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Estimation of surrounding the spillage time

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Abstract

This article presents the model of oil spill on the grid graph. We consider the three types of grid which parameters are corresponding to condition on the sea. The firefighter problem gives the algorithm to bound the oil spill. We compare the action starting time in model described by sum of N variables with the uniform probability distribution and the action starting time in real situation described by sum of two triangular random variables. For every type of grid we determine the Time Average of Going Around of the Spill.

1. Introduction

The risk of pollution accidents in the Baltic Sea is increasing, because the maritime transport and oil transportation is growing. The Baltic Sea is one of the busiest seas in the world therefore marine safety and response to accidents at sea have high priority. On average *7%* of shipping accidents in the Baltic Sea resulted in some kind of pollution [2]. The number of pollution incidents from 2000 to 2010 is growing (*Figure 1*, *Figure 2*), but there have been no major oil spills since 2003. Recent major pollution accidents in the Baltic Sea were in 2001 and in 2003. On 29 March 2001 during a storm the accident occurred at the maritime border between Germany and Denmark. The oil spill was result from the collision between the bulk carrier “Tern” and the oil tanker “Baltic Carrier”. In the accident was destroyed the tank that contained approximately 2700 tons of oil, around *50%* of oil recovered at sea. On 31 May 2003 north of Bornholm, the Chinese bulk carrier “Fu Shan Hai” collided with the Cypriot container ship “Gdynia” resulting in the release of 1200 tonnes of fuel oil, around 1100 tonnes of oil recovered at sea.

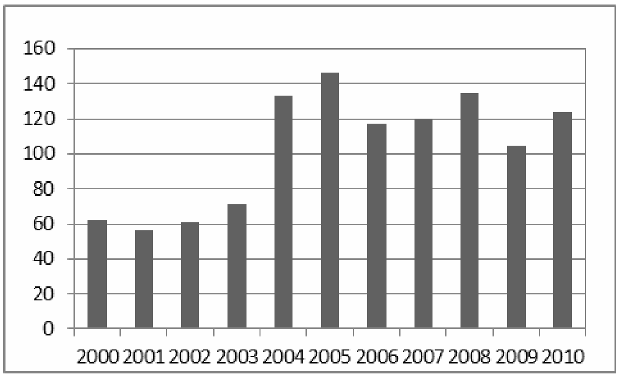

Figure 1. Number of shipping accidents [8]

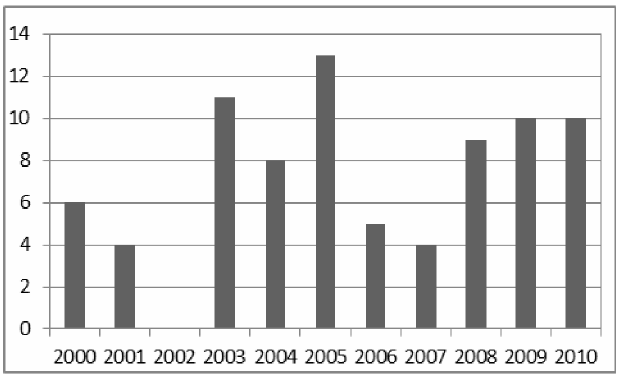

Figure 2. Number of shipping accidents resulting in pollution [8]

The ill effects of the oil spill are being borne not just by people living in the nearby coastal areas but also by marine animals, birds and the vegetation in polluted parts. Oil spills are becoming a major hazard to marine life.

The interest of all the Baltic countries is to improve maritime safety and to prevent pollution from maritime transport. One of the projects in field of oil spill is the Baltic Master II. Baltic Master II, a follow up to Baltic Master I, is a flagship project in the European Union that brings partners from 9 different countries around the whole Baltic Rim; from local, regional and national authorities to research institutes, pan-Baltic organizations and universities. One of the aims of Baltic Master II is to improve the on-land response capacity to oil spills in the Baltic Sea by integrating local and regional perspectives with cross-border cooperation [9].

There is a lot of various ways of oil polluted water purification but the most common the oil spill liquidation method is collection on surface water. The first and the most important stage of the collection is to reduce a spillage. The *Figure 3* presents the action of spillage limit by using the oil barriers.

