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MULTI-ATTRIBUTE UTILITY THEORY ANALYSIS FOR BURN-IN PROCESSES COMBINED WITH REPLACEMENT

ANALIZA POŁĄCZONYCH PROCESÓW SZTUCZNEGO STARZENIA I WYMIANY PROWADZONA W OPARCIU O WIELOATRYBUTOWĄ TEORIĘ UŻYTECZNOŚCI

Components from a heterogeneous population may result in non-well behaviour in the failure rate function. This paper considers a population of components that consists of two different sub-populations: a population of weak components and a population of strong components. This component heterogeneity is treated using a mixture distribution for the components' lifetimes. This mixture models two distinct behaviours: a short characteristic lifetime for the weak components and a long characteristic lifetime for the strong components. Simple policies may not be effective to address the distinct behaviours of failures for these components. Thus, combined preventive replacement and a burn-in procedure based on a multi-criteria perspective are proposed in order to suitably integrate the different objectives from the burn-in and preventive replacement procedures, taking into account the preferences of the decision-maker. We consider the cost and the mean residual life as the criteria of the proposed model. Multi-attribute Utility Theory (MAUT) allows alternatives that are more aligned with the preferences of the decision-maker to be developed.

Keywords: burn-in, Multi-attribute Utility Theory, replacement, residual life.

Elementy składowe tworzące niejednorodną populację mogą prowadzić do nieprawidłowości funkcji intensywności uszkodzeń. W prezentowanej pracy badano populację komponentów składającą się z dwóch różnych subpopulacji: populacji komponentów słabych i populacji komponentów mocnych. Niejednorodność komponentów opisano za pomocą rozkładu mieszanego ich czasu pracy. Rozkład mieszany pozwala modelować dwa różne zachowania: krótki czas pracy charakterystyczny dla słabych elementów i długi czas pracy charakterystyczny dla elementów mocnych. Proste strategie konserwacyjne mogą nie dawać oczekiwanych efektów w przypadku komponentów, które różnią się pod względem charakteru uszkodzeń. Aby odpowiednio powiązać odmienne cele procedur sztucznego starzenia (wygrzewania, docierania) i wymiany profilaktycznej elementów składowych zaproponowano, w oparciu o podejście wielokryterialne, procedurę łączącą sztuczne starzenie i wymianę profilaktyczną, która uwzględnia także preferencje decydenta. Jako kryteria proponowanego modelu rozważano koszty i średnią trwałość resztkową. Wieloatrybutowa teoria użyteczności (MAUT) pozwala na tworzenie alternatyw, które licują z preferencjami osoby odpowiedzialnej za podejmowanie decyzji eksploatacyjnych.

Słowa kluczowe: sztuczne starzenie, wieloatrybutowa teoria użyteczności, wymiana, trwałość resztkowa.

1. Introduction

1.1. The context

The search for highly reliable systems has been intensifying. Such systems can be identified by implementing practices that reduce losses due to failures. Thus, maintenance plays a key role in organizations because it can significantly contribute to reducing both costs and failures, irrespective of the timing of failures during the useful life of the component. Preventive maintenance takes effect when the use of a particular policy enables the reduction of potential failures. Because these policies widely influence the safety and economy of operations, they are very important to improve the performance and reduce the cost of any producing systems.

Burn-in procedures consist of testing a new component for a given period before its active life in order to prevent early failures.

Burn-in has been studied by several researchers, including Kuo and Kuo[24], who presented a review of the main aspects of the procedure. Furthermore, Block and Savits [2] provided optimization examples and criteria. Regarding the adopted criteria, Block et al. [3] balanced residual life and variation (conditional survival) in the criterion of burn-in via the residual coefficient of variation, whereas Baskin [1] analysed burn-in using the general law of reliability. Perl-

stein et al.[29] analysed the cost of the optimal duration of burn-in, the components of which were characterized by hybrid exponential distributions using Bayesian theory. Restrictions can be added, as demonstrated by Chi and Kuo [13], who proposed a model to minimise costs under two restrictions, reliability and capacity.

