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THE SIMPLEST MODELS OF THE MACROECONOMICS DYNAMICS IN A COMPOSITION WITH THE PRINCIPLE OF THE SUBJECTIVE ENTROPY MAXIMUM

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Abstract

This paper proposes a solution to a certain macroeconomic model. A multi-alternative problem of aviation transportation optimal organisation in conditions of uncertainty of the subjective preference functions is considered. Conditional optimisation of the objective functional containing the entropy of the individuals' operational effectiveness functions preferences is carried out in the framework of the simplest macroeconomic problem. The principle of the *Solow* and *Cobb–Douglas* models, similar to the approach adopted for the estimation of economic growth, is modified with the subjective entropy maximum principle. The advantages of the described optimisation approach are demonstrated in the generalised terms of the operational effectiveness functions for aviation transportation organisation.

Keywords: aviation transportation; operational effectiveness; objective functional optimisation; simplest macroeconomic problem; entropy
Type of the work: research article

1. INTRODUCTION

Aviation industry and airlines are undergoing hard times due to the SARS COVID-19 pandemic period as well as the tragedy of the heroic rebuff of Ukraine to the fascist–russist full-scale warfare invasion.

The current circumstances require indispensable measures to be taken in the major macroeconomic airline industry components. The presented paper is dedicated to the simplest macroeconomic problem setting in the framework of the *Solow* [1–3] and *Cobb–Douglas* [4,5] models, similar to the approach adopted for the estimation of economic growth [6], together with consideration of the individuals' subjective preferences functions of the available alternatives obtained based on the subjective entropy maximum principle, which has been previously proposed in the literature [7–10]. This principle was applied to the simplest problems concerning the macroeconomics dynamics, and those were the continuous models; they were in the type of the *Walras–Leontief* ones [11].

Nevertheless, some important problems are neither included within nor converge into those classes of the simplest macroeconomics models: *Solow* [1–3], *Mankiw–Romer–Weil* [6] and others.

A combination of macroeconomics models incorporating the principle of the subjective entropy maximum is tried in the present work. The principle was developed during 1990–2010. Although this principle formally hardly deviates from the *Jaynes'* principle [12–14], the combination utilised in

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the present study widens the horizon of the practical applications of the obtained results, especially in psychology [7–10], economics [7–10,15], theory of conflicts, etc. [16–23]. Further, the approach could be recommended for implementation in other spheres of study that share similar characteristics [24–32].

2. PROBLEM STATEMENT

Let us consider a macroeconomics problem by *Solow*.
The production function is given by the expression:

$$Y_{t+1} = A\left(K_t\right)^{\alpha}\left(L_t\right)^{\beta}, \tag{1}$$

where Y_{t+1} is the production function Y at the time (step of iteration, recursion stage) $t + 1$, the time following the previous time$t$; A is the coefficient; K_t is the capital gained at the preceding point of time t; α is the power index for the capital K_t; L_t is the labour component L of the production process Y at the earlier point of time t; and β is the power index for the labour component L, analogously to α coefficient.

For some reasons, the power indices of α and β can be bound by the relation of

$$\beta = 1 - \alpha. \tag{2}$$

For the components of K_t and L_t there should be satisfied the balance equation:

$$Y_t = K_t + L_t. \tag{3}$$

The division envisaged by Eq. (3) is provided with the use of the individuals' subjective preferences π:

$$\pi_t^K \text{ and } \pi_t^L. \tag{4}$$

3. PROPOSED SOLUTION

3.1. Entropy conditional optimisation

The objective functional for the individuals' subjective preferences π, as ascertained in Eq. (4), is determined using the expression:

$$\Phi_\pi = -\sum_{i=1}^{N=2} \pi_t^i \ln \pi_t^i + \beta_\pi \left[\pi_t^K K_t + \pi_t^L L_t\right] + \gamma\left[\sum_{i=1}^{N=2} \pi_t^i - 1\right], \tag{5}$$

where

$$-\sum_{i=1}^{N=2} \pi_t^i \ln \pi_t^i \tag{6}$$

is the subjective entropy of the individuals' subjective preferences π, representing the measure of the available alternatives' effectiveness preferences functions uncertainty; and β_π and γ are the corresponding cognitive coefficients.

