac{\beta_0\Theta}{1+B_1\Theta+B_2\Theta^2} - \frac{\gamma_0\Theta^3}{1+C_1\Theta+C_2\Theta^2} \quad . \tag{5}$$

The third term of Eq. (5) has usually negligible impact on the character of dynamic viscosity changes. In order to simplify the calculations, it was finally accepted the following class of functions describing the dynamic viscosity change depending on the shear rate [12]:

$$\eta_p \equiv \eta_o - \frac{\alpha_0\Theta}{1+A_1\Theta+A_2\Theta^2} - \frac{\beta_0\Theta}{1+B_1\Theta+B_2\Theta^2}. \tag{6}$$

The method presented in work [12] requires, that the considered range of shear rates should be divided into subintervals in the following manner:

$$\Theta_1 \equiv \Theta_{min} \approx 0 < \Theta_2 < \Theta_3 < \Theta_4 < \Theta_5 < \Theta_6 \equiv \Theta_{max}\,. \tag{7}$$

For each value of shear rate we determine the value of experimentally obtained dynamic viscosity of investigated lubricant, i.e. [12]:

$$\begin{aligned}
&\eta = \eta_1 \cong \eta_0 \quad \text{for } \Theta = \Theta_1 \equiv \Theta_{min}, &&\eta = \eta_2, &&\text{for } \Theta = \Theta_2,\\
&\eta = \eta_3 \quad \text{for } \Theta = \Theta_3, &&\eta = \eta_4, &&\text{for } \Theta = \Theta_4,\\
&\eta = \eta_5 \quad \text{for } \Theta = \Theta_5, &&\eta = \eta_6 \cong \eta_\infty, &&\text{for } \Theta = \Theta_6 \equiv \Theta_{max}.
\end{aligned} \tag{8}$$

The calculated values of pseudo-viscosity are valid, when curves described by the constitutive relationships (3) will be in accordance with the viscosity curves obtained experimentally. Substituting the conditions (8) to relation (6) we get the following system of equations [12]:

$$\begin{aligned}
&\frac{\alpha_0\Theta_1}{1+A_1\Theta_1+A_2\Theta_1^2} + \frac{\beta_0\Theta_1}{1+B_1\Theta_1+B_2\Theta_1^2} = \eta_o - \eta_1,\\
&\frac{\alpha_0\Theta_2}{1+A_1\Theta_2+A_2\Theta_2^2} + \frac{\beta_0\Theta_2}{1+B_1\Theta_2+B_2\Theta_2^2} = \eta_o - \eta_2,\\
&\frac{\alpha_0\Theta_3}{1+A_1\Theta_3+A_2\Theta_3^2} + \frac{\beta_0\Theta_3}{1+B_1\Theta_3+B_2\Theta_3^2} = \eta_o - \eta_3,\\
&\frac{\alpha_0\Theta_4}{1+A_1\Theta_4+A_2\Theta_4^2} + \frac{\beta_0\Theta_4}{1+B_1\Theta_4+B_2\Theta_4^2} = \eta_o - \eta_4,\\
&\frac{\alpha_0\Theta_5}{1+A_1\Theta_5+A_2\Theta_5^2} + \frac{\beta_0\Theta_5}{1+B_1\Theta_5+B_2\Theta_5^2} = \eta_o - \eta_5,\\
&\frac{\alpha_0\Theta_6}{1+A_1\Theta_6+A_2\Theta_6^2} + \frac{\beta_0\Theta_6}{1+B_1\Theta_6+B_2\Theta_6^2} = \eta_o - \eta_6.
\end{aligned} \tag{9}$$

