Krzysztof WIERZCHOLSKI*

SLIDE BEARING WEAR PROGNOSIS FORMULATED AS THE SOLUTIONS OF RECURRENCE EQUATIONS

PROGNOZA ZUŻYCIA ŁOŻYSK ŚLIZGOWYCH FORMUŁOWANA JAKO ROZWIĄZANIE RÓWNANIA REKURENCYJNEGO

Key words:

wear depth in operating times, HDD micro-bearings, recurrence algorithm

Słowa kluczowe:

głębokość zużycia w czasie, mikrołożyska w HDD, algorytmy rekurencyjne

Summary

In this paper was elaborated the simulation method of determination of anticipated wear of HDD micro-bearings by virtue of Bharat Bhushan and Chizhik experiments. Measurements of micro-scale and macro-scale wear depth show that values on both scales of all the silicon samples are about the same among different silicon materials and higher than of SiC. In both materials the wear depends of normal load, number of time cycles and dynamic conditions. On the basis of experiments it is visible that the tendencies of wear depth

* Institute of Mechatronics, Nanotechnology and Vacuum Technique, Technical University Koszalin, 75-620 Koszalin, ul. Racławicka 15–17, Poland; e-mail: krzysztof.wierzcholski@wp.pl.

increases versus time periods or time cycles are different for various materials. On the ground of mentioned measured values and using the recurrence dependencies, was elaborated the algorithm of wear values in HDD micro-bearing in arbitrary long time period after exploitation beginning.

Coefficients of recurrence equation are obtained in experimental way. These coefficients contain wear corrections resulting from the vibrations, unsteady load and surface roughness asperities measured from the nominal mean level. The obtained solutions of wear recurrence equation are assigned to the regions of coefficient-values for which the wear process converges to stability limit wear value or tend to larger and larger wear values during the exploitation process. This fact enables to see a connection between roughness, material properties in one side and the wear values or wear increases in succeeding time units of exploitation process in other side. Moreover on the basis of mentioned analysis we can anticipate and optimize the least wear of slide HDD micro-bearing.

INITIAL INFORMATION ABOUT MICROBEARING WEAR PROGNOSIS

The wear studies in the micro- and nano-scale have been conducted on actual or simulated MEMS components. The MEMS devices in which wear issues are critical are HDD micro-bearing and micro-motor [**L. 1**–**5**]. Solutions of the presented problem can be possible on the basis of recently obtained measurements of micro-bearing and micro-motors wear during the first number cycles or time periods (may be month or years) of exploitation. The function between wear depth and number of cycles or time periods depends on normal load, and the kind of lubricant, and the kind of material of cooperating sample or bearing surfaces [**L. 1**–**3**, **5**]. The experimental dependencies are presented in **Fig. 1.**

After experimental AFM measurements it follows that the sequence of discrete wear values $\{f_{n+2}\}$ for n = 1,2,3,…, i.e. the values of a decrease in nm of HDD micro-bearing journal diameter equal to the sum $(f_{n+1} + f_n)$ of wear in two foregoing successive time units (possibly in months) multiplied by a dimensionless average stochastic coefficient $0 < a \le 1$ plus some exponent dimensional stochastic function bD^n **[L. 1, 2, 5].** Coefficients D, a, b depend on the experimentally obtained normal loads, the roughness of surfaces, the micro-hardness of micro-of bearing material, the journal angular velocity, and the frequencies of vibrations **[L. 1, 2, 4, 5].** Variable n is numbered by natural numbers 1, 2, 3,…. The dimensionless constant and n independent random parameter $0 < a \le 1$ denotes any coefficient which averages the wear in two succeeding foregoing time units **[L. 5]**. Power term bD^n with index n as exponential function make up the negative or positive differences between the real wear in the succeeding time unit and wear $a(f_{n+1} + f_n)$ obtained from the mentioned average values **[L. 1, 2].** In this case wear of HDD micro-bearing, this can be described by the following recurrence equation:

$$f^*_{n+2} = a(f^*_{n+1} + f^*_n) + bD^n \quad \text{for} \quad n = 1, 2, 3,.... \tag{1}$$

Recurrent Equation (1) determines analytical formula $\{f^*_n\}$ presenting a sequence of wear values numbered for n = 1, 2, 3, … time units, if we know dimensionless values D[1], a [1] and dimensional value b[nm]. To solve this problem, it is necessary to add the boundary conditions **[L. 5]**. Hence, we assume that in two first time units (possibly in months) and the wear by virtue of experiments attains dimensional values W_1[pm], W_2[pm].

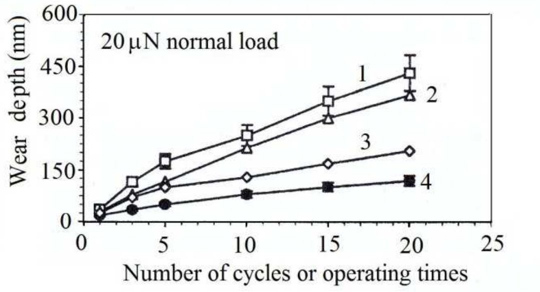

Fig. 1. Wear depths as a function of normal load and as a function of cycles or operating times for various samples: 1 – undoped Si(100), 2 – undoped polysilicon film, 3 – n-type polysilicon film, 4 – SiC film

Rys. 1. Głębokość zużycia jako funkcja obciążenia normalnego oraz jako funkcja liczby cykli czasu eksploatacji dla różnych próbek: 1 – nieaktywny krzem Si(100), 2 – nieaktywna błona poślizgowa, 3 – n-ty rodzaj błony polisilikonowej, 4 – błona węglika krzemowego

THE BASIC PRINCIPLES OF SOLUTION

The general solution of Eq. (1) has the form **[L. 6]**:

$$f^*_n = C_1\chi_1^n + C_1\chi_2^n + f^b_n, \text{ for } n = 1, 2, 3,\dots \tag{2}$$

with
$$\chi^2 - a\chi - a = 0 \tag{3}$$

where C_1, C_2 arbitrary constants and $\{f^b_n\}$ denotes particular solution of non-homogeneous equation. Symbols χ_1, χ_2:

$$\chi_{1,2} = \frac{a}{2} \pm \sqrt{a + \frac{a^2}{4}}, \quad \text{for} \quad 0 < a \le 1, D_2 \le \chi_{1,2} \le D_1, \quad D_{1,2} = \left(1 \pm \sqrt{5}\right)/2 \tag{3}$$

denote only real roots of characteristic equation (3). By imposing the boundary conditions $f_1^* = W_1$, $f_2^* = W_2$ on the general solution (2), we obtain the following system **[L. 7]:**

$$C_1\chi_1 + C_2\chi_2 + f_1^b = W_1, \quad C_1\chi_1^2 + C_2\chi_2^2 + f_2^b = W_2 \tag{4}$$

The system of equations (4) determines the unknown constants C_1 and C_2.

THE CASE OF WEAR STABILIZATION AND CONTINUOUS INCREASES

For the following assumptions **[L. 1**, **2]**:

$$\chi_1,\chi_2 \neq D,\quad \chi_{1,2} = \frac{a}{2} \pm \sqrt{a + \frac{a^2}{4}}, 0 < a < \frac{1}{2},\ \frac{1}{2} < a \leq 1 \quad \text{i.e.} \quad a \neq \frac{D^2}{D+1},\ D_2 \leq D \leq D_1 \tag{5}$$

the sum of solutions (2) after N time units has the following form [**L. 6**, **7**]:

$$\sum_{n=1}^{N} f_n^* = C_1\chi_1 \frac{1-\chi_1^N}{1-\chi_1} + C_2\chi_2 \frac{1-\chi_2^N}{1-\chi_2} + \frac{bD}{D^2 - aD - a}\,\frac{1-D^N}{1-D} \tag{6}$$

EXAMPLE 1

In two successive time units, the HDD micro-bearing journal diameter decreases W_1 and W_2 nm **[L. 1, 2]**. Now we can determine the wear after infinite time units using measurements and stochastic data presented by a[1] = 1/6, D[1] = 1/3 for arbitrary b [nm]. From formulae (4), (5), (6), we obtain the following exploitation parameters:

$$a \neq \frac{D^2}{D+1} = \frac{1}{12}, \chi_1 = \frac{1}{2}, \chi_2 = -\frac{1}{3},\quad C_1 = \frac{24}{5}b + \frac{4}{5}W_1 + \frac{12}{5}W_2,\quad C_2 = -\frac{9}{5}b - \frac{9}{5}W_1 + \frac{18}{5}W_2 \tag{7}$$

By putting the data (7) in Formula (6), the sum of wear, i.e. the sum of the solutions of recurrence Equation (2), in following time units, we attain the following form of convergent wear value sequence:

$$\text{for} \quad N=1, \quad f_1^* = W_1,$$
$$\text{for} \quad N=2, \quad f_1^* + f_2^* = W_1 + W_2$$
$$\text{for} \quad N=3, \quad \sum_{n=1}^{3} f_n^* = 1.16666(6)W_1 + 1.16666(6)W_2 + 0.33333(3)b$$
$$\text{for} \quad N=4, \quad \sum_{n=1}^{4} f_n^* = 1.19444(4)W_1 + 1.36111(1)W_2 + 0.500000b$$
....
$$\text{for} \quad N=10, \quad \sum_{n=1}^{10} f_n^* = 1.249211W_1 + 1{,}497671W_2 + 0{,}745381b$$
.... (8)
$$\text{for} \quad N=\infty, \quad \sum_{n=1}^{\infty} f_n^* = 1.250000W_1 + 1{,}500000W_2 + 0{,}750000b$$

In this case, the increases of the wear has a limit in succeeding time limits during the exploitation and, in a sufficient number of time units, we obtain constant wear values presenting in last row in formula (8).

REMARK 1

Using measurements and data presented by a[1] = 4/5, D[1] = 3/2<D_1 for arbitrary b [nm], we obtain [**L. 1**, **2**, **6**]:

$$\chi_1 = \frac{2}{5}\left(1 \pm \sqrt{6}\right) > \pm 1,\ a \neq \frac{D^2}{D+1} = 2{,}25 \tag{9}$$

In this case, from Formula (6), it follows that, if time unit N = 1, 2, 3,… increases, then wear always increases.

THE CASE OF WEAR STABILIZATION

For the following assumptions **[L. 1**, **2**, **6]**:

$$0 < a < \frac{1}{2}, \quad \chi_1 = D, \quad \chi_2 = -\frac{D}{D+1}, \quad a = \frac{D^2}{D+1}, \quad 0 < D < 1 \tag{10}$$

the sum of solutions (2) after N time units has the following form [**L. 6**, **7**]:

$$\sum_{n=1}^{N} f_n^* = C_1\chi_1 \frac{\chi_1^N - 1}{\chi_1 - 1} + C_2\chi_2 \frac{\chi_2^N - 1}{\chi_2 - 1} + \frac{b\left[D^N(ND - N - 1) + 1\right]}{(D-1)^2(2D - a)} \tag{11}$$

After infinitely many time units, the sum of wear (11) is convergent and tends to the following value:

$$\lim_{N\to\infty} \sum_{n=1}^{N} f_n^* = \sum_{n=1}^{\infty} f_n = C_1 \frac{D}{1-D} - C_2 \frac{D}{2D+1} + \frac{b(D+1)}{(D-1)^2 D(D+2)} \tag{12}$$

THE CASE OF WEAR CONTINUOUS INCREASES

For the following assumptions **[L. 1**, **2**, **.3]**:

$$\left(D = 1, a = \frac{1}{2}\right) \Rightarrow \left(\chi_1 = 1, \chi_2 = -\frac{1}{2}\right) \tag{13}$$

The sum of solution (2) in successive time units has the following form **[L. 7]**:

$$\sum_{n=1}^{N} f_n^* = W_1\left\{\frac{1}{3}N - \frac{4}{9}\left[\left(-\frac{1}{2}\right)^N - 1\right]\right\} + W_2\left\{\frac{2}{3}N + \frac{4}{9}\left[\left(-\frac{1}{2}\right)^N - 1\right]\right\} + \tag{14}$$

$$+ b\left\{-\frac{10}{9}N - \frac{8}{27}\left[\left(-\frac{1}{2}\right)^N - 1\right] + \frac{N(N+1)}{3}\right\},$$

for N = 1, 2, 3, … .

From formula (14), it follows that the wear after the following time units is always divergent, and in the presented case, we attain the following values:

$$\text{for} \quad N = 1, \quad f_1^* = W_1$$

$$\text{for} \quad N = 2, \quad f_1^* + f_2^* = W_1 + W_2$$

$$\text{for} \quad N = 3, \quad \sum_{n=1}^{3} f_n^* = 1.500000 W_1 + 1.500000 W_2 + b$$

$$\text{for} \quad N = 4, \quad \sum_{n=1}^{4} f_n^* = 1.750000 W_1 + 2.250000 W_2 + 2.500000 b \tag{15}$$

....

$$\text{for} \quad N = 12, \quad \sum_{n=1}^{12} f_n^* = 4.44444(4) W_1 + 7.55555(5) W_2 + 38.962(962) b$$

....

In this case, the increases of the wear has no limit in succeeding time limits during the exploitation, and we obtain larger and larger wear values in succeeding time units.

CONCLUSIONS

The algorithm of HDD micro-bearing wear calculations in succeeding time units (possibly in months or years) had been generated on the basis of experimental achievements.

The following new achievements are obtained:

1. The algorithm enables one to find the wear at the in arbitrary time units after the beginning of exploitation.
2. The knowledge of the operating parameters, including the algorithm formula, enables one to recognise the function of wear distribution as a function of continually increasing wear values or as a function presenting stabilising wear values after a sufficient number of time units after the beginning of exploitation.

REFERENCES

1. Bhushan B., Nano-tribology and Nano-mechanics of MEMS/NEMS and Bio-MEMS, Bio-NEMS materials and devices. Microelectronic Engineering, 84, 2007, pp. 387–412.
2. Bhushan B,. Handbook of Micro/Nano Tribology, second ed. CRC Press, Boca Raton, London, New York, Washington D.C., 1999.
3. Jang G.H., Seo C.H., Ho Scong Lee, Finite element model analysis of an HDD considering the flexibility of spinning disc-spindle, head-suspension-actuator and supporting structure. Microsystem Technologies, 13, 2007, pp. 837–847.
4. Wierzcholski K., Enhancement of memory capacity in HDD micro- bearing with hyperbolic journals, Journal of Kones Powertrain and Transport, Vol. 15, No. 3, 2008, pp. 555–560.
5. Wierzcholski K., Chizhik S., Trushko A., Zbytkova M., Miszczak A., Properties of cartilage on macro and nano-level. Advance in Tribology, Hind. Pub. Corp. NY, 2010, http://www.hindawi.com/journals/at/2010/ 243150/.
6. Koźniewska I., Równania rekurencyjne. PWN, Warszawa 1973.
7. Kącki E., Partial differentia equations in physics and technique. WNT, Warszawa, 1989.

Streszczenie

Zużycie mikrołożysk w napędach HDD w kolejnych okresach eksploatacyjnych, takich jak kolejne miesiące, lata jest bardzo istotnym problemem procesu eksploatacji komputerów. Jak wynika z badań doświadczalnych, wartość zużycia zależy od wartości obciążenia, rodzaju materiału trących

się powierzchni oraz ich chropowatości, środków smarujących, a także czasu trwania eksploatacji. W zależności od wymienionych parametrów rozwój zużycia odbywa się mniej lub bardziej intensywnie. Niniejsza praca za pomocą zależności rekurencyjnych przedstawia algorytm symulujący zużycie mikrołożysk HDD w dowolnym czasie trwania eksploatacji, mając do dyspozycji pomierzone wartości zużycia po dwóch kolejnych latach lub miesiącach pracy łożyska oraz dysponując wiedzą o zmianach i wzroście wartości zużycia zaczerpniętą z dokonanych pomiarów doświadczalnych. Współczynniki rozpatrywanych zależności i równań rekurencyjnych są ściśle uzależnione od wymienionych parametrów eksploatacyjnych.