Then, on condition of

$$\frac{\partial \Phi_\pi}{\partial \pi_t^i} = 0 \tag{7}$$

it is possible to find

$$-\ln \pi_t^K - 1 + \beta_\pi K_t + \gamma = 0. \tag{8}$$

This yields

$$\ln \pi_t^K = \gamma - 1 + \beta_\pi K_t. \tag{9}$$

Thus,

$$\pi_t^K = e^{\gamma - 1 + \beta_\pi K_t}. \tag{10}$$

On the other hand,

$$-\ln \pi_t^L - 1 + \beta_\pi L_t + \gamma = 0. \tag{11}$$

This yields

$$\ln \pi_t^L = \gamma - 1 + \beta_\pi L_t. \tag{12}$$

Thus,

$$\pi_t^L = e^{\gamma - 1 + \beta_\pi L_t}. \tag{13}$$

The procedure outlined in Eqs (7)–(13) leads to the expression:

$$\pi_t^K + \pi_t^L = e^{\gamma - 1 + \beta_\pi K_t} + e^{\gamma - 1 + \beta_\pi L_t} = 1. \tag{14}$$

The normalising condition, i.e. Eq. (14), means that

$$e^{\gamma - 1} = \frac{1}{e^{\beta_\pi K_t} + e^{\beta_\pi L_t}}. \tag{15}$$

Because of Eq. (15),

$$\pi_t^K = \frac{e^{\beta_\pi K_t}}{e^{\beta_\pi K_t} + e^{\beta_\pi L_t}}. \tag{16}$$

In turn

$$\pi_t^L = \frac{e^{\beta_\pi L_t}}{e^{\beta_\pi K_t} + e^{\beta_\pi L_t}}. \tag{17}$$

3.2. Simulation

In the simplest case, the recursive system could be used:

$$\begin{pmatrix} Y_{t+1} \\ K_{t+1} \\ L_{t+1} \end{pmatrix} = \begin{pmatrix} A(K_t)^{\alpha} (L_t)^{\beta} \\ \pi^K Y_{t+1} \\ Y_{t+1} - K_{t+1} \end{pmatrix}. \tag{18}$$

For the accepted data:

$$A = 1.872,\ \alpha = 0.3,\ \beta = 0.7. \tag{19}$$

The initial conditions would then be represented by:

$$\begin{pmatrix} Y_0 \\ K_0 \\ L_0 \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \\ 4 \end{pmatrix}; \tag{20}$$

and

$$\pi^K = 0.2\,; \tag{21}$$

The results of the computer simulation are shown in Figs 1–3.

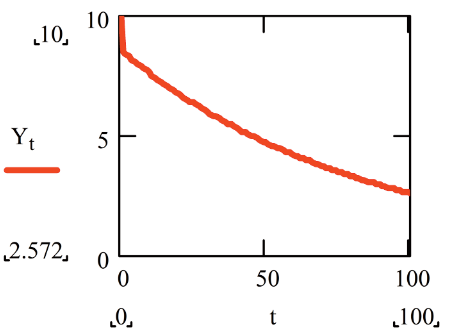

Figure 1. Production function.

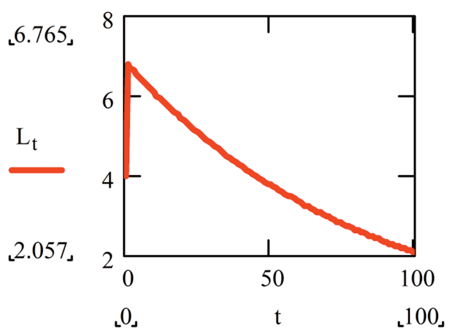

Figure 3. Labour function.

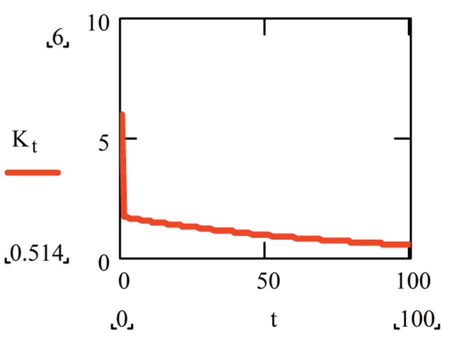

Figure 2. Capital function.

In the case when

$$\pi^{K} = 0.34, \tag{22}$$

the picture drastically changes.

The results of modelling for Eqs (1)–(20) with Eq. (22) instead of Eq. (21) are illustrated in Figs 4–6.

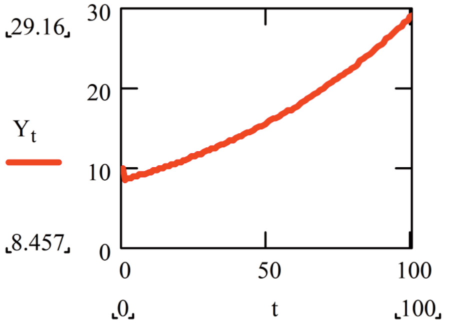

Figure 4. Production function.