The use of a combined policy is sometimes more effective than the use of a pure policy [13]. An example of combined policy is presented by Golmakani and Fattahipour [19], who aimed to determine the best period for inspection and replacement in the condition-based maintenance. However, research studies involving combined policies of burn-in with replacement are rare, and the ones that stand out are those by Jiang and Jardine [21], Canfield [5], Thangaraj and Rizwam [32], Drapella and Koznik [15].

A heterogeneous population of components that are present early and later fail related to exclusives failure models demands that the policy is adaptable. Therefore, a pure policy or the use of only one procedure is not effective.

1.2. The present contribution

Decisions regarding the burn-in and maintenance decisions are of great interest to researchers and decision-makers, and finding alternatives that improve the performance of managed systems is a challenge.

The vast majority of research studies addresses the problem of burn-in with a single objective function. Although the adoption of combined policies represents an advance, combined procedures sometimes do not provide improvements, especially when the analysis that drove the combination of processes was focused on only one criterion. In fact, each process is characterized by unique features and is applied with specific objectives. Measuring the effect of combination based on only one aspect does not reflect the actual impact of the combined policy [25]. Thus, the use of a multi-criteria approach to build combined policies becomes very prominent.

The use of multi-criteria methods to make decisions on maintenance has been growing exponentially, as shown in a previous study [16]. Therefore, multi-criteria methods constitute an area of interest to the theoretical and practical realms, and this article aims to address this currently poorly explored problem with a joint model of burn-in and replacement based on a multi-criteria approach.

Cavalcante [7] presents a paper that considers a multi-criteria model for a combined burn-in and replacement process for a simple system with the cost and post-burn-in reliability.

This present research differs from the previous paper by considering the cost and mean residual life criteria on the development of a multi-criteria model to improve the burn-in and preventive replacement combined process. The decision is based on the values of burn-in (b) and replacement (y) that maximize the global utility function, furthermore is presented a methodology to apply burn-in and replacement from the MAUT approach. In addition is performed a comparison of three policies: maximizing the residual mean life (Policy I), Minimizing the cost (Policy II) and using the methodology proposed in this study MAUT (Policy III) to aggregate the two criteria. The result of this comparison brings to the decision-maker important insights with managerial impact, what we consider, besides the other aspects, an essential contribution that was not present in previous works.

In addition to the introduction, this article consists of five sections. Section 2 provides a brief review of the concepts of burn-in and replacement. Section 3 presents the multi-attribute utility theory (MAUT) for burn-in and replacement analysis. A numerical application for the model developed with a discussion of management insightsis presented in section 4, and section 5 lists the conclusions.

2. Combined burn-in and replacement procedure modelling literature

The burn-in procedure is based on a screening process that utilizes accelerated aging or simulates the conditions of use of all or a set of these items. The application of this procedure is justified by the assumption that the population of a given set items can be divided into sub-populations of weak and strong items. The weak items tend to fail more quickly, whereas strong items fail due to wear-out much later than weak items.

The burn-in consists of operating the systems or components in situations that simulate the real operating conditions of equipment and / or extreme conditions to which they may be subjected, as demonstrated by [2]. During this process, the items are subjected to high temperatures and a high degree of vibration. Thereafter, items that have resisted and items that have failed (especially the representatives of the weak sub-population) can be identified. Items that resisted are considered to be of good quality, whereas those that failed are discarded or repaired in addition to having their causes of failure analysed.

A series of decisions must be made when implementing burn-in. The first and most important one is the decision to applythe technique, which should be based on the potential effect of burn-in on the items in the system. Specifically, Cha and Finkelstein [11] warn that an unduly long burn-in time may damage items of good quality. The distribution of failures should have a high initial value for the application of the method to be justified. Furthermore, the reduction

in 'infant mortality' (early failures), the changes in the life expectancy of items and application to the entire system or specific components should be considered.

Burn-in is aimed precisely at the period of infant mortality, which is a worrying phenomenon that is responsible for greatly dissatisfied clients who do not accept that a newly acquired asset should present failures. Manufacturers also do not tolerate this poor performance because discontented consumers harm the company's image, as noted by Tangaraj and Rizwam [16] and Wu and Clements-Croome [35].

