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Original research paper

Artificial Neural Networks and Fuzzy Inference Systems for line simplification with Extended WEA Metric

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Abstract: The issue of line simplification is one of the fundamental problems of generalisation of geographical information, and the proper parameterisation of simplification algorithms is essential for the correctness and cartographic quality of the results. The authors of this study have attempted to apply computational intelligence methods in order to create a cartographic knowledge base that would allow for non-standard parameterisation of WEA (Weighted Effective Area) simplification algorithm. The aim of the conducted research was to obtain two independent methods of non-linear weighting of multi-dimensional regression function that determines the "importance" of specific points on the line and their comparison to each other. The first proposed approach consisted in the preparation of a set of cartographically correct examples constituting a basis for teaching a neural network, while the other one consisted in defining inference rules using fuzzy logic. The obtained results demonstrate that both methods have great potential, although the proposed solutions require detailed parameterisation taking into account the specificity of geometric variety of the source data.

Keywords: generalization of geographic information, line simplification, computational intelligence, ANN, FIS

1. Introduction

Cartographic generalisation is a way to create a model generalisation of geographic information, with the aim to achieve the intended objective, i.e. to learn about the space indirectly. Considering thecomplexity of geographical space, obtaining knowledge about it requires an adaptation of thecreated model to the perception and cognitive possibilities of the recipient by means of intentional generalisation of source data. Molenaar (1993) defines generalisation as "a process of abstracting the representation of geographic information when the scale of map is changed". This process usually consists of two phases: conceptual generalisation, which involves the abstracting of information along with determining the rules and graphic generalisation, i.e. the application of geometric shape simplification algorithms and graphic symbolisation.

Traditional cartography (Imhof, 1982), uses the so-called phenomenal approach as an important aspect of the generalisation process. When simplifying the shape of a line representing a topographic object, one should consider the nature and spatial context of the original field object. Thus, the process of generalisation of geographic information should not be analysed only on the elementary level of geometric operations performed on points, lines and polygons, as the geometric elements are just a representation of topographic objects that create geographic reality. So, the actual subject of generalisation is not the simplification of geometry of elementary objects, but modelling. Cartographic modelling consists not only in the abstraction of objects, but also of phenomena, as well as noticing large-scale relationships between individual objects and creating a large-scale representation of the geographic space that will match the aims and purposes of the given study.

One of the methods of generalisation of geographic information that takes into account the model-related and cognitive aspect of the process is the creation of a cartographic knowledge base that allows for the automation of the cartographic generalisation process while at the same time maintaining the subjectivity of the process (White, 1985; McMaster, 1986; Wang and Müller, 1998; Veregin, 1999; Balboa et al., 2005; Olszewski, 2010; Visvalingam and Whelan, 2014; Visvalingam, 2015).

Generalisation of geographic information may be implemented in many ways, which differ with respect to the assumed methodology, as well as to the level of automation of the modelling process. Therefore, the answer to the question concerning the possible compromise between the cartographic modelling process and its automation, is an important challenge of contemporary cartography.

In the authors' opinion, it is possible to find a compromise solution using the so-called computational intelligence (CI), and, in particular, artificial neural networks and fuzzy inference systems. This solution comprises a specific "transfer" of subjective, cartographic knowledge to a digital tool, which will automatically specify the adequate method and its parameters, in the process of spatial data modelling for a given level of details (LoD).

Following the IEEE Computational Intelligence Society, computational intelligence deals with "the theory, designing and utilisation of biologically inspired computational methods, with particular respect to neural networks, genetic algorithms, evolution programming, fuzzy inference systems and hybrid systems" (http://ieee-nns.org/). As opposed to the conventional meaning of the artificial intelligence, computational intelligence does not utilise algorithms based on symbolic representation of knowledge.

2. Inspiration

2.1. Computational intelligence methods in the generalisation

The studies on the use of computational intelligence methods in the generalisation of geographic information were inspired both own publications (Olszewski, 2010; 2011) and works published by other authors. Modules of generalisation of geographic information using expert systems, artificial intelligence. Neural networks and machine learning were the subject of studies by various authors (Muller, 1990; Buttenfield and McMaster, 1991; Armstrong, 1991; Mark, 1991; Laurini and Thomson, 1992; Meyer, 1986; Schylberg, 1993; Meng, 1993; Liu et al., 2001; Han and Miller, 2009). These authors pointed out the necessity to formalise cartographic knowledge that is necessary for the creation of properly functioning expert systems. Nickerson (1991) suggested that such knowledge should be obtained through the analysis of existing maps.