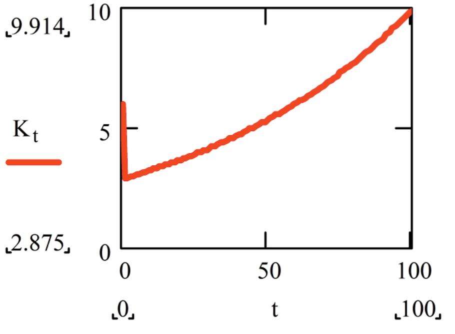

Figure 5. Capital function.

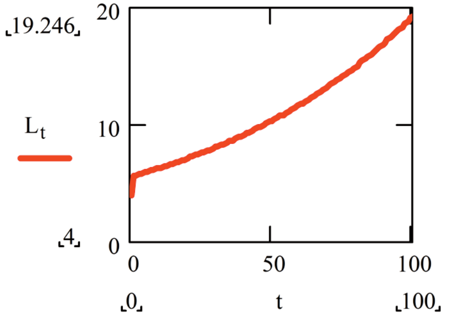

Figure 6. Labour function.

The results of calculations obtained in pursuance of Eqs (1)–(22) are portrayed in Figs 1–6, and they prompt a conclusion that there must be some preferences function value that ensures neither decrease nor increase of the characteristics, but stable magnitudes.

Indeed, at the value of

$$\pi^K = 0.22255 \tag{23}$$

there is a situation of stability.

The results indicated by Eq. (23) prove the supposition described above. It is represented in Figs 7–9.

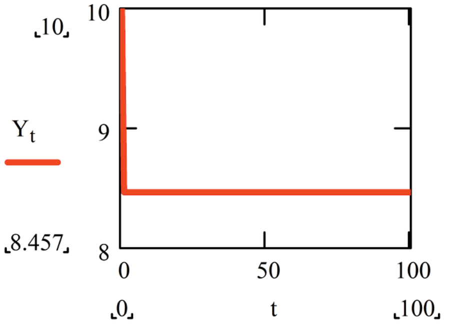

Figure 7. Production function.

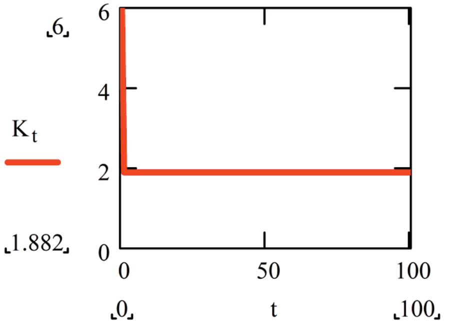

Figure 8. Capital function.

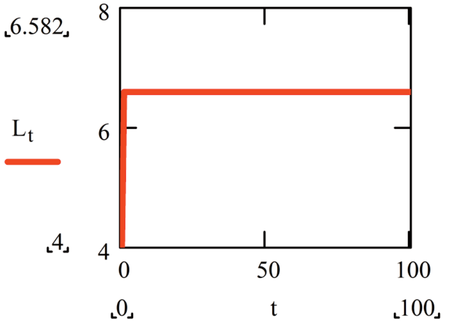

Figure 9. Labour function.

Moreover, at the value of

$$\pi^K = 0.386\,, \tag{24}$$

there is one more situation of stability.

The results arising pursuant to Eq. (24) are shown in Figs 10–12.

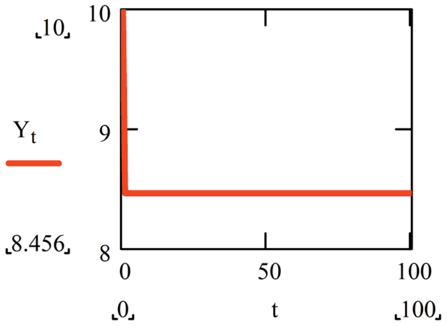

Figure 10. Production function.

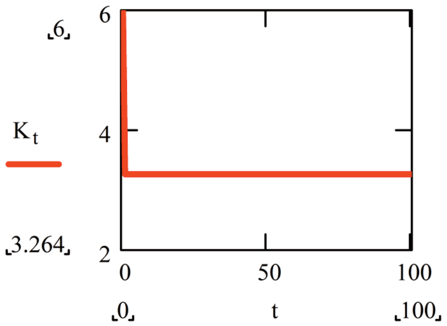

Figure 11. Capital function.