Furthermore, additional costs are related to the replacement of defective items or repairing them in the field, which increases the cost of warranty services. Generally, the causes of early failures are linked to problems of design, process control, finishing and installation.

Parts should only be replaced if operating costs increase, reliability decreases (a growing failure rate) or items become obsolete over time. Replacements can be conducted in blocks or by age. These approaches are each characterized by specific characteristics and circumstances in which they are more suitable. According to Castro and Alfa [10], block replacement consists of changing a set of items in a certain period, whereas age-based replacement consists of replacing the item after a certain period of time since the start of its useful life or due to the occurrence of failures (whichever comes first). Block replacement is easier to apply but more expensive. Despite the divergences, both modalities are assumed to renew the system.

If cost is adopted as the sole criterion of interest, a replacement policy can be implemented by constructing a model that defines the balance between the cost of preventive replacement and its benefits by determining the age at which the equipment should be changed in order to minimize the total cost expected of substitutions per unit of time. For a model to be able to describe more realistic situations, integrating replacement policies and burn-in by considering multiple criteria can bring significant benefits.

3. Proposed decision making modeling for burn-in and replacement

MAUT is based on axioms established by Von Neumann and Morgenstern [33] and searches to gather important objectives for making the decision based on multiple objectives. It employs the utility function to assess relevant objectives, allowing it to assess trade-offs. The use of MAUT yields a structuring approach to decision-making that can create a consistent decision model.

The key feature of MAUT is the use of utility functions for modelling attributes. Utility theory describes the preference attributes on a scale of 0 (undesirable) to 1 (desirable), transforming the attribute measures to a utility scale in order to allow different attributes that can be compared with a common measurement scale [16].

MAUT maximizes the utility function, and this maximum is found via an elicitation process and must meet the conditions and axioms of the utility function. This approach as been widely used in various contexts, such as in policy analysis and health services, including quality of life analyses, political decisions, and environmental decisions [32, 17].

Chelst and Canbolat [14] proposed six steps for applying MAUT that facilitate the implementation and understanding of the method; these steps are being used in other studies [24] and are described below:

1) Recognize the decision alternatives. The alternatives should reflect the decision problem that is being analysed.
2) The objectives must be listed and reflect the decision problem.
3) Measurably establish the attributes to measure objectives; each objective should have its own attribute.
4) Elicit the decision-maker's preference for each objective of decision based on the importance of each goal. Clarify the pref-

erences of each stakeholder, with reference to the objectives, while reflecting your preferences.

5) Establish a utility function to characterize the decision-maker's preferences regarding the alternatives, having established the function preference for each decision objective. A single utility function is derived and scaled from 0 to 1 to find the global utility function.

6) Analyse decision alternatives for calculating a global utility function: the optimal decision is made by optimizing the global utility function.

MAUT includes both mathematical theory and a series of evaluation techniques. The information obtained from the evaluation serves to classify alternatives, make choices or clarify a situation for the decision-maker [34, 18]. MAUT has been used to aggregate the objective cost and mean residual life in a combined policy involving burn-in and replacement, the cited characteristics of which are perfectly suited to the problem.

3.1. Assessment the attributes

The criteria to be assessed need to be defined in order to define a maintenance policy involving burn-in and replacement with a multi-criteria approach. Thus, the role of the decision-maker is essential. We will discuss the criteria residual average life and cost; the decision variables are represented by b (the time of burn-in) and y (age for replacement).

In this article, the following notation is adopted:

C_f – Cost of replacement per failure
C_p – Cost of programmed replacement
C_r – Cost of repairs during burn-in
C_b – Expected cost of the burn-in
β – Parameter of form
η – Parameter of scale
b – Burn-in time
$C(t)$ – Mean cost per unit of time
$R(t)$ – Reliability of Function
$F_s(t)$ – Function of simple accumulated distribution
$F(t)$ – Function of mixed accumulated distribution
y – Replacement time
λ_1 – utility function parameter
k_1 – scale constant of cost
γ_1 – utility function parameter
k_2 – scale constants of MRL

3.2. Attribute mean residual life

The mean residual life can be interpreted by the decision-maker as an indication of customer satisfaction: the client is more likely to acquire products of the same brand if such products have a long service life and good performance over their lifetime; as a result, the customer is more satisfied with products and plays his role for a given period [22].

