Monitoring technical systems with the use of relational fuzzy cognitive maps

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In this work, a certain approach to the modeling dynamic states in technical systems is presented. This approach consists in a replacing the classic differential model with a model based on relational fuzzy cognitive map. Described method is illustrated on a practical example of a simple electrical RLC circuit. The method of preparation of the normalized reference data was described. The process of building a relational fuzzy cognitive map with concepts crucial from the modeling purpose point of view is presented; also the results of such a map learning process are shown. At the end a partial comparison of simulation results of work of models: classic – based on the set of differential equations and proposed – based on relational fuzzy cognitive map is performed.

1. Introduction

Cognitive maps in variants: crisp and fuzzy can be applied in intelligent modeling of imprecise objects. Different aspects of such an approach were presented e.g. in [1]-[10], where possibilities of a synthesis [2, 4, 9, 10] as well as analysis [1, 5, 7] of such type models (building, learning, fuzzyfication, etc.) were taken into consideration.

The paper is devoted to designing and applying the relational fuzzy cognitive maps at the modeling of precise technical systems on the example of a simple electro technical circuit with transient states.

Application of traditional modeling methods (based on e. g. sets of differential equations) sometimes involves, at improper parameters selection, a risk of arising instability of described process and, in consequence, difficulties in the analysis. Dynamic relational cognitive maps, especially in terms of fuzzy, can represent a finiteness of real processes, which fulfils (through description using fuzzy operators) real object stability conditions.

It needs to be stressed that some aspects of applying the fuzzy cognitive maps in modeling electro technical systems were presented in [8].

Comparative results of the application of a relational fuzzy cognitive map and classic method of the modelling, on the example of a specific RLC circuit, and conducted simulation analysis short summary are presented further in the chapter.

2. Theoretical background

Generally, fuzzy cognitive map (FCM) [2, 4, 5] is a mathematical tool to model the systems that are characterized by large level of incompleteness and imprecision of data. The incompleteness refers to the incomplete knowledge about the internal structure of the system and the physical phenomena, which take place in the system, what makes difficult to construct its classic mathematical description. The imprecision is understood as the use of subjective measures of the system parameters, such as e.g. temperature (“it’s cold”, “it’s warm”, ... instead formal Celsius scale), pressure (“high pressure”, “low pressure”, ... instead values in Pacals), current (“low current”, medium current”, ... instead values in Ampers), etc. The systems with these characteristics (economic, geographic, medical, sociological, and others) are very common in the world around, and therefore innovative mathematical tools, such as the FCM models are in high demand. In FCMs the problem of imprecision of information is solved through a specific, graph-based structure of the model in which nodes are the main concepts of the modeled system, indicated by the experts, and edges reflect causal relationships between these concepts. As for the imprecision of determining the values of concepts and relationships between them, FCMs use fuzzy logic [1, 5]. General structure of the classic FCM shows Fig. 1.

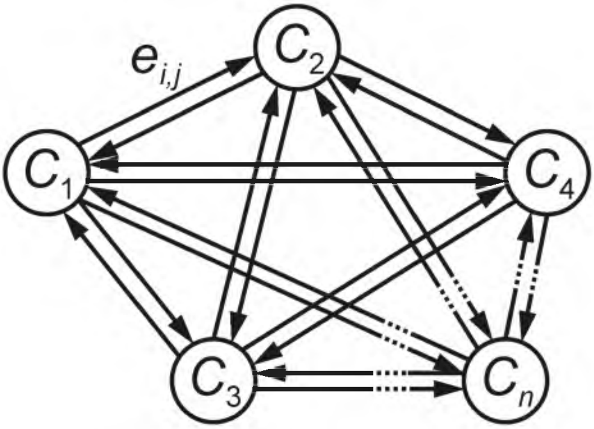

Fig. 1. General structure of a fuzzy cognitive map, where: C_i – fuzzy concepts (i = 1,..., n); $e_{i,j}$ – fuzzy causal connections between concepts i and j; n – number of concepts

FCM technique has been successfully developed for over 25 years, but, in its classical sense, has some features (especially in the context of different approaches to adaptation of parameters) inconsistent to the philosophy of fuzzy model, because in most applications it operates on numerical representations of fuzzy quantities instead of their linguistic values.

