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Calculating Extreme Criteria of Eigenvalue Problems of a Medical Robot for Soft Tissue Surgery Using the Multi-objective Optimization

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Abstract

Medical robots with an instant center of rotation mechanism in a trocar are used for operating a human body or servicing artificial organs. The result of the work is the development of a multi-criteria optimization model of a discussed medical robot, considering safety factor, first eigenfrequency and buckling coefficient as a criteria. The article also analyzes two issues of mechanics, the natural frequency and linear buckling. A discrete mesh model of a novel robot design with ten degrees of freedom and ended with a scalpel was developed based on finite element method. For the given loads and supports, a multi-criteria optimization model was evolved, which was solved by using the response surface method and the multi-objective genetic algorithm. The results section shows the Pareto fronts for the criteria and geometrical dimensions of the kinematic chain. The courses of resonant vibrations and buckling strains were also characterized. The solved optimization model gives correct values for the adopted criteria. The values of resonance were defined, which makes it possible to select mechatronic drive systems in terms of the input they generate. Variability of the resonant vibrations phenomena, as well as shapes and directions of buckling, provide information about the displacements taking place in the medical robot system.

Keywords: medical robot, natural vibration, buckling, optimization, genetic algorithm, Pareto front

1. Introduction

Vibrations of the medical robot structure are unacceptable due to the lack of accuracy at positioning and repeatability. The causes of vibrations of the medical robot are low stiffness, backlash, force input from a tissue cutting tool or servomotors. The operation of a medical robot in the ranges of dangerous arguments from the resonance curve cause an increase in the ordinate of the vibration amplitude and a decrease in accuracy at positioning and repeatability of the effector.

The outcome of the work is to identify a mathematical model enabling the optimization of the first eigenvalue, describing free vibration assuming additional optimized criteria considered as significant, i.e. buckling factor and safety factor. Moreover, shapes of natural vibrations for next eigenvalues (resonance) and strains for subsequent values of buckling coefficients are analyzed.

So far, researchers in various research centers have conducted some investigation in this area. For example, in research [1], the first natural frequency for the PUMA 560 industrial robot, used to operate the human body, is appointed and it is stated to be 12.5

[Hz]. In some work, the authors show a medical robot with a constant point mechanism and in a numerical way define its first natural frequency as 2.5 [Hz]. The author of this work considers this frequency to be too low and criticizes the design due to the work possibilities of this robot in the undesirable ranges of resonance curve, resulting from the movements enabling the functionality in the operating field. This results in a significant loss of accuracy at positioning and repeatability, and a loosening of the robot's mechanical system. Next work [2] states that the first natural frequency of industrial robots is about 10 [Hz].

A modern overview of robots currently used in medicine is shown in work [3]. The appliance of the finite element method (FEM) in the stress analysis of a modern medical robot with silicone elements based on the von Mises hypothesis is shown in work [6]. The articles [4,5] illustrate numerical modeling based on the computer aided design (CAD) model using the FEM. Optimization is performed and a significant reduction in the robot's displacement for surgical oncology is obtained. Topology optimization and the FEM are used in the article [7] and a 10.4% reduction in the weight of the rehabilitation robot, to the upper limb with five degrees of freedom is acquired. Multi-criteria optimization, where the criteria is the natural frequency, speed value and static stiffness, is performed for a hybrid robot with five degrees of freedom in work [8]. In the paper [9], a genetic algorithm is used to solve the optimization model of the DELTA robot mechanism with a parallel geometry.

Null-space-based optimization is used to simultaneously optimize the mass and friction of the robot in the article [10].

The following work aims to create a rational multi-criteria model for the adopted criteria and constraints. The optimization model is solved with the usage of a genetic algorithm. The practical effect of this work is to be an innovative, useful, and safe to use construction for the given criteria and limitations.