The mean residual life function is given by the following expression [4]:

$$\mu(t) = \begin{cases} E\{X - t | X \geq t\} = \int_t^\infty \dfrac{R(x)dx}{R(t)}, & \text{if } R(t) > 0 \\ 0, & \text{if } R(t) = 0 \end{cases} \quad (1)$$

The optimal period of burn-in can be defined by the point at which the corresponding mean residual life reaches its maximum [4]. Because X is the lifetime, we must find a b that maximizes E[X−b|X>b].

Thus, maximizing the MRL (Mean Residual Life) will increase the possibility of the item remaining in working order for longer.

3.3. The attributes of cost

Jiang and Jardine [9] proposed a model that can determine the burn-in period and the optimum replacement interval given the heterogeneous (mixed) population.

The simple accumulated distribution function is given by

$$F_s(t) = 1 - e^{-(\frac{t}{\eta})^\beta} .$$

However, because we are analysing a heterogeneous distribution, $F(t) = p_1 F_1(y) + p_2 F_2(t)$, in which F1(t) and F2(t) are two simple distributions, and p1 and p2 are the ratios of each of these functions in the total population. The Cost Function is given by the following:

$$C(b,y) \frac{c_r F(b) + c_b \int_0^b R(t)dt + c_p R(b) + (c_f - c_p)\left[F(y+b) - F(b)\right]}{\int_0^y R(x+b)dx}$$

$$(2)$$

Figures 1 and 2 show the behaviour of the mean residual life and the cost for different periods of burn-in and replacement intervals; these figures can be used to verify the conflict between the analysed criteria.

The figures show that increasing the burn-in period allows the MRL to reach a maximum at a given y. Thereafter, the MRL begins to de-
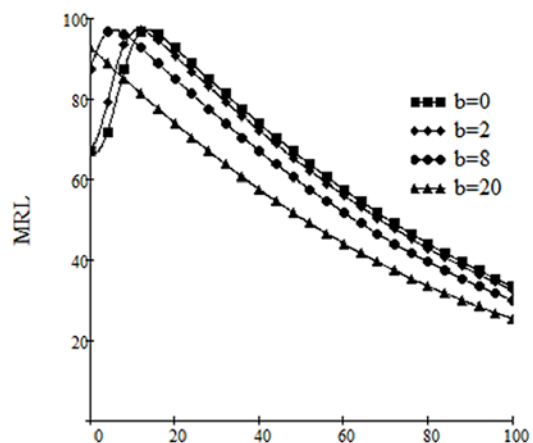

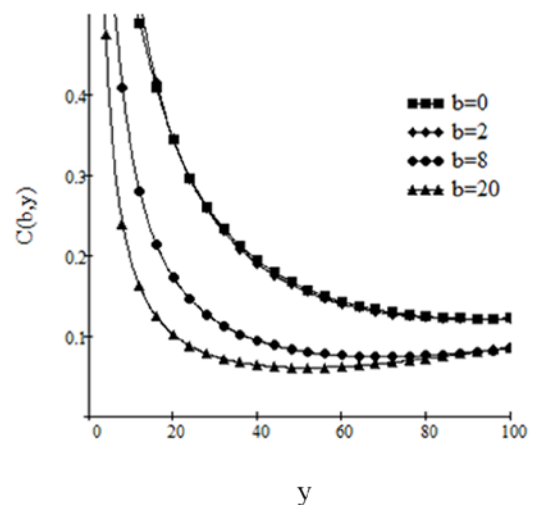Fig. 1. *Behaviour of mean residual life (MRL) – y (replacement time) - b (burn-in time)*

Fig. 1. *Behaviour of mean residual life (MRL) – y (replacement time) - b (burn-in time)*

crease, and the cost function is minimized at another value of y. These trends clearly demonstrate the conflict between the decision criteria.

The age of a component directly correlates with its likelihood of failure. Depending on the parameters adopted, a combination of (b, y) that maximizes the MRL of the component is used, which is identifiable in the graphs.