Figure 3. The action of spillage limit [2]

In this article we use graph model of oil spill [3]. The model shows how to use available forces and resources to reduce a spill. The primary aim is analysis of time to prepare the rescue action in the case maritime oil spill disasters at the sea. This is one of the elements of working on a tool to support shares relating with oil spill. This tool may be useful for a decision maker, provide the necessary information, help in decision making.

2. The stochastic model

We consider the model of oil spill on the graph. In this model there will be utilized the infinite grid, G. A grid consists of a vertex set, $V(G)$, and an edge set, $E(G)$. An every vertex corresponds to rectangular area of the sea and an every edge represents contact between two areas. We have three possibilities to identify contact between two areas. We consider three grids: the Cartesian grid, C, the triangular grid, Tr, the strong grid, S. The Cartesian product of two paths of infinite length forms the Cartesian grid, (*Figure 4*). The triangular grid, also called an isometric grid, is a grid formed by tiling the plane regularly with equilateral triangles, (*Figure 5*). The strong product of two paths of infinite length forms the strong grid, (*Figure 6*). In the grids C, Tr, S an every vertex has 4, 6 and 8 the neighbors, respectively. Note that $V(C)=V(Tr)=V(S)$ and $E(C)\subset E(Tr)\subset E(S)$. The choice of a grid depends on hydro-meteorological conditions. The size and the dimensions of area relating with a one vertex depends on the type of substance spill.

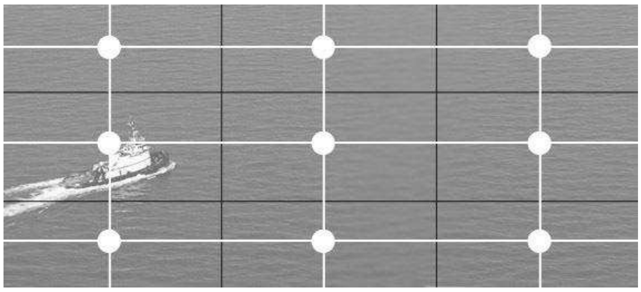

Figure 4. The Cartesian grid model

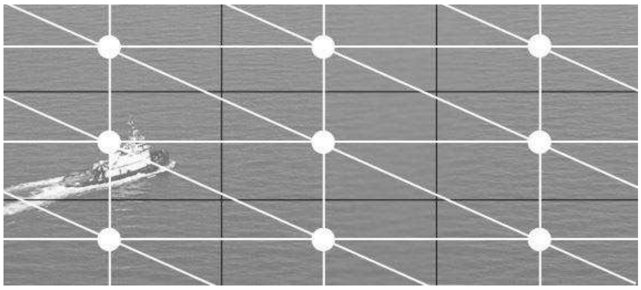

Figure 5. The triangular grid model

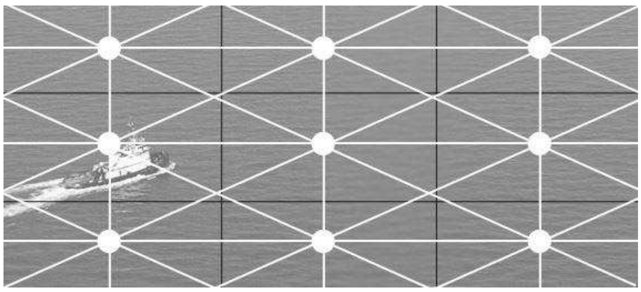

Figure 6. The strong grid model

We define several possible states for nodes. A node affected by oil is a vertex that stays all time in this state. A protected node is a vertex where a dam was placed. Empty node is a vertex neither affected by oil nor with a dam.

In model we use the firefighter problem where the time is measured using cycles. We consider the number of cycles before and after start the action. The oil spill starts at a given empty node (or a set of nodes) at cycle 0. The number of cycles from appearance the spill to starting the action is described by N independent cycles. The number of cycles N is the random variable dependent on many factors. We do not have any information necessary to determine the number of cycles N. We do not know how many cycles are needed to detect the spillage, how many cycles takes to reach the place of spill or how many cycles are needed to prepare the action.