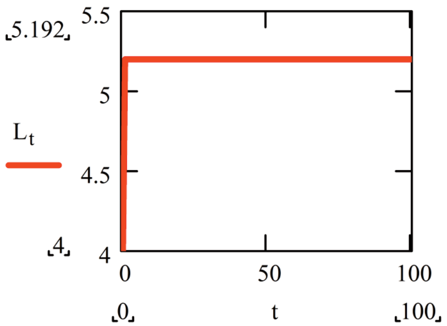

Figure 12. Labour function.

Thus, there are at least two situations of the stability; and moreover, at practically the same production (compare Figs 7 and 10), the distribution between the components of Y_t, namely K_t and L_t, is different (compare Figs 8 and 11 with 9 and 12, respectively).

The other decrease is for the values of

$$\pi^K > 0.386. \tag{25}$$

For instance,

$$\pi^{K} = 0.3986. \tag{26}$$

The results are represented in Figs 13–15.

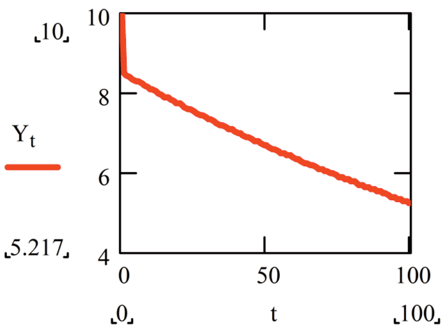

Figure 13. Production function.

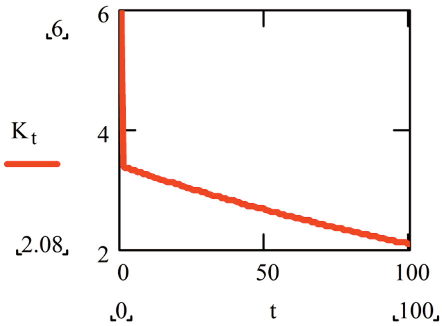

Figure 14. Capital function.

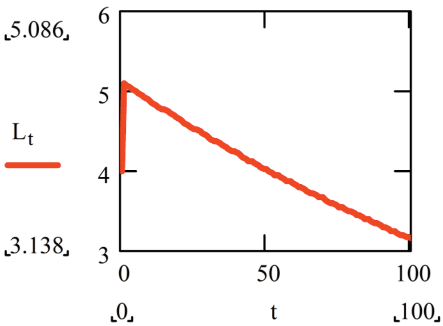

Figure 15. Labour function.

Therefore, we have arrived at a means to formulate an optimisation problem for the production maximum.

3.3. Further developments

One way in which the development of the model can be linked with the results following from Eqs (1)–(26) arises when the preferences are considered not constant.

For example,

$$\begin{pmatrix} Y_{t+1} \\ K_{t+1} \\ L_{t+1} \\ \pi_{t+1}^{K} \end{pmatrix} = \begin{pmatrix} A(K_t)^{\alpha}(L_t)^{\beta} \\ \pi_t^{K} Y_{t+1} \\ Y_{t+1} - K_{t+1} \\ \dfrac{e^{\beta_\pi K_t}}{e^{\beta_\pi K_t} + e^{\beta_\pi L_t}} \end{pmatrix}. \tag{27}$$

The initial value is given as:

$$\pi_0^{K} = 0.3. \tag{28}$$

For the parameter,

$$\beta_\pi = 0.3. \tag{29}$$

The computer simulation results arising pursuant to Eqs (27)–(29) are shown in Figs 16–20.

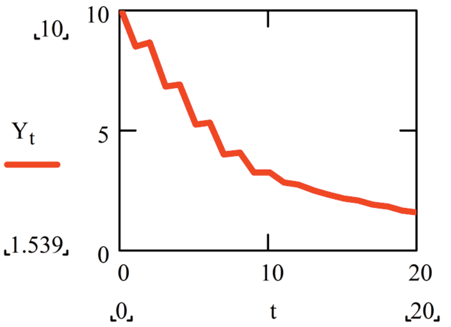

Figure 16. Production function.

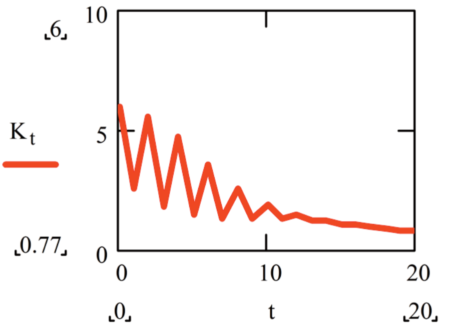

Figure 17. Capital function.