The mean residual life varies significantly; therefore, it should be adopted as a performance criterion.

A relationship between the points of change of the failure rate and the mean residual life function can be established [28]. Thus, any failure rate that follows the bathtub curve has a mean residual life function with an inverted bath-tub shape, with a single point of change that precedes the point of change of the failure rate [27]. When the failure rate is unimodal, i.e., it grows to a certain point and thereafter begins to decrease, the MRL is consequently unimodal in the form of an inverted bath-tub and may be smaller than the MTTF (Mean time to failure) after the burn-in. Thus, the MRL after burn-in must not exceed the MTTF as a constraint. Nevertheless, burn-in may not be necessary or economical for unimodal rates. For more details, see Chang [12].

3.4. Multi-attribute utility theory for MRL and cost

The preference of the decision-maker is described by utility functions that consist of the attribute cost, which should be minimized, and the mean residual life, which must be maximized. The utility function models the preference of the decision-maker, where in the utility function dimension costs and mean residual life will be measured on the same scale, the utility scale. Each alternative measure for the time of burn-in (b) and replacement interval (y) can be evaluated by the global utility function, and the trade-off between cost and mean residual life is evaluated by the decision-maker in utility levels.

The utility function for each of the attributes must be known to obtain the multi-attribute utility function [23].

To convert the cost function (function 2) and average residual life (function 1) in to utility values, mathematical functions that describe the utility function need to be studied [7,19,34].

We consider an exponential function for the utility function of the Cost, U(C), and a logistical function for the utility of MRL, U(μ). This choice is justified by the desire to minimize cost and maximize the MRL. This article proceeds to utility function return; the greater the cost, the lower the value of its utility function, and the higher the mean residual life return, the greater the value of its utility function.

Thus, the utility function for the cost can be represented by an exponential function:

$$U\big(C(b,y)\big) = \lambda_1 e^{-\gamma_1 C} \tag{3}$$

The utility function for the mean residual life can be represented by a logistical function:

$$U\big(MRL(b,y)\big) = \lambda_2 e^{-\frac{\gamma_2}{\mu}} \tag{4}$$

The values of λ_1 and γ_1 as well as the values of λ_2 and γ_2 must be adjusted to represent the function value such that function 3 approaches 1 when the cost function is minimized and 0 when the cost function is maximized. Furthermore, function 4 should approach 1 when the residual life is maximized and 0 when the residual life is minimized.

To use the aggregation procedure, the conditions of preferential independence need to be identified. In this study, we have assumed independence in the utility function, and additive independence for this assumption aims to generate the least restrictive model. Nevertheless, other preference structures can be used [19, 14]. With this assumption,

the additive model may be used, in which k_1 and k_2 represent the scale constants. These constants may represent your preference by eliciting the preference of the decision maker, where $k_1 + k_2 = 1$. When k_1 exceeds k_2, the decision-maker prioritizes cost over the average residual life, and when k_2 exceeds k_1, the decision-maker gives priority to the average residual life. The global utility function can be described for an additive function, as shown in equation 5 below.

$$U(C, \mu) = k_1 U(C) + k_2 U(\mu) \tag{5}$$

The alternatives are represented by the length of the burn-in and the replacement interval. Thus, each pair (b, y) is associated with a cost and a mean residual life and, consequently, a corresponding utility function. Therefore, the pair of values (b, y) that maximizes the global utility function is optimal.

3.5. Methodology to apply MAUT for burn-in analysis

The steps proposed by Chelst and Canbolat [14] can be applied to implement MAUT for burn-in analyses as follows:
1) Recognize the decision alternatives; alternative decisions for the analysed problem are the time of burn-in (b) and the time (age) to replacement (y).
2) Establish the objective of analysis; in this study, we proposed two objectives: mean residual life and cost
3) Establish the attributes measurably; the mean residual life can be measured using equation (1), and cost can be measured using equation (2);
4) Apply an elicitation procedure with the decision-maker to obtain the parameter utility analysis; we simulated values in numerical approaches (section 4) demonstrating its variations.
5) Establish a utility function to characterize the decision maker's preferences in order to obtain the parameter utility analysis with equations (3) and (4).
6) Analyse decision alternatives for calculating global utility function; the best action maximizes the global utility described by equation (5).