With this system we determine the following six unknowns:

$$A_1, A_2, B_1, B_2, \alpha_0, \beta_0. \qquad (14)$$

3. Examples of calculations

The variable pseudo-viscosity coefficients were determined for 4 operated engine oils and for 2 ferro-oils. The investigation concerned the operated Castrol GTX SAE15W40, the operated Shell Helix Ultra AV-L 5W30, the operated Delo®1000 Marine 30, the operated Superol CC-40 and ferro-oil based on the mineral oil which satisfies the SAE15W40 standard [4, 5]. All engine oils have been used for the normative operating time. The ferro-oil was a colloidal mixture of mineral oil, Fe_3O_4 magnetic particles (1.4% and 8%) and an anti-surfactant, which prevents the particle clustering. The measurement of viscosity changes as a function of shear rate was performed on Hakke Mars III rheometer with the high shear chamber configuration at temperature 60°C. The dynamic viscosity changes as a function of shear rate are shown in Fig. 1. The used engine oil Shell Helix has a lower value of viscosity than the viscosity value of the new oil, while the other examined engine oils have a viscosity greater than the viscosity of the corresponding samples of the non-used oils.

The dynamic viscosity measurements were carried out with the change of shear rate in the range of from 0 s^{-1} to 135000 s^{-1}. In the case of ferro-oil the measurement was automatically interrupted before obtaining the shear rate at 135000 s^{-1} due to exceeded allowable torque of measuring device. For the ferro-oil, which contains 1.4% of magnetic particles the measurement was terminated at the shear rate Θ=123400s^{-1}, while for ferro-oil which contain 8% of magnetic particles the measurement was finished at the shear rate about Θ=50000s^{-1}.

In Fig. 1 the values of shear rate limits are indicated as $\Theta_1, \Theta_2, \Theta_3, \Theta_4, \Theta_5, \Theta_6$ and the corresponding values of the dynamic viscosity $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6$ at the ends of this ranges. The subdivision may be uniform (Fig. 1a, b) or irregular (Fig. 1c). The irregular division allows for a more accurate curve fitting for large changes in the dynamic viscosity in dependence on shear rate. The values of the shear rate and the values of the dynamic viscosity at these points are given in Tab. 1.

Tab. 1. The values of shear rate and dynamic viscosity at the ends of the considered ranges

Θ_1 [s^{-1}]	Θ_2 [s^{-1}]	Θ_3 [s^{-1}]	Θ_4 [s^{-1}]	Θ_5 [s^{-1}]	Θ_6 [s^{-1}]	η_1 [mPas]	η_2 [mPas]	η_3 [mPas]	η_4 [mPas]	η_5 [mPas]	η_6 [mPas]
Castrol GTX											
0.25	27029	54075	81043	107989	134256	30.78	27.82	26.44	25.60	24.74	24.51
Shell Helix											
0.15	27043	54088	81054	107997	134267	12.99	12.48	12.32	12.27	12.23	12.19
Delo®1000 Marine 30											
0.03	27024	54071	81038	107980	134251	34.09	31.64	30.22	29.19	27.91	27.68
Superol CC-40											
0.04	27025	54070	81038	107983	134254	32.65	30.63	29.67	29.11	28.33	28.03
Ferro-oil 1.4%											
129	24749	49550	74505	99207	123463	69.31	58.46	53.84	50.14	48.29	46.23
Ferro-oil 8%											
10.00	3990	7997	15007	35044	49924	484.34	277.98	245.75	215.50	172.87	163.11