Relational fuzzy cognitive map (RFCM) [7, 9, 10] is a novel approach to the modeling and monitoring with the use of FCMs in which fully fuzzy method of inference as well as the model learning is applied. It can be reached because all elements of the model, i.e. concepts and connections (relations) between concepts have fuzzy nature, with the values of the concepts described using fuzzy numbers [3] and the connections – with the use of fuzzy relations instead of the traditional fuzzy linguistic values.

RFCM, similarly to FCM, is a kind of mathematical model in which physical quantities of the modeled system are substituted by concepts crucial for the modeling purposes and the flow of the information between quantities runs by relations that connect individual concepts. Its specific attribute is the use of normalized values of individual concepts and relations. General form of RFCM is also in a certain sense similar to FCM and can be written like in (1).

$$\langle \mathbf{X}, \mathbf{R} \rangle \tag{1}$$

where: $\mathbf{X}$ – set of fuzzy values of concepts ($\mathbf{X} = [X_1, ..., X_N]^T$); $\mathbf{R}$ – set of fuzzy relations between concepts ($\mathbf{R} = \{R_{i,j}\}_{i \neq j}$); $i, j = 1, ..., N$; N – number of concepts.

Applying the fuzzy numbers to represent the concepts means, that fuzzy sets used for this purpose must meet special requirements [3], which in turn impose certain restrictions on the membership functions of these sets [3]. One of possible forms of such a function is a Gauss-based function shown by equation (2):

$$\mu_{\overline{X}_i}(u) = e^{-\left(\frac{u-\overline{X}_i}{\sigma_i}\right)^2} \tag{2}$$

where: $\overline{X}_i$ – center of the membership function of the i-th linguistic variable; $i = 1, ..., K$; K – number of linguistic variables; σ_i – fuzziness coefficient of the i-th linguistic variable; u – the point of universum, where the function $\mu_{\overline{X}_i}$ is calculated.

An example of run of the function (2) is shown in Fig. 2, where $\overline{X}_i$ = 0,5.

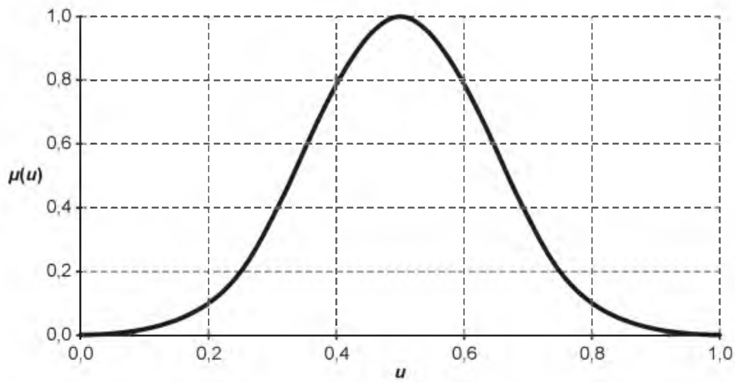

Fig. 2. Hypothetical run of the Gauss-based membership function

Fuzzyfication of the relations between concepts is a more complex issue. This can be done using a wider range of membership function types, but it is convenient to use the form of Gaussian function in accordance with equation (3):

$$\mu_{R_{i,j}}(u_1,u_2)=e^{-\left(\frac{x_2-r_{i,j}(u_1)}{\sigma_{i,j}}\right)^2} \tag{3}$$

where: $\mu_{R_{i,j}}$ – member function of the fuzzy relation between concepts i and j; u_1, u_2 – axes of universum for fuzzy relation; $\sigma_{i,j}$ – fuzziness coefficient of the fuzzy relation between concepts i and j; $r_{i,j}$ – strength of the fuzzy relation between concepts i and j ($r_{i,j} \in [-1, 1]$).