A numerical application to illustrate the use of this methodology to apply MAUT to a burn-in analysis is presented below.

4. Numerical application and management insights

The parameters adopted and results of a numerical application of MAUT are shown in Table 1, where the results obtained for each set of parameters by applying the three policies: maximizing the mean residual life (Policy I), minimizing the cost (Policy II) and using MAUT (Policy III) to aggregate the two criteria.

The maximum mean residual life is shown for each parameter, and the cost associated with the maximum residual life is also shown when the cost function is minimized. This value corresponds to the mean residual life when the cost is minimized. Finally, the optimization of the global utility function, the respective cost values, the mean residual life, and the values of b and y in weeks are given, where

$$Fs(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}} \quad \text{and} \quad F(t) = p_1 F_1(y) + p_2 F_2(t).$$

The adoption of a criterion related to performance results in a more effective policy. As observed in the results, considering only the cost results in a lower mean residual life compared with considering a mixed policy. However, a higher mean residual life results in higher costs.

Based on the numerical analysis, the following observations should be emphasized for the proposed model:

1) Adopting a minimum cost for Policy I results in a small mean residual life. However, maximizing the mean residual life increases the cost. Therefore, a policy that considers both criteria is more efficient.

Table 1: Parameters adopted and results obtained when adopting policies I(maximizing the residual life), II(minimizing the cost) and III(applying MAUT to optimize the cost and mean residual life (MRL))