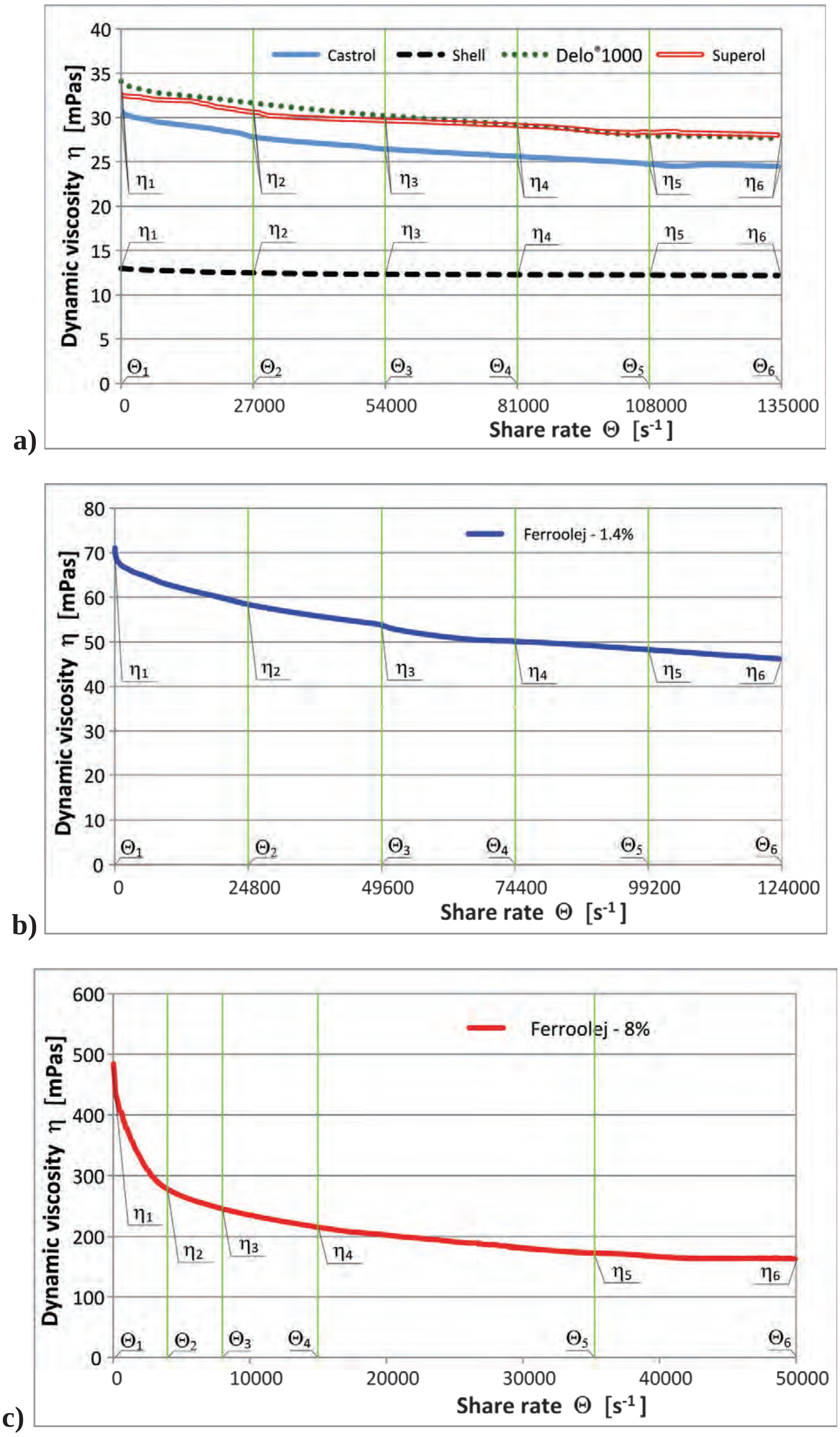

Fig. 1. The viscosity curves of the investigated lubricants at temperature T=60°C: a) used engine oils b) new ferro-oil – 1.4% of magnetic particles, c) new ferro-oil – 8% of magnetic particles

The system of equations (9) was solved with the use of the Mathcad 15 software. Such systems are highly non-linear, resulting in a multiple solutions with respect to the coefficients A_1, A_2, B_1, B_2, α_0, β_0. Most of the solutions does not meet the requirement of properly fitted curve described by the equation (6) to the viscosity curve obtained experimentally. After repeatedly performed calculations, the values of requested coefficients, which allowed for a relatively accurate fit of considered curves, have been obtained. These values and the value of the greatest error of fitting Δ that occurred in the whole length of the curves, are given in Tab. 2 .