2. Materials and methods

The structure has ten degrees of freedom. It can be divided into maxi part having four degrees of freedom RTRR used to perform regional movements and mini part, ended with a scalpel, having six degrees of freedom RRRS to perform local movements near the tissue. Figure 1 shows the instantaneous center of rotation and marks the successive degrees of freedom. The finite element method was adopted to solve the computational model. The robot has been divided into finite elements SOLID187 and is a finite system with a total number of 33832 degrees of freedom.

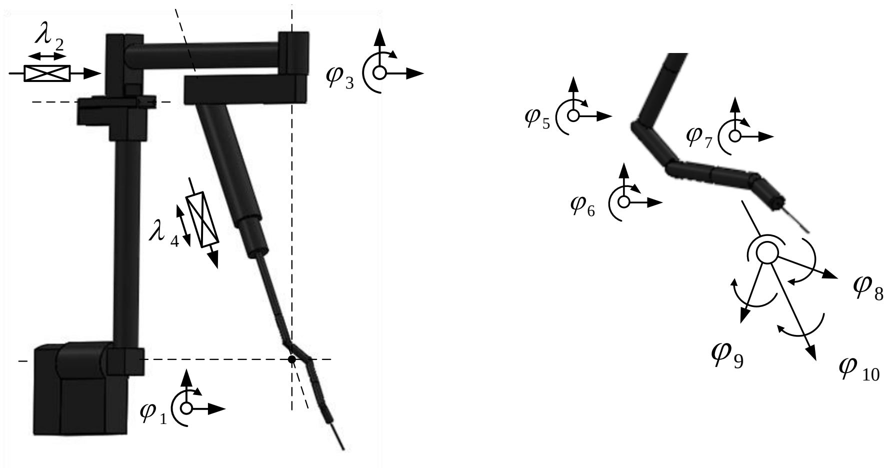

Figure 1. The kinematic chain of a medical robot

The calculation of the necessary stresses to determine the pre-stress phenomenon, appearing in the FEM analysis of the linear buckling phenomenon was performed based on the von Mises hypothesis. The Lanczos method for large symmetric systems [11] was used to solve the frequency and buckling eigenproblems. There are also other methods of solving eigenvalues issues, which were analyzed in work [12].

2.1. Eigenvalue problem for the linear buckling using finite element method

The solution of the buckling issue comes down to the determination of eigenvalues, like buckling coefficients and eigenvector, which are buckling shapes, taking into account the static equilibrium equation and small displacements in the mechanical system. By determining the critical load ratio to the applied load as λ, it can be stated that buckling will occur in the mechanical system of the medical robot when λ <1.

The equation describing the static equilibrium has the following form:

$$[\boldsymbol{K}] \cdot \{\boldsymbol{u}\} = \{\boldsymbol{F}\}, \tag{1}$$

where: $[\boldsymbol{K}]$ – stiffness matrix, $\{\boldsymbol{u}\}$ – nodal displacement vector, $\{\boldsymbol{F}\}$ – vector of nodal forces. The stiffness matrix is defined as:

$$[\boldsymbol{K}] = \int_V [\boldsymbol{B}]^T [\boldsymbol{D}][\boldsymbol{B}]\, dV, \tag{2}$$

where: $[\boldsymbol{B}]$ – linear strain - displacement matrix, $[\boldsymbol{D}]$ – constitutive matrix.

Stress-stiffness matrix is determined by:

$$[\boldsymbol{K_G}] = \int_V [\boldsymbol{G}]^T [\boldsymbol{S}][\boldsymbol{G}]\, dV^2, \tag{3}$$

where: $[\boldsymbol{G}]$ – obtained from shape functions by appropriate differentiation, $[\boldsymbol{S}]$ – initial stresses. Knowing that:

$$([\boldsymbol{K}] + [\boldsymbol{K_G}]) \cdot \{\boldsymbol{u}\} = \{\boldsymbol{F}\}, \tag{4}$$

it is stated simultaneously that while losing stability other states of equilibrium are possible.