Cost Parameters				Mixed Failure distribution parameters						Utility Function Parameters								Optimum Results		
C_0	C_r	C_p	C_f	$\eta1$	$\beta1$	$p1$	$\eta2$	$\beta2$	$p2$	$k1$	$\lambda1$	$\gamma1$	$k2$	$\lambda2$	$\gamma2$		b	y	C	MRL
0.2	0.9	2	13	7	1.2	0.4	125	4	0.6	0.55	1.389	2.6	0.45	2.18	73	Max μ	2.639	12.227	0.589	93.448
																Min C	8.874	79.96	0.126	37.033
																Max U	7.543	40.786	0.184	66.657
0.3	0.9	2	13	7	1.2	0.4	125	4	0.6	0.55	1.437	2.6	0.45	2.183	73	Max μ	2.639	12.227	0.618	93.448
																Min C	6.292	85.62	0.14	35.301
																Max U	4.747	43.909	0.204	66.371
0.1	0.9	2	13	7	1.2	0.4	125	4	0.6	0.55	1.321	2.6	0.45	2.183	73	Max μ	2.639	12.227	0.561	93.448
																Min C	12.03	72.034	0.107	39.863
																Max U	10.561	36.169	0.159	68.067
0.2	1.8	2	13	7	1.2	0.4	125	4	0.6	0.55	1.4	2.55	0.45	2.184	73	Max μ	2.639	12.227	0.6	93.448
																Min C	8.128	81.988	0.132	36.303
																Max U	6.629	41.694	0.193	66.663
0.2	0.45	2	13	7	1.2	0.4	125	4	0.6	0.55	1.38	2.61	0.45	2.183	73	Max μ	2.639	12.227	0.584	93.448
																Min C	9.219	78.923	0.123	37.432
																Max U	7.943	40.163	0.18	66.853
0.2	0.9	4	13	7	1.2	0.4	125	4	0.6	0.55	1.498	2.8	0.45	2.183	73	Max μ	2.639	12.227	0.732	93.448
																Min C	8.454	90.84	0.144	31.439
																Max U	7.023	46.67	0.208	62.046
0.2	0.9	1	13	7	1.2	0.4	125	4	0.6	0.55	1.351	2.6	0.45	2.183	73	Max μ	2.639	12.227	0.518	93.448
																Min C	9.128	74.66	0.116	40.033
																Max U	7.897	38.175	0.169	68.649
0.2	0.9	2	26	7	1.2	0.4	125	4	0.6	0.55	1.571	2.5	0.45	2.183	73	Max μ	2.639	12.227	0.969	93.448
																Min C	12.786	64.397	0.181	44.262
																Max U	12.168	40.548	0.22	62.872
0.2	0.9	2	9	7	1.2	0.4	125	4	0.6	0.55	1.306	2.6	0.45	2.184	73	Max μ	2.639	12.227	0.473	93.448
																Min C	6.013	90.353	0.103	32.922
																Max U	2.383	39.937	0.176	72.023
0.2	0.9	2	13	3.5	1.2	0.4	125	4	0.6	0.55	1.29	2.6	0.45	2.054	73	Max μ	2.324	7.191	0.911	101.401
																Min C	7.014	74.585	0.098	41.396
																Max U	6.572	37.302	0.146	70.623
0.2	0.9	2	13	9	1.2	0.4	125	4	0.6	0.55	1.43	2.6	0.45	2.254	73	Max μ	2.761	14.171	0.507	89.811
																Min C	8.271	83.087	0.137	35.607
																Max U	5.537	42.158	0.204	67.19
0.2	0.9	2	13	7	2.4	0.4	125	4	0.6	0.55	1.35	2.6	0.45	2.08	73	Max μ	6.471	5.885	1.22	99.64
																Min C	10.606	75.553	0.115	38.598
																Max U	10.274	38.953	0.165	65.874
0.2	0.9	2	13	7	0.6	0.4	125	4	0.6	0.55	1.358	2.6	0.45	2.254	73	Max μ	5.646	5.142	0.877	89.784
																Min C	4.023	82.237	0.118	38.517
																Max U	3.162	38.757	0.176	70.877
0.2	0.9	2	13	7	1.2	0.8	125	4	0.2	0.55	2	2.65	0.45	2.35	73	Max μ	8.7	14.799	1.195	85.416
																Min C	16.8	96.442	0.262	25.205
																Max U	15.89	55.605	0.329	48.181
0.2	0.9	2	13	7	1.2	0.3	125	4	0.7	0.55	1.327	2.6	0.45	2.147	73	Max μ	2.502	10.038	0.543	95.578
																Min C	6.229	78.26	0.109	39.604
																Max U	4.516	39.065	0.164	70.884
0.2	0.9	2	13	7	1.2	0.4	250	4	0.6	0.55	1.163	2.54	0.45	1.433	73	Max μ	2.884	16.147	0.477	202.803
																Min C	10.527	163.762	0.06	76.03
																Max U	9.745	80.307	0.09	139.174
0.2	0.9	2	13	7	1.2	0.4	80	4	0.6	0.55	1.716	2.6	0.45	3.72	73	Max μ	2.456	9.293	0.709	55.547
																Min C	7.028	50.787	0.208	23.154
																Max U	4.692	26.215	0.306	42.614
0.2	0.9	2	13	7	1.2	0.4	125	8	0.6	0.55	1.288	2.6	0.45	2.113	73	Max μ	2.652	12.434	0.582	97.616
																Min C	9.57	84.638	0.097	27.266
																Max U	7.653	40.283	0.179	69.816
0.2	0.9	2	13	7	1.2	0.4	125	2	0.6	0.55	1.534	2.6	0.45	2.204	73	Max μ	2.705	13.28	0.569	92.423
																Min C	8.258	108.75	0.165	49.375
																Max U	7.48	51.63	0.198	69.775
0.2	0.9	2	13	7	1.2	0.4	125	4	0.6	0.7	1.389	2.6	0.3	2.185	73	Max μ	2.639	12.227	0.589	93.448
																Min C	8.874	79.96	0.126	37.033
																Max U	8.353	52.841	0.149	55.919
0.2	0.9	2	13	7	1.2	0.4	125	4	0.6	0.4	1.389	2.6	0.6	2.185	73	Max μ	2.639	12.227	0.589	93.448
																Min C	8.874	79.96	0.126	37.033
																Max U	5.997	29.929	0.253	77.913

2) Policy III, which uses MAUT to consider both the cost and mean residual life, indicates that the ideal burn-in and replacement interval (b + y) values for the defined structure of preferences (highlighted grey) are 7.543 and 40.786 weeks, respectively. This alternative offers a cost that represents approximately one third of the cost obtained by applying policy I and a mean residual life that represents an increase of almost 30% compared with the value obtained using policy II, which yields a utility of 0.801. Without a multi-criteria analysis, the decision-maker cannot visualize this result, i.e., cannot realize the trade-off between objectives.