Tab. 2. The values of α_0, β_0, A_1, A_2, B_1, B_2 coefficients for investigated lubricating oils

	Castrol GTX	**Shell Helix**	**Delo®1000 Marine 30**	**Superol CC-40**	**Ferro-oil 1.4%**	**Ferro-oil 8%**
α_0	$13.0339\cdot10^{-5}$	$1.3600\cdot10^{-5}$	$11.0686\cdot10^{-5}$	$10.8022\cdot10^{-5}$	$67.7685\cdot10^{-5}$	0.083350395
β_0	$5.9786\cdot10^{-6}$	$8.9821\cdot10^{-6}$	$2.1415\cdot10^{-6}$	$3.7480\cdot10^{-7}$	0.000278921	0.083364735
A_1	$4.2665\cdot10^{-6}$	$-9.4214\cdot10^{-6}$	$7.1888\cdot10^{-6}$	$1.5314\cdot10^{-5}$	$8.9675\cdot10^{-5}$	$56.1805\cdot10^{-5}$
A_2	$2.4286\cdot10^{-10}$	$6.1697\cdot10^{-10}$	$1.0074\cdot10^{-10}$	$5.9025\cdot10^{-11}$	$2.4521\cdot10^{-10}$	$-1.5835\cdot10^{-9}$
B_1	$-1.3025\cdot10^{-5}$	$7.5078\cdot10^{-6}$	$-1.5121\cdot10^{-5}$	$-1.5801\cdot10^{-5}$	$7.3921\cdot10^{-6}$	$58.7898\cdot10^{-5}$
B_2	$5.4914\cdot10^{-11}$	$-5.5060\cdot10^{-12}$	$6.3580\cdot10^{-11}$	$6.4754\cdot10^{-11}$	$2.016\cdot10^{-12}$	$-1.6602\cdot10^{-9}$
Δ	1.32%	0.47%	1.62%	1.55%	2.46%	5.26%
$\Delta=[(\eta_{exp}-\eta_{num})/\eta_{exp}]\cdot100\%$; η_{exp} - measured value of dynamic viscosity (Fig. 1); η_{exp} - calculated value of dynamic viscosity obtained from the equation (6);						

In Fig. 2 are shown the sample plots of viscosity curves obtained experimentally and numerically. The results of numerical calculations of changes in the apparent viscosity versus shear rate are obtained from the equation (6) and the coefficients are determined from the equation (9).

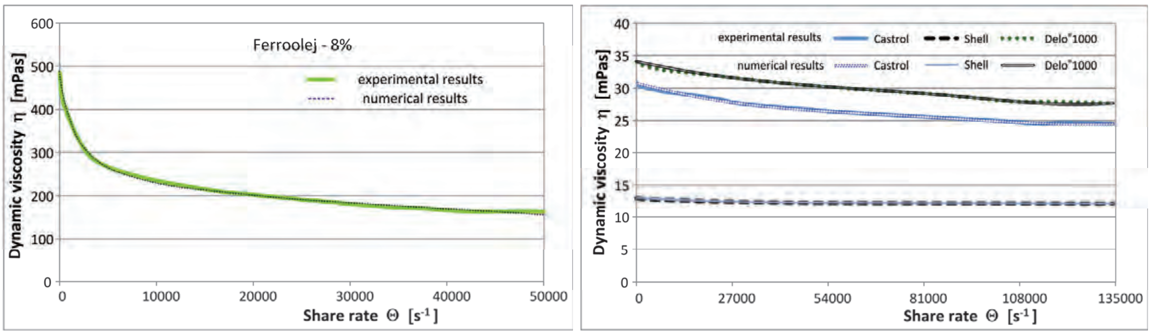

Fig. 2. The comparison of the viscosity curves obtained in the experiment and in the numerical calculations

The nature of changes in the variable pseudo-viscosity coefficients α(Θ), β(Θ) obtained for the analyzed oils, is shown in Fig. 3. These curves are represented by the equation (4) and the auxiliary coefficients given in Tab. 2. In the graphs on the left side of Fig. 3, one can observe the changes of α(Θ) coefficient, while on the right side changes of β(Θ) coefficient.