$$([\boldsymbol{K}] + \boldsymbol{\lambda}[\boldsymbol{K_G}]) \cdot \{\boldsymbol{u}\} = \{\boldsymbol{F}\}. \tag{5}$$

$$([\boldsymbol{K}] + \boldsymbol{\lambda}[\boldsymbol{K_G}]) \cdot \{\boldsymbol{u} + \boldsymbol{\delta u}\} = \{\boldsymbol{F}\}. \tag{6}$$

After subtracting the equations, the symmetrical problem that defines the stability of the substitution system is obtained to solve:

$$([\boldsymbol{K}] + \boldsymbol{\lambda}[\boldsymbol{K_G}]) \cdot \{\boldsymbol{\delta u}\} = \{\boldsymbol{0}\}. \tag{7}$$

where**:** $\boldsymbol{\lambda}$ – eigenvalues, which are load coefficients, $\boldsymbol{\delta u}$ – eigenvector, which is the shape of buckling.

2.2. Eigenvalue problem for the natural frequency using finite element method

Solving the eigenfrequency problem is to discover the eigenvalues that are resonant frequencies and the eigenvectors that are the shapes of the resonant vibrations. When the value of the natural frequency is equal to the value of the harmonic excitation frequency, a dangerous phenomenon of resonance and a significant increase in the amplitude of vibration takes place.

Disregarding the Rayleigh damping, the equation describing the natural vibrations can be formed as:

$$[\boldsymbol{M}] \cdot \{\ddot{\boldsymbol{u}}\} + [\boldsymbol{K}] \cdot \{\boldsymbol{u}\} = \{\boldsymbol{0}\}. \tag{8}$$

The mass matrix can be written as:

$$[\boldsymbol{M}] = \int_V \rho[\boldsymbol{N}]^T\, [\boldsymbol{N}] dV, \tag{9}$$

where:ρ – density, [**N**] – matrix of shape function.

The general solution of equation (8) has a following form:

$$\{\boldsymbol{u}\} = \boldsymbol{u_A} \cdot \cos(\omega t) + \boldsymbol{u_B} \cdot \sin(\omega t), \tag{10}$$

$$\{\ddot{\boldsymbol{u}}\} = -\boldsymbol{\omega}^2\{\boldsymbol{u}\}. \tag{11}$$

After substituting (11) to (8) the equation called eigenequation was obtained:

$$([\boldsymbol{K}] - \omega^2[M]) \cdot \{\boldsymbol{u}\} = \{\boldsymbol{0}\}. \tag{12}$$

where: $\boldsymbol{\omega}$ – eigenvalues, which are natural frequencies of the effector, $\boldsymbol{u}$ – eigenvectors, that are mode shapes of the effector.

2.3. Multi-objective optimization model

The optimization model formulated in this work assumes three criteria: safety factor, first natural frequency and buckling coefficient, characterizing the ratio of the load applied to the mechanical system to the Euler's critical load. Restrictions were imposed on the model in the form of ranges for variability of dimensional parameters, value of the safety factor and the values of the frequency. The assumed issue is to find the values of decision variables $\boldsymbol{d}$, for which the vectorized objective function will reach the optimal value. Therefore, from a mathematical point of view, the vector optimization problem should be solved.

The objective function was adopted as follows:

$$\boldsymbol{f}(\boldsymbol{d}) = \{f_1(\boldsymbol{d}) \longrightarrow min, f_2(\boldsymbol{d}) \longrightarrow max, f_3(\boldsymbol{d}) \longrightarrow min\}. \tag{12}$$

the following restrictions were adopted:

$$\begin{gathered} min \leq d_1 \leq max, min \leq d_2 \leq max, min \leq d_3 \leq max, \\ f_1(\boldsymbol{d}) \geq 4, 30Hz \leq f_2(\boldsymbol{d}) \leq 40\ Hz, 10 \leq f_3(\boldsymbol{d}) \leq 40, \end{gathered} \tag{13}$$

where: $f_1(\boldsymbol{d})$ – safety factor criterion, $f_2(\boldsymbol{d})$– first natural frequency criterion, $f_3(\boldsymbol{d})$ – buckling coefficient, d_1 – dimension related to the diameter of the vertical column, d_2 – dimension related to the diameter of the horizontal column, d_3 – dimension of sleeve of the last link.