In cycle number $N+1$ begins the action of a spill surrounding. The action happens the same way as a fire surrounding in the firefighter algorithm. At every subsequent time step (cycle), we place available dams and the oil spill spreads deterministically to all empty nodes that have already neighbor affected by oil. According to the algorithm the spillage movements and the action movements are going to alternate. It happens as long as the oil spill is surrounded which means that the spill is unable to spread any further.

Before starting the action one cycle means only the move of spill but after starting the action one cycle means the move of preventive action and the move of spill.

Incompleteness and uncertainty of the data do not allow to determine when started the action and how long is one cycle so we consider the probability distribution every cycle is uniform with parameters a and b, where a and b are the minimum and the maximum duration respectively. The probability density function of the every cycle is equal

$$f_j(x) = \begin{cases} 0 & x \notin (a,b) \\ \dfrac{1}{b-a} & a \le x \le b \end{cases} \qquad (1)$$

where j is number of cycle.

3. The action starting time

In this paragraph we consider the factors having influence on the action starting time in real situation. We assume that the time since oil spill occurs to starting the action is composed of two stages. The first stage is related to information about the spillage and this time is represented by the variable T_1. Master of the ship or other crew member should notify authorities like port state control, oil clean up team. In exceptional situation the accident is not reported immediately. The spillage is detected by rescue unit like Maritime Regional Unit of the Border Guard. When rescue units are given information about oil spill, begins next steps. The time of second stage T_2 depends on the following elements: the time needed to locate the spill by the rescue units, the time needed to collect forces and resources, the time needed to get to the spill space.

We use the triangular distribution to describe the action starting time. The triangular distribution is typically used as a subjective description, in decision making, in simulations, generally for model in which there is an interval defined by minimum value, maximum value and the most common value in an expert's opinion. The *Figure 7* presents the density function of the triangular distribution.

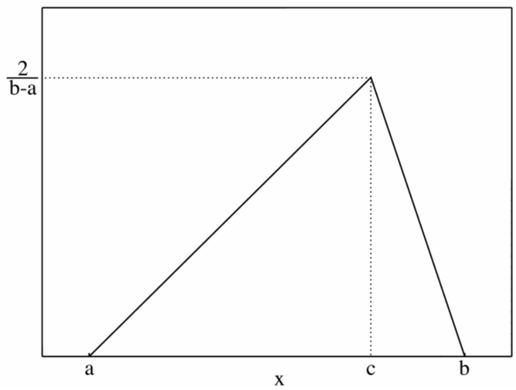

Figure 7. The density function of the triangular distribution

We assume the action starting time is the sum of two triangular random variables T_1 and T_2 [7]. A triangular distribution of a random variable T_1 is a continuous probability distribution with lower limit a_1, mode m_1, upper limit b_1. The lower limit $a_1 = 0$ because at best the information about the spillage comes immediately. The probability density function variable T_1 on the support $0 \le x \le b_1$ is defined as

$$f_1(x) = \begin{cases} 0 & x \notin (0,b_1) \\ \dfrac{2x}{b_1 m_1} & 0 \le x \le m_1 \\ \dfrac{2(b_1 - x)}{b_1(b_1 - m_1)} & m_1 \le x \le b_1 \end{cases} \qquad (2)$$

The probability density function variable T_2 on the support $a_2 \le x \le b_2$ having mode m_2 is defined as

$$f_2(x)=\begin{cases} 0 & x \notin (a_2,b_2) \\ \dfrac{2(x-a_2)}{(b_2-a_2)(m_2-a_2)} & a_2 \le x \le m_2 \\ \dfrac{2(b_2-x)}{(b_2-a_2)(b_2-m_2)} & m_2 \le x \le b_2 \end{cases} \qquad (3)$$

In case $a_1 \le x \le m_1$, $a_2 \le x \le m_2$ the probability density function of the variable $T = T_1 + T_2$ is described by formula

$$f_{12}(x)=k_1\left[\frac{x^3}{6}-\frac{x^2}{2}(a_1+a_2)+\right.$$

$$-x\left(\frac{m_1^2}{2}+\frac{m_2^2}{2}-a_1m_1-a_2m_2-a_1a_2\right)$$

$$-\frac{m_1^2}{2}(a_1-a_2)+\frac{m_2^2}{2}(a_1-a_2)$$

$$-a_1a_2(m_1+m_2)+\frac{m_1^3}{3}+\frac{m_2^3}{3} \qquad (4)$$

where $k_1=\dfrac{4}{(b_1-a_1)(b_2-a_2)(m_1-a_1)(m_2-a_2)}$ for $x \ge m_1+m_2$.