The application of ANN and FIS for the purposes of generalisation of linear objects was directly inspired by the works (Zhou and Jones, 2005). The WEA algorithm developed by these authors requires the determination of at least 4 method parameters for each of the analysed points that are characteristic for the line subject to simplification. This allows for the creation of a four-dimensional non-linear regression function with use of ANN and FIS. Thus, the creation of a cartographic knowledge base in such case means the optimisation of a multi-dimensional response function of the generalisation system. Computational intelligence methods are perfectly suitable for this purpose.

The objective of the performed research works was to develop a methodology of the simplification of linear objects with use of the well-known WEA (Weighted Effective Area) algorithm (Zhou and Jones, 2005) and the selected computational intelligence methods: artificial neural networks and fuzzy inference systems. The authors proposed to modify the multi-parameter WEA algorithm, substituting theweights calculated by multiplication with the weights computed by knowledge base, integrated with a computational engine, which utilises the fuzzy logic and/or artificial neural networks (Olszewski et al., 2011). This approach allows for the development of a knowledge base concerning cartographic generalisation methods, basing on two different approaches:

– the approach which utilises machine learning methods (implicite methods) consisting in the acquisition of examples of correct solutions,
– the approach which defines open, however purposefully "fuzzy" generalisation rules (explicite methods).

Utilisation of both approaches allows for comparing the obtained results. Basing on the creation of the knowledge base, which utilises computational intelligence methods, the proposed solution enables the selection of nodes basing on the weights determined with the knowledge base.

The inference engine, in both approaches, is the knowledge base, which utilises computational intelligence methods. Depending on the knowledge acquisition ways and types of its formal representation in the knowledge base, the computational system is based on the utilisation of artificial neural networks (the implicite knowledge specified as a set of examples of correct solutions) or on fuzzy inference (the explicite knowledge specified in the form of open, fuzzy rules, which utilises so-called, linguistic variables and membership functions) The duality of the proposed solution allows not only for a diversification of the way of acquisition and representation of cartographic knowledge required for the automation of the generalisation process, but also for an objective comparison of obtained results.

2.2. Fuzzy Inference Systems (FIS)

The theory of fuzzy sets, proposed by Zadeh (1965, 1973) assumes that the following components are used for description of the system operations:

- the so-called, linguistic variables (e.g. big, small, about half, enough, rather important),
- the so-called, fuzzy conditional clauses, which expresses relations between linguistic variables in the form of IF-THEN rules, e.g. if the number of inhabitants is big, then the city is important,
- the so-called, complex inference rules that allow for the induction of resulting values basing on the knowledge of the primary variable (e.g. if dinner served in a restaurant is tasty) and relations between variables (e.g. if dinner is tasty and the waiter is nice, the tip will be high).

The vast majority of cases in which fuzzy logic is applied are connected with the widely understood process of control. The so-called fuzzy rules calculus is applied, in which relations are expressed in the form of IF-THEN rules with predecessors and consequents, which contain linguistic variables.

2.3. Artificial Neural Networks (ANN)

Neural networks have become more widely used as a result of their useful values for modelling and calculations. Advantages of the ANN may be specified as follows (Tadeusiewicz, 1998; Patterson, 1996):

- the ANN allows for a relatively simple creation of complex, non-linear models, "learning" at the same time with the use of presented examples. This method does not require to assume, a priori, any presupposition concerning the shape and the non-linearity level of the created regression function,
- neural networks are characterised by a high level of resistance to errors. Information used for learning may be incomplete or erroneous. The neural network, which has been taught correctly, may effectively filter noises and perform calculations basing on resultant tendencies and trends,
- an important advantage of utilisation of neural networks is the simplicity of their use. ANNs operate as the "black box": question – answer. In practice, models required by the user are constructed by the neural networks themselves, since they learn using the specified examples. The process of learning substitutes programming. As a result, computational tasks may be solved without the knowledge of algorithms, having the test set, containing "questions" and "answers" only,
- an appropriately prepared (taught) ANN is characterised by the ability to generalise, i.e. the ability to generalise the acquired knowledge,
- many types of neural networks exist, which differ by the structure and rules of operations. The mostly applied regression networks include so-called multilayer perceptrons (MLP) and networks of radial basis functions (RBF).