3) Increasing the burn-in cost decreases the optimal burn-in length because the improvement in the performance of the items will not compensate for the cost of the procedure. In order to reduce the impact, the time until the replacement tends to be longer. The same occurs if the cost of preventive replacement and repairs during the burn-in increase. If these costs are reduced, the situation will be reversed. However, if the replacement by failure cost increases, the policy will tend to indicate a longer burn-in time and a shorter interval until replacement, which results in lower costs.

4) The effects of k1 and k2 were also analysed. For a k1 value higher than k2, the cost of the optimal policy is lower. However, the mean residual life cannot be high. For a k2 value higher than k1, the mean residual life is high, which increases costs. Nevertheless, the utility function balances the two previous policies. Thus, it can reflect the preference of the decision-maker

5) For example, the decision-maker can be committed to deliver a longer residual life and clearly know the cost associated with this decision. However, if the decision-maker has a preference for minimizing the cost, he will know the corresponding residual life. The decision-maker can assess the consequences and evaluate the attributes by comparing both the maximum and the minimum for a trade-off, creating the possibility of an improved decision that incorporates his preference.

The results shown in Table 1 indicate that variations in cost parameters alter only the solutions found when the optimization is based on cost or the overall utility function because the approach that considers the maximization of the mean residual life does not take costs into account. The shape parameter is known to be associated with the degradation of the population. A higher $\beta 1$ value promotes frequent early failures. Therefore, burn-in procedures need to be prioritized, and components need to be replaced after a short time. If $\beta 2$ is reduced, degradation by age will decrease, allowing the definition of longer replacement periods.

We also varied the distribution parameters, which modified the results obtained by the three policies presented. Reducing the scale parameter of the first portion of the distribution ($\eta 1$) indicates that more frequent replacements because failures tend to occur sooner. Thus, replacements are required to prevent malfunction. The behaviour of the

policies is similar if $\eta 2$ increases, which will imply an increase in the replacement period, with a lower cost and higher mean residual life.

Changing the proportion of the different distributions that comprise the mixed distribution of items yields interesting results. For a higher value of p1, the length of the burn-in period is extended. As the proportion of the second distribution decreases, the replacement period increases. Increasing the F2 proportion produces similar results because this population primarily consists of "strong items" that need to be replaced sooner.

5. Conclusions

Decisions regarding time of burn-in and replacement intervals are complex, and decision-makers should not ignore the influence of the decision on the customers' perceptions of products. Burn-in aims to avoid early failures associated with negative consequences with customers; however, the product is placed on the market after tests, and its residual life performance will be evaluated by consumers, which is represented in the MAUT by the mean residual life attribute. The mean residual life may play a more important role for complex equipment that is difficult to access and highly available to any outfit. However, decision-makers responsible for competitively priced products may be more interested in minimizing the cost.

In this study, we developed a model based on MAUT to address the decision problem related to burn-in with replacement. Until fairly recently, models that aimed to minimize costs predominated such decisions. However, performance criteria are very important and may be more important than cost in some cases. Several objectives may be important, and these objectives may sometimes conflict. When modelling the problem, more than one aspect can be considered by using a multi-criteria approach. A range of methods is available for modelling, and the decision-maker must select the method best suited for the situation. Thus, the decision-maker will be able to make decisions that are more on target because a maintenance policy is obtained that can prevent different types of faults by combining applicable procedures that consider multiple objectives.

The preventive maintenance of items characterized by the occurrence of failures both at the beginning and the end of useful life is best used under a combined policy of burn-in and replacement that considers the cost and residual life of the components. In accordance with the degree of preference defined by the decision-maker, the application of MAUT may well represent the situations encountered in real life. The MAUT modelling for the decision to use burn-in with replacement proved to be an effective approach, wherein the decision maker can easily assess the maximum and minimum alternatives for each attribute and explicitly evaluate trade-offs that are crucial to the modelling goal.

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