For $a_1 = 0$ the probability density function of the variable T is equal

$$f_{12}(x)=k_1\left[\frac{x^3}{6}-a_2\frac{x^2}{2}-x\left(\frac{m_1^2}{2}+\frac{m_2^2}{2}-a_2m_2\right)\right.$$

$$\left.+a_2\frac{m_1^2}{2}-a_2\frac{m_2^2}{2}+\frac{m_1^3}{3}+\frac{m_2^3}{3}\right] \qquad (5)$$

where

$$k_1=\frac{4}{b_1(b_2-a_2)m_1(m_2-a_2)} \text{ for } x \ge m_1+m_2.$$

We assume that the time from appearance the oil spill to starting the action consist of N cycle where every cycle has uniform probability distribution with density function described by formula (1) and in real condition this time is described by formula (5) so the question is how many cycles are needed to start the action?

The distribution of the number of cycles which pass from start time of spill to action start time is given by formula

$$P(\textit{number of cycle} = N)=$$

$$P\left(\sum_{j=1}^{N} X_j < T < \sum_{j=1}^{N+1} X_j\right)=$$

$$P(X < T < X + X_{N+1})=$$

$$P(0 < T - X < X_{N+1})$$

where

$$T = T_1 + T_2,\ X=\sum_{j=1}^{N} X_j\ ,\ \{X_j : j = 1,2,\ldots\}$$

is a sequance of i.i.d. random variable with the probability density function of the form described by (1). The probability density function of the X is given by

$$f_X(x)= f^{(n)}(x)=$$

$$\begin{cases} \dfrac{1}{(n-1)!(b-a)^n} \cdot \displaystyle\sum_{i=0}^{\overline{n(n,x)}}(-1)^i\binom{n}{i}[x-na-i(b-a)]^{n-1} & na \le x \le nb \\ 0 & \textit{otherwise} \end{cases} \qquad (6)$$

If $Y = T - X$ then

$$P(\textit{number of cycle} = N)=$$

$$P(0 < Y < X_{N+1})=$$

$$P(\{Y>0\}\cap\{Y<X_{N+1}\})=$$

$$\iint_D f_Y(x) f_{X_{N+1}}(x+y)dxdy$$

The probability density function of Y is given by formula

$$f_Y(y)=\int_0^\infty f(x, y+x)dx$$

where $f(x,t)$ is the joint density function of variables T and X.

4. The action time and the spillage area

The firefighting problem has been investigated on the Cartesian grid C in [1], on the triangular grid Tr in [5], and on the strong grid S in [6]. We use the obtained results for analysis the action time of spill surrounding and the spillage area.

Assuming we start the action in $N+1$ cycle and we have 2 dams at each cycle on the grid C, 3 dams at each cycle on the grid Tr and 4 dams at each cycle on the grid S, we know the number of cycles needed to go around the spillage.

Theorem 1 On the Cartesian grid the number of cycles needed to go around the spillage is equal $32N+1$ and the number of vertices of a spill area is equal $318N^2+14N+1$.

Theorem 2 On the triangular grid the number of cycles needed to go around the spillage is equal $18N+3$ and the number of vertices of a spill area is equal $172N^2+58N+5$.

Theorem 3 On the strong grid the number of cycles needed to go around the spillage is equal $24N+7$ and the number of vertices of a spill area is equal $340N^2+218N+35$.

The number of cycles when the action started is the random variable with probability density function given by (6) also the number of cycles needed to go around the spillage is the random variable.

We can use expected value to estimate the Time Average of Going Around of the Spillage, TAGAS, for every type of grid.