3. Research – Extended WEA Metric

3.1. Idea

During the research works the authors developed the knowledge bases, using two (implicite and explicite) methods for the system of generalisation, which support operations of the WEA algorithm. This algorithm, being the extension of a standard, one-parameter EA procedure (Visvalingam-Whyatt, 1993), uses several parameters for the needs of evaluation of the importance of particular vertices (Zhou and Jones, 2005) – Figure 1:

- the elementary triangle area,
- the triangle's flatness (calculated on several ways),
- the triangle's skewness,
- the convexity.

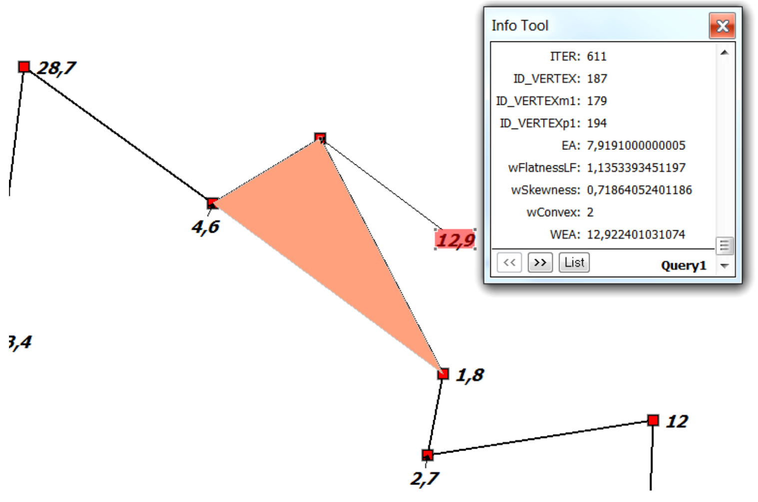

Fig. 1. Determination of the geometrical parameters of individual triangles (WEA value was calculated by means of "classical" multiplication of weight

In the original approach, Zhou and Jones (2005) propose to express the resulting weight of particular characteristic points of a line object using the formula (1):

$$\mathrm{WEA} = \mathrm{W}_{\mathrm{Flat}} * \mathrm{W}_{\mathrm{Skew}} * \mathrm{W}_{\mathrm{Convex}} * \mathrm{EA}. \tag{1}$$

Research works performed by the authors of the present work were based on the assumption that instead of determining WEA by multiplying particular parameters, it is possible to develop the cartographic knowledge base of the generalisation system, which can substitute multiplication with more sophisticated method. This base, after the required "transfer" of the cartographic knowledge to the system (based on either: neural network or fuzzy rules), will allow for implementation of the expert system, which will determine the importance of particular points creating the linear object.

3.2. Data

The authors used the following data to conduct the analyses:

– as source data subject to the generalisation process: SABE (Seamless Administrative Boundaries of Europe) 200 data of a geometrical accuracy equivalent to that of maps in the scale 1:1 000 000,
– as reference data: small scale digital maps for Europe: 1:5 000 000 and 1:17 500 000.

SABE data were chosen because of their accessibility and comparability at the unified LoD. 4 test areas were selected to represent different types of costal lines: from a simple one to very complex fjords.

The authors transformed all input data to a homogenous set of coordinates with use of the conformal representation for the territory of Europe. This enabled us to unify the values of the determined geometrical coefficients characterising individual elementary triangles.

Small-scale reference data were used both as a source of examples for the teaching of artificial neural networks and as a reference cartographic image to which the obtained generalisation results were compared.

Access to vector data for a fragment of borders of four European countries enabled us to create a set of the number of vertices creating individual coast lines. This in turn allowed for precise control of the line simplification process with use of the modified WEA algorithm, by means of selecting the number of iterations corresponding to the number of removed points, as the objective of the test was to obtain a cartographic representation of the coast line consisting of the same number of characteristic points as in the case of the reference data.

The analysis of the Table 1 clearly demonstrates that, due to the high variety of source data, the process of reducing the geometrical complexity of individual coast lines is of a non-linear nature. This also means that the determination of the parameters of the generalisation method requires a different approach for each specific test area.