The optimization model is based on Pareto front. Pareto-optimality assumes solutions that are not dominated by a subset of the possible solutions of the model. The problem of searching for the optimal solution is solved by using the MOGA genetic algorithm. The solution is also based on a meta-model in the form of a response surface, i.e. a function created based on discrete data from numerical experiments using the FEM [13]. The use of the response surface method reduces the search area for the genetic algorithm. This decreases the computation time.

3. Results

Changes in resonant vibrations (eigenfunctions) for next natural frequencies of the mechanical model are presented in Figure 4. The first transverse in the XY plane and the second longitudinal in the XZ plane of free vibration shape have a bending harmonic character with amplitudes of 37.2 and 40.52 [mm].

In the third and fourth XZ natural frequency, the corresponding longitudinal XY and transverse XZ bending vibrations begin with the amplitudes 107.74 and 141.54 [mm]. High amplitude resonance vibrations may be risky for the operated patient.

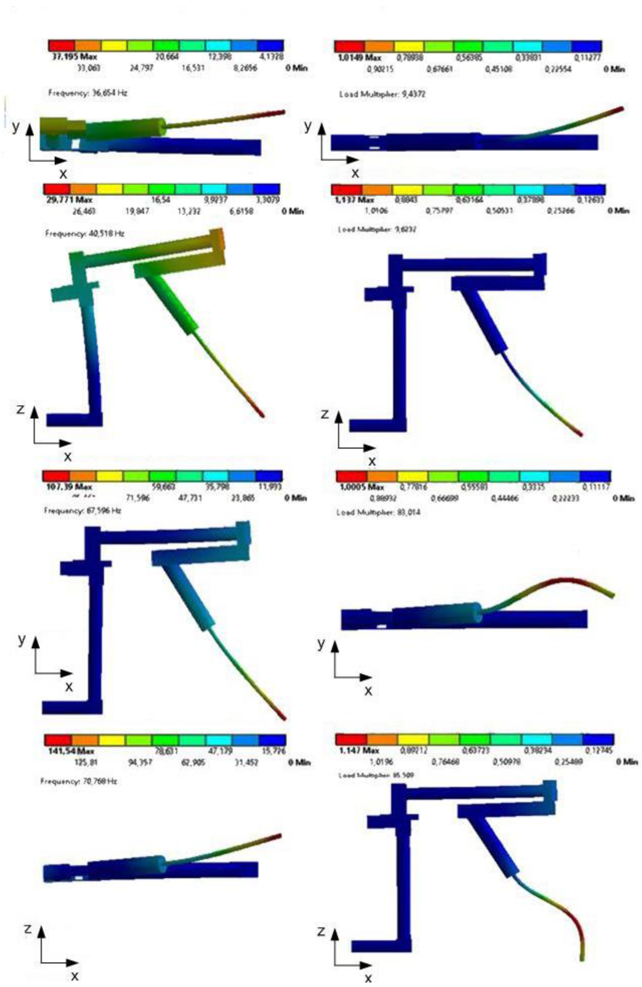

Figure 4. Forms of natural vibrations of a medical robot $\omega_{1..4} = 36.65$, 40.52, 67.6, 70.77 [Hz]. Buckling shapes of medical robot for $\lambda_{1..4} = 9.44, 9.62, 83.01, 85.5$

The linear buckling of the model was analyzed and it was found that the most exposed element was the last link of the regional movement mechanism. First and second degree of freedom tubular sections are not exposed to buckling.

For the first buckling value, transverse deformations appear in the XY plane with an amplitude of 1.01 [mm]. For the second value, longitudinal strains appear in the XZ orthogonal direction with an amplitude of 1.14 [mm]. The third lateral buckling deformation takes place in the XY plane and has an amplitude of 1 [mm]. The amplitude of the fourth transverse deformation in the XZ plane is 1.15 [mm].