4. Conclusions

1. The analyzed method of determination of the variable pseudo-viscosity coefficients allows to obtain these values with a high accuracy.
2. The disadvantage of this method is that it gets many of solutions. In such case, one should select such values of coefficients, which gives as accurately fitted viscosity curves, as it possible.
3. The nature of change of pseudo-viscosity coefficients of the used engine oil Castrol GTX SAE15W40, Delo®1000 Marine 30, Superol CC-40 is similar. The dynamic viscosity of these oils was greater then dynamic viscosity of corresponding new oils. The used oil Shell Helix Ultra AV-L 5W30 has lower viscosity than corresponding sample for new oil. This may be resulted from getting the fuel into the engine oil (the passenger car, in which this oil was used, had a diesel particulate filter). The nature of change of pseudo-viscosity coefficients for this oil is different from the three other cases.

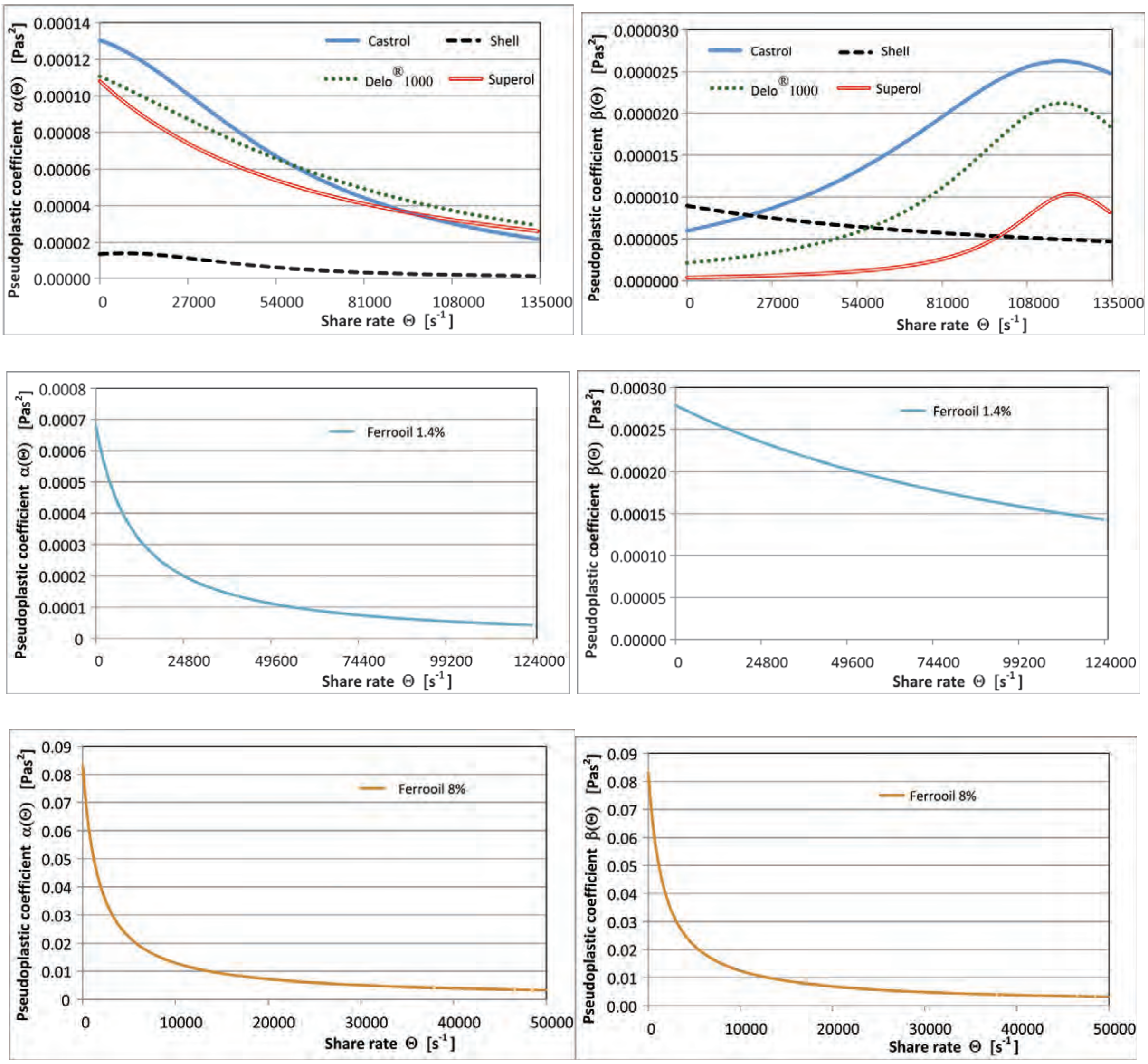

Fig. 3. The curves illustrating the variations in α(Θ), β(Θ) coefficients as a function of shear rate

4. The nature of change of pseudo-viscosity coefficients of ferro-oils, which contained 1.4% and 8% of magnetic particles is similar. The values of the corresponding pseudo-viscosity coefficients differ, because the measured values of dynamic viscosity of ferro-oil with 1.4% of magnetic particles varies considerably from the dynamic viscosity values measured for the ferro-oil with 8% of magnetic particles.
5. The adoption of the variable pseudo-viscosity coefficients should result in including these coefficients in the equations of momentum.

References

[1] Astarita, G., Marrucci, G., *Principles of non-Newtonian fluid mechanics*, McGraw Hill Co. 1974.

[2] Bhushan, B., *Nano-tribology and Nano-mechanics of MEMS/NEMS and Bio-MEMS, Bio-NEMS materials and devices*. Microelectronic Engineering, 84, pp. 387-412, 2007.

[3] Böhme, G., *Strömungsmechanik nicht-Newtonscher Fluide*. Teubner Studienbücher Mechanik, Stuttgart 1981.

[4] Czaban, A., Miszczak, A., *Wyznaczanie współczynników pseudolepkości olejów silnikowych.* Tribologia, 4 (244), pp. 33-40, 2012.

[5] Frycz, M, *Effect of Temperature and Deformation Rate on Dynamic Viscosity*, Solid State Phenomena, Vol.199, pp.137-142, 2012.

[6] Markova, L.V., Myshkin, N.K., Kong, H., Han, H.G., *On-line acoustic viscometry in oil condition monitoring*. Tribology International, Vol. 44, pp. 963-970, 2011.

[7] Miszczak, A., *Analiza hydrodynamicznego smarowania ferrocieczą poprzecznych łożysk ślizgowych*, Fundacja Rozwoju Akademii Morskiej w Gdyni, dysertacja habilitacyjna, 2006.

[8] Teipel, I., *Die räumliche Staupunktströmung für ein viscoelastisches Fluid*. Reologica Acta, 25, pp.75-79, 1986.

[9] Truesdell, C. A., *First Course in Rational Continuum Mechanics*, John Hopkins Univ., Baltimore Maryland 1972.

[10] Wierzcholski, K., *The Viscoelastic Lubrication Problem of Micro-Bearing*. Tribologia, 3 (231), pp. 231-240, 2010.

[11] Wierzcholski, K., *Determination of Pseudo-viscosity Coefficients for Visco- elastic Lubricants*, Journal of Kones Powertrain and Transport, Vol.17, No.2, pp.491-496, 2010.

[12] Wierzcholski, K., Miszczak, A., *Algorytm wyznaczania zmiennych współczynników pseudolepkości olejów na bazie eksperymentu*, Tribologia, in printing, 2013.