Table 1. Number of vertexes in the test data of different levels of details

Test area	LoD1 (SABE 200)	LoD2	LoD3
Iceland	695	85	18
Great Britain	506	49	15
Norway	1245	77	32
Poland	115	36	16

3.3. Application

The authors of this article developed a custom application that enables to determine the geometrical parameters of individual vertices for the input data, to select parameters for the “classic” WEA algorithm in compliance with the description presented in the article by Zhou and Jones (2005): M, N, KS, KH, type of filter (LP, HP), SM, SK and the convexity coefficient. Moreover, the tool developed by the authors also enables to control the generalisation process with use of an external inference engine using artificial neural networks or fuzzy inference systems.

For each of these approaches the authors used completely different methods of creation of a cartographic knowledge base for the system.

3.4. Implicite Knowledge Base Using the ANN

The original concept of the authors was to develop a unified neural network of a complex structure that would enable both a cartographically correct generalisation of the relatively simple Polish coast line and a simplification of the geometrically complex Norwegian fiords. In order to perform these tasks values of the geometrical parameters of individual triangles were determined for each reference line in all four test areas, on two generalisation levels LOD2 and LOD3. This allowed us to obtain nearly 500 learning examples for teaching MLP and RBF type neural networks. The applied input data included: the elementary triangle area, the triangle's flatness, the triangle's skewness, the convexity, and the denominator of the scale of the reference map was used as the input variable. The original set of teaching data obtained from the analysis of LoD2 and Lod3 was extended by several hundred examples obtained in the course of analysis of selected LoD1 points, which allowed to obtain a total of 896 records for teaching the artificial neural network.

Basing on a set of input data prepared in such a way the authors separated teaching and validation data and developed over ten neural networks of varied architecture (MLP, RBF), number of hidden layers (1, 2, 3) and a varied number of neurones in the hidden layers (between 8 and 326). However, the obtained results were not completely satisfactory, as the differentiation of the learning data resulted in the fact that the network that was intended to be "general" did not differentiate between the specificity of coast lines of Poland, Norway, Iceland and Great Britain in the course of the generalisation process (Figures 5, 6).

Thus, the authors decided to develop four sets of neural networks whose specific features resulted from the geometry of individual coast lines. For teaching each of these neural networks a set of 30% source points was selected from the original LoD1 line, to which the WEA value was assigned manually in the process of iteration-based determination of algorithm coefficients (EA, skewness, flatness, convexity), preferring specific vertices of the coast line (for the coast line of Poland, Norway, Iceland and Great Britain). Such grouping of teaching data allowed for the proper training of neural networks dedicated to the specific geometry of Polish coast line. In order to obtain an adequate response function ANN for the remaining test areas it was necessary to add some further examples to the teaching data. It should be noted that the complexity of the neural network and the learning time increased along with the growth of the set of teaching data and the computational efficiency decreased (as more complex networks operate more slowly).

3.5. Explicite knowledge base using FIS

The fuzzy rules system was based on fuzzified values of the coefficients used in the original WEA algorithm (Figure 2). WFlat, WSkew, WConvex, EA, resulting from the geometry of the analysed triangle. In the first approach, each of the coefficients was assigned "small" and "large" linguistic variables determined with use of the Gauss curve, of the maxima within the limits of the range of variable data value (determined basing on the first iteration of the classic WEA parameter for all test areas). Only the convexity filter, which adopts discreet values (here: 1 and 5) was determined by triangular membership functions without a common part. However, the Wconvex variable has been finally eliminated from the fuzzy rules, as it interfered with the obtained results by preferring points located on convex "turns" of the coast lines and overly eliminating points from the concave areas.

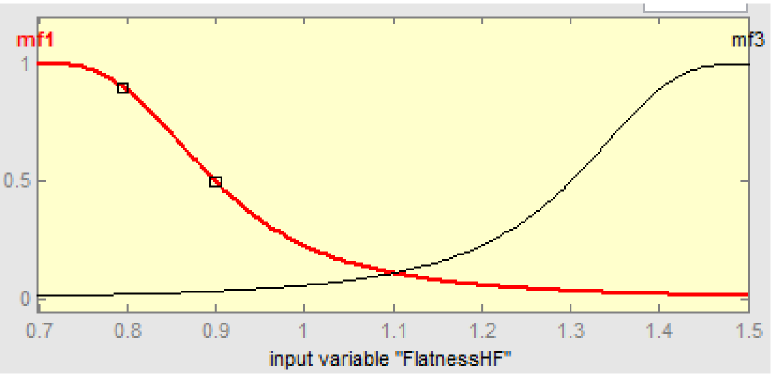

Fig. 2. Membership functions describing the linguistic variables of the flatness filter