The buckling shapes and directions of the buckling strains, of which the amplitude is periodic, are shown in Figure 4.

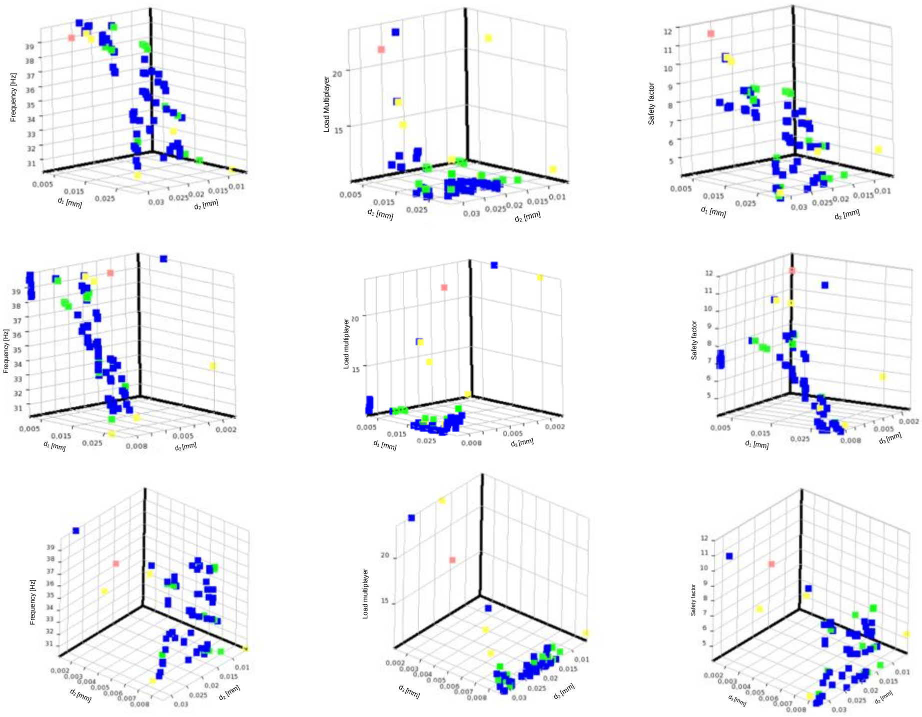

Figure 6. Pareto fronts for criteria and decision variables d_1, d_2 and d_3

The optimization model was solved after 9 iterations. Figures 6 show the Pareto fronts, giving the basis for the optimal inference. The following results were obtained: $f_1(\boldsymbol{d}) = 4.95$, $f_2(\boldsymbol{d}) = 34.77$ [Hz], $f_3(\boldsymbol{d}) = 10.04$.

4. Conclusions

The optimization model enables the calculation of correct, i.e. the best possible values for the adopted criteria and constraints. The best calculated safety factor gives the certainty that there is no danger from the strength point of view. The maximum natural frequency ensures the maximum robot’s rigidity. The buckling coefficient greater than unity guarantees that there will be no buckling phenomenon in the medical robot structure for the assumed boundary conditions.

The vibrations of the robot's structure as a result of compatible input forces with resonance were examined. The shapes of vibrations inform about the nature of resonance deformations, directions of vibrations and their type. The numerical experiment made it possible to identify longitudinal and transverse bent vibrations. Dangerous places in the construction of the medical robot have been identified. Risky vibrations of the robot's endoscopic tool with large amplitudes were indicated, which definitely put the health of the operated patient at risk. The conducted analysis also provides the basis for the selection of drives from the point of view of the generated forces on the kinematic chain.

The buckling phenomenon was also assayed and the type of strains was found, which could occur during this phenomenon.

The next stage of this work will be the optimal selection of balances for the subsequent degrees of freedom of the robot; a future prototype device intended to be used in hospitals.

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