Graphical representation of such a relation is presented in Fig. 3.

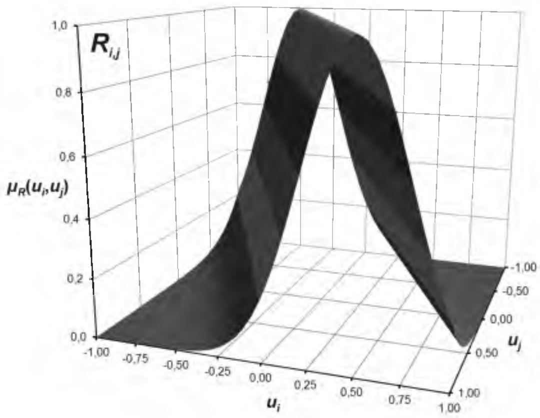

Fig. 3. Hypothetical shape of the Gauss-based membership function of a fuzzy relation

One can apply different working models of fuzzy relational cognitive maps. One of them is a dynamic model described by equation (4):

$$X_i(t+1)=X_i(t)\oplus\bigoplus_{j=1}^{N}[(X_j(t)\oplus X_j(t-1))\circ R_{j,i}] \tag{4}$$

where: i – the number of considered output concept (i = 1, ..., N); t – discrete time; $\oplus$ – fuzzy addition operation; $\oplus$ – fuzzy subtraction operation; $R_{j,i}$ – single fuzzy relation between fuzzy concepts j and i; $\circ$ – maxmin fuzzy composition operation; N – number of concepts.

Characteristic feature of a model based on relational fuzzy cognitive map is possibility of the creation relations (connections) between concepts, which are purely abstract, not reflecting physical flow of signals between elements of a real system. It is associated with ignoring the concepts that are irrelevant from the point of view of the modeling purpose. At this point, it should be noticed that, dependently on assumed purposes, cognitive maps containing different groups of concepts can be built for the same system. Moreover, these concepts can be physical quantities occurring in a real system as well as quantities with qualitative nature ”added” to the model for the needs of achieving the specific purpose. Independently of the number and the nature of concepts, proper selection of relations between them plays a crucial role. These relations can be assigned to initial values basing on expert knowledge, but mostly it is necessary to additionally teach the model (e.g. through the comparison of the model work results with the reference results obtained from a real system).

Wider description of methods of designing the RFCMs, fuzzyfication of concepts and relations, and arithmetic operations on fuzzy numbers are contained, among others, in [7, 9, 10].

3. The problem formulation

Modeling technical systems is not fundamentally different from modeling the other systems. First of all, it should be defined whether the modeling purpose is gaining the information on the model state at certain point of time, or reproducing the time courses of values of selected quantities. Then, concepts crucial from the modeling purpose point of view should be selected. In extreme cases, the number of these concepts can correspond with the number of quantities existing in a real system, but usually the assumed purpose can be reached for significantly lower number of considered concepts.

Below, on a selected example of simple RLC circuit (shown in Fig. 4), certain approach to the build of model of technical object basing on relational fuzzy cognitive map, is presented.

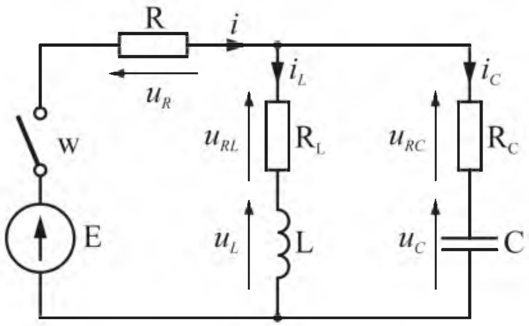

Fig. 4. Tested RLC circuit. $R = 10\ \Omega$; $R_L = 0.5\ \Omega$; $R_C = 0.5\ \