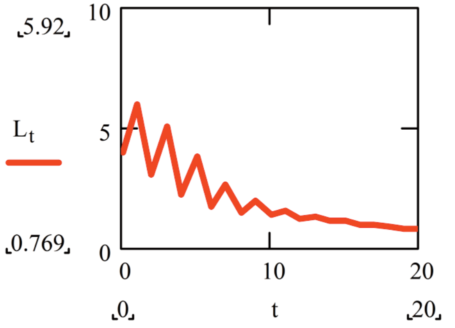

Figure 18. Labour function.

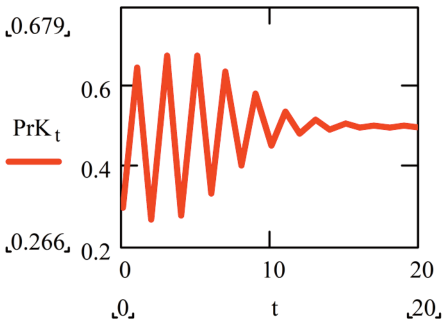

Figure 19. Preferences function.

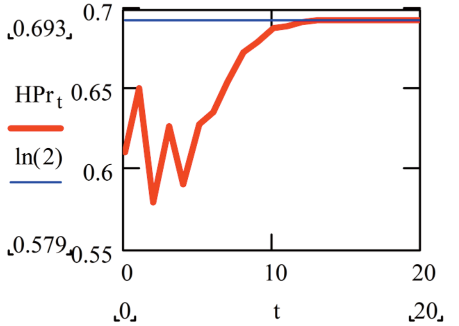

Figure 20. Subjective entropy.

4. ANALYSIS

It is discernible from the results of the simulation that, when the preferences functions obtained the values belonging to the diapason from Eqs (23)–(24), the production and its characteristic components increase (see Figs 1–20).

It is supposed that the parameters of the income (the result of production) Y, physical capital K and labour L are known and controlled in the moment of time$t$.

Moreover, consumption C is also assumed to be known; and labour L is some function of C, being such that if $C = 0$, then $L = L_{min}$ or $L = 0$. In addition, L has a zone of saturation.

The logistic function is a good one for a description of similar processes.

In such a problem, setting consumption C would be rendered possible after the procurement of production Y and capital K.

The problem is that consumption C can be determined from the balance equation or the formula by *Cobb–Douglas* if the latter is accepted as a part of the model. These two values do not coincide in the general case.

The *Solow* model contains a detailed description of all characteristics depending on the following two varying factors: physical capital K and labour L. Consumption C could be present in such types of models. However, it may ideally be considered via the labour L model. There are two conditions that are accepted:

1. When consumption C tends to 0 then labour L tends to L_{min}. There is the so-called slavery labour that enables the sustenance of this relation.
2. If consumption C tends to ∞ then labour L tends to L_{max}, and this could be obtained through the constraint for physical capital K.

In the framework of the above constraints, the dynamical model has been constructed, which realises some properties of the process:

1. There is some value of the labour L tension when the economic is characterised with the decrease of the system through time.
2. On the other hand, there are some values ensuring the increase of the economical characteristics.
3. Also, it is a very important moment when the labour L tension belongs to a certain diapason, for then, the economical characteristics are stable in time.

Qualitative investigations based on the *Mankiw–Romer–Weil* economics model give a larger number of such diapasons and boundaries. This issue is a problem for further study. Moreover, it is possible to construct models where the areas of physical capital K and human capital H are separated into the spheres of the special economic problems. For example, this approach could be applied to the agricultural problems or the transportation ones (including aviation), civil engineering and construction business systems, computer technologies and finally to the area of various scientific technologies.

It is important to bear in mind that the procedures outlined in the present study need not be confined in their scope to the choosing of the values of the preference function occurring in the examples discussed herein; rather, the preference functions are obtained in the explicit view following from the optimisation. In the examples discussed here, it is possible to show the change in the subjective entropy of the processes. This would explain the behaviour of the processes under study in the article but it could be further elaborated in further research.

5. CONCLUSIONS

At the current stage, studies into the creation of the macroeconomics classes of models, as well as their qualitative and quantitative investigations, are still underway. In addition, the applicability of these ideas extends predominantly to the higher educational issues.

The seminal feature is the production of the scientific product, which relates mostly to the intellectual product rather than equipment. Production of 'discovery' that can be evaluated though the years ahead is the peculiarity of such kind of human activity.

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