Another element that posed some difficulties was the determination of value ranges of the remaining variables. The initially used approach (minimum and maximum values calculated for all sets) led to the fact that in some areas one of the variables always remained "small" (this refers to the area for which the minimum value of the variable was adopted and the remaining values did not diverge significantly from the minimum) and in other areas only "large" (analogically). This resulted in the fact that the weights obtained as a result had identical values for all or for a significant majority of the points. Thus, in this form they were useless. Moreover, the variable value ranges differed significantly not only between different test areas but also between individual iterations. This referred in particular to the area of the triangle – a smaller number of points in subsequent iterations implied a significant increase in areas.

Due to the related problems with obtaining unambiguous results, the authors have decided to dynamise the value ranges. In this approach, the minimum and maximum value of WFlat, WSkew, EA were calculated each time (with each subsequent iteration). Values of linguistic variables determined in such dynamic way constituted "input material" for the steering file and FIS inference engine containing the rules and definitions of linguistic variables. Such solution enabled to adapt the definitions of linguistic variables not only to the test area but also to the iteration, i.e. to the specificity level, while the automation of this action ensured higher universality of the rules.

The finally adopted system of rules was extremely simple, as it consisted of only 3 rules, each using only one variable (Figure 3). Sample decision areas resulting from two of the variables are presented in Figure 4.

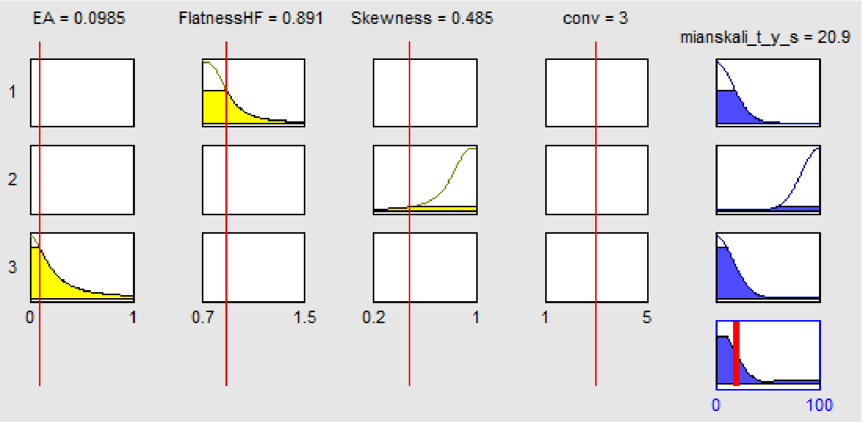

Fig. 3. Graphical illustration of fuzzy interference rules

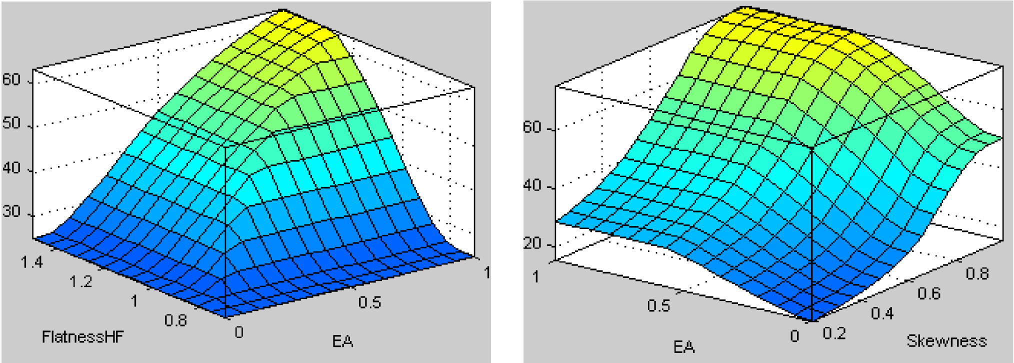

Fig. 4. Decision surfaces of FIS for different pairs of attributes

Apart from the experiments connected with changes in the membership functions and fuzzy rules, the influence of the general settings of the fuzzy system on the obtained results was also tested (although the methods of realisation of fuzzy operators “and” and “or” were insignificant due to the simplicity of the applied rules). In most of the cases the best results were obtained using the default settings, i.e.:

– the implication method – minimum
– the aggregation method – maximum,
– the defuzzification method – centroid.

Only in the case of the generalisation of Norwegian coast line to the accuracy level LOD3 the implication method was changed (to: "product" which scales the output fuzzy set) as well as the aggregation method (to: "probabilistic OR"), which improved the generalisation result.

4. Discussion

The obtained results (Figure 5, 6) demonstrate that a potentially simple optimisation of a four-dimensional regression function determining the value of "importance" of specific points on coast lines is by no means a trivial issue, regardless of whether neural networks or fuzzy inference systems are used.

In comparison to manual generalisation, which characterises small-scale reference data (LoD2 and LoD3 represented, respectively, in red and in brown), all tested automatic generalisation algorithms differ quite significantly. Obviously, for Polish coast line, which is quite uncomplicated from the geometrical point of view, these differences are small and in most cases negligible, but for the remaining test areas, in particular the Norwegian coast line, the automatic and manual approaches yield different results. The results of generalisation with use of knowledge bases NEURO and FUZZY were compared not only to classical cartographic manual generalisation but also to the commonly used global line simplification algorithm of Douglas–Peucker (DP) and the "classic" WEA algorithm with the application of "medium" parameters used in this method.

Although the use of a very simple structure of fuzzy inference system based on only three linguistic variables and the simplest rules enables to obtain a non-linear system response function, and thus the generalisation of individual coast lines, the obtained results are far from satisfactory. This is particularly noticeable at a high level of geometric complexity reduction (LoD3) and for areas with a complex geometry (Figure 5, 6).

However, an advantage of this approach is the simplicity of the created model – the explicit, yet purposefully fuzzy determination of inference rules and a high computational efficiency that also results directly from the simplicity of the system.

However, obtaining cartographically correct results would require a significant parameterisation of the process by means of:

- creating FIS systems dedicated to individual objects (categories of geometric complexity),
- parameterisation of such systems through an appropriate selection of membership functions and inference rules.

A visual comparison of the obtained results with classical cartographic maps (on an analogical scale) allows to state that the artificial neural networks approach brings much better results in the aspect of cartographic accuracy then fuzzy approach. However, the results of ANN "looks better" then FIS, it is also much more complex and time-consuming. This is due to the fact that the appropriate functioning of the neural network required the preparation of huge sets of teaching data, numerous adjustments and supplementations of these sets and teaching MLP and RBF type networks. The high complexity of these networks (the number of hidden layers and neurones in these layers) leads in turn to the fact that the computational efficiency of such system is lower by a range from that of the classic WEA algorithm and/or the application of the FUZZY approach.

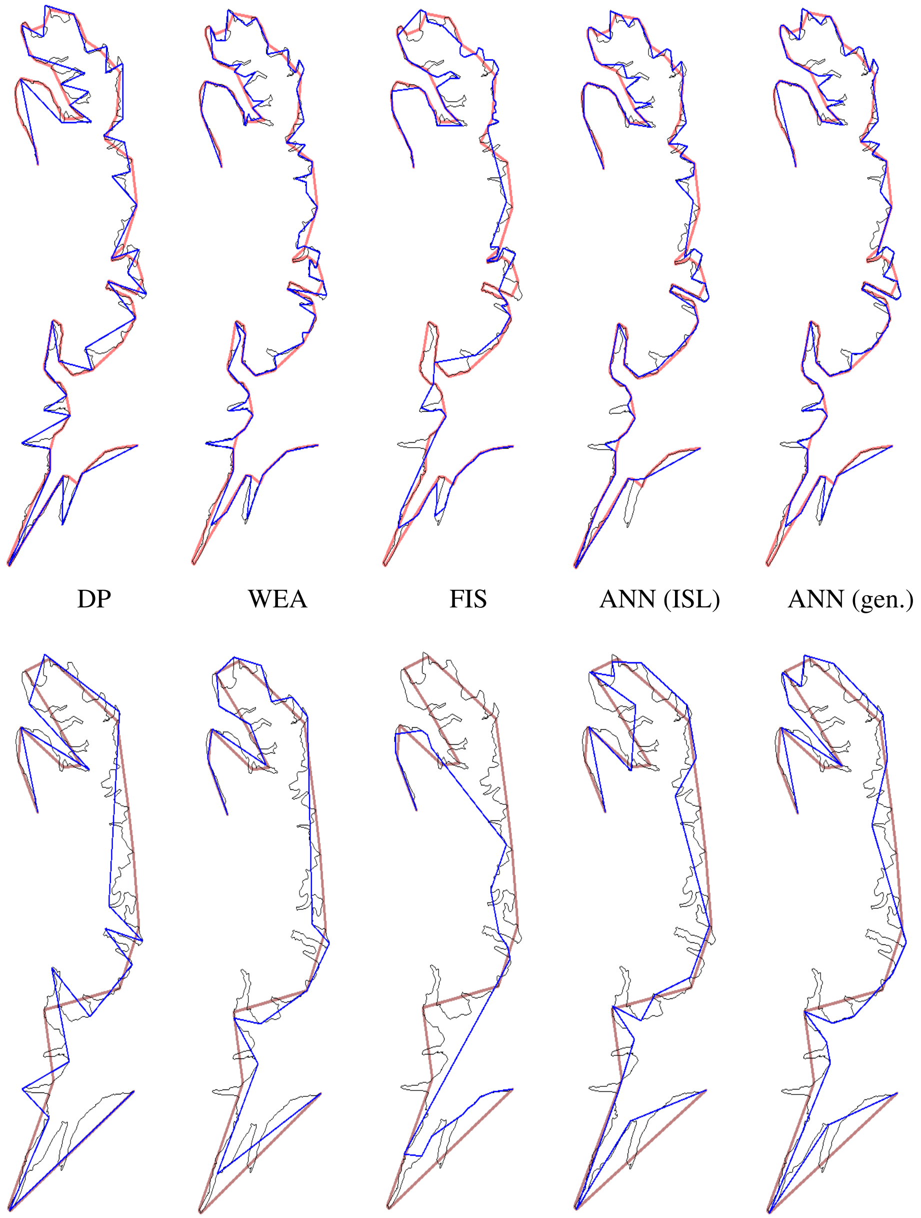

Fig. 5. Achieved results – Island: upper row – LOD2, lower row – LOD3; black – source data, red – model solution, blue – experiments results. Different kind of neural networks were used depending on the LOD: for LOD2 – MLP, for LOD3 – RBF

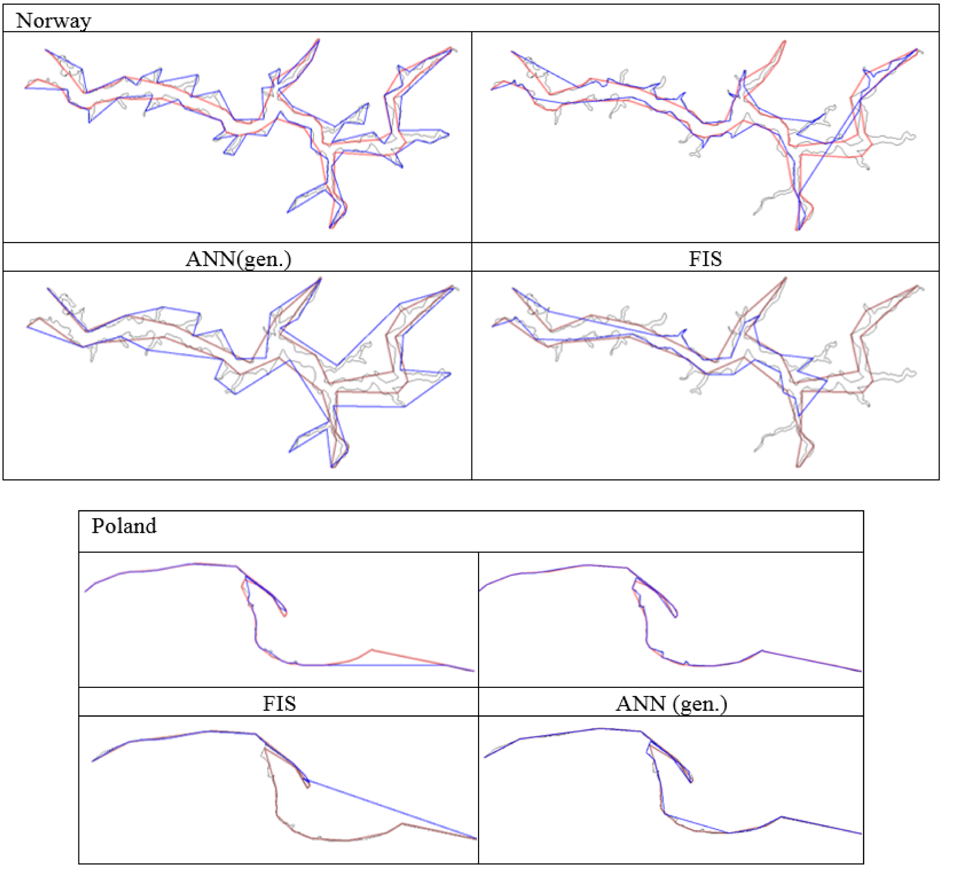

Fig. 6. Achieved results – Norway (top), Poland (down): upper row – LOD2, lower row – LOD3; black – source data, red – model solution, blue – experiments results. Different kind of neural networks were used depending on the LOD: for LOD2 – MLP, for LOD3 – RBF

5. Conclusion

Although the original objective of the authors of the study, which was to develop a complex cartographic knowledge base explicite (with use of FIS) and implicite (basing on ANN) that would allow for a cartographically correct simplification of any geometrical objects has not been achieved, the obtained results should nevertheless be considered as valuable.

The performed research works proved the potential usefulness of computational intelligence and construction of the knowledge base in the geographic information generalisation process. The obtained results are strongly influenced not only by the selection of teaching examples (their amount and quality), but also by the complexity of the neural network, its architecture (MLP, RBF), the number of neurones in hidden layers, applied algorithms, time of teaching the ANN, division of data into teaching and validation sets, the level of smoothing of radial functions, the nature of source data etc. The determination of these parameters, being essential for the formalisation of the system knowledge base, is to a wide extent a subjective process, which may be automated only partially. It should be emphasised that the application of artificial neural networks enables to create very complex, multi-dimensional non-linear regression functions, which constitutes a basis for controlling the generalisation process and for obtaining credible results (although at the cost of a significant amount of computation time). Further research should provide an answer to the question whether it is more effective to create complex, multi-layered neural networks of a "general" nature, allowing for the generalisation of various types of objects or to construct ad-hoc relatively simple networks for the generalisation of specific objects or classes of objects (such as lowland rivers, borders of forest areas etc.). The other interesting aspect would be a comparison of analyzed methods with contemporary simplification algorithms for instance Raposo. The authors are also planning to analyse other data mining techniques, such as regression decision trees, in the aspect of their usability with respect to multi-criteria simplification of objects.

On the other hand, the analysis of fuzzy inference systems demonstrates their high computational efficiency and easiness of interpretation. Individual rules explain the importance of the parameters of the method in an explicit (although purposefully fuzzy) way. However, the selection of the membership function, defuzzification method etc. require specific expert knowledge not only in cartography but, first of all, in the fuzzy logic domain. The initially obtained results presented in this study point to the necessity to construct FIS systems dedicated to specific object classes (such as the lowland rivers or borders of forest areas listed hereinabove), characterised by a more complex structure and a higher number of decision rules.

Both fuzzy inference systems and artificial neural networks may be applied for the purposes of wide analyses and spatial data mining (Olszewski, 2009). Each of these approaches has its advantages. Artificial neural networks are characterised by a natural ability to generalise knowledge acquired as a result of analyses of specified examples; however, it is difficult to interpret the process leading to results generated by the appropriately trained ANN. In order to simplify interpretation, the process of extraction of rules, which openly describe operations of the modelled system, is often applied. It is an approach similar to defining IF-THEN relations in the fuzzy inference method. This type of approach is referred to as neuro-fuzzy modelling. Extracted symbolic rules enable to neglect unimportant factors and to extract the basic components of the model being constructed (Gopal et al., 2001). However, the optimisation of obtained solutions is important. It is particularly visible in rule based systems. It is much easier to optimise several rules, which create the computational engine of the system is much easier than to optimise the operations of a multi-layer, complex and specially trained artificial neural network.

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