For the Cartesian grid:

$$E[TAGAS]=E\left(\sum_{j=1}^{32N+1}X_j\right)$$

$$=E(32N+1)E(X_j)$$

$$=[32E(N)+1]\frac{a+b}{2}$$

For the triangular grid:

$$E[TAGAS]=E\left(\sum_{j=1}^{18N+3}X_j\right)=[18E(N)+3]\frac{a+b}{2}$$

For the strong grid:

$$E[TAGAS]=E\left(\sum_{j=1}^{24N+7}X_j\right)=[24E(N)+7]\frac{a+b}{2}$$

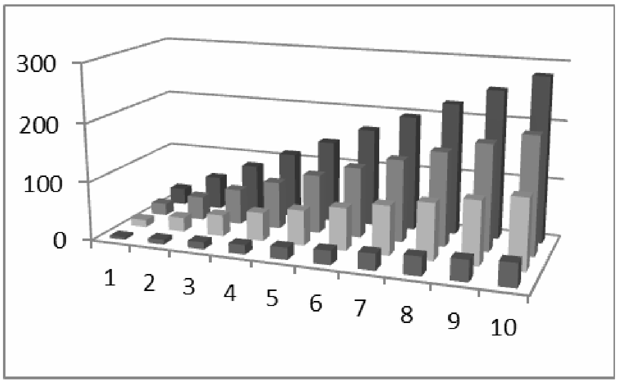

Figure 8. The TAGAS - the C grid

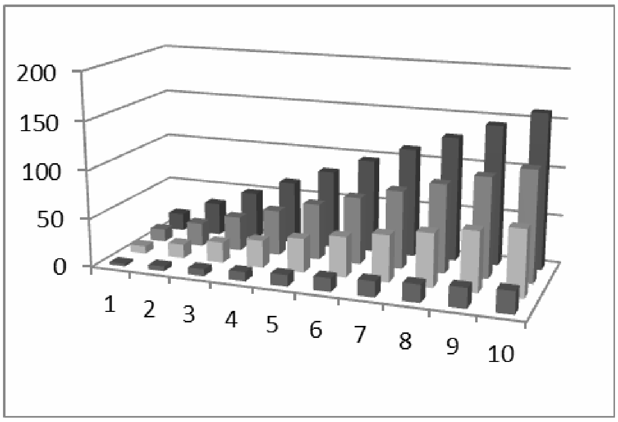

Figure 9. The TAGAS - the Tr grid

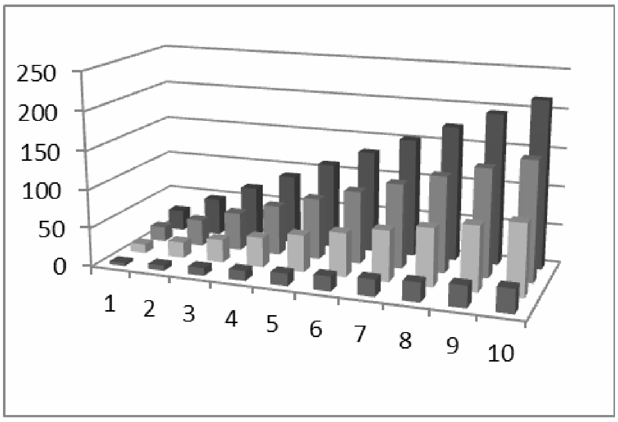

Figure 10. The TAGAS – the S grid

The *Figures 8-10* present plots of the TAGAS being a function of expected number of cycles $E(N)=1,2,\ldots,10$ and depending on the parameters a, b.

In plot we consider following cases of a and b:

$a=0h$ and $b=0,25h$,

$a=0,25h$ and $b=0,5h$,

$a=0,5h$ and $b=0,75h$,

$a = 0{,}75h$ and $b = 1h$.

5. Conclusion

The presented model provides the tools allowing to describe the variety of situation at the sea in case of an oil spill. The three types of grids allow to take into account the prevailing conditions at the sea, a type of substance, hydro-meteorological conditions. The algorithm also provides an opportunity to choice the right amount forces and resources to reduce a spill, corresponding to the real possibilities of available rescue units. The paper presents the Time Average of Going Around of the Spill depending on the parameters associated with the action starting time and the number of forces and resources. Described model allows also to consider issues such as Max Save and Min Budget. The main aspect of Max Save is to minimize a spill area regardless of amount of used forces and cost of a share. The problem Min Budget is to reduce a spill by a minimum cost of share.

This article presents issues that are a part of the work on a tool to support shares relating with oil spill. This tool is built for decision makers, it should allow for an appropriate choice of rescue strategy and help in estimating effects of maritime oil spill disasters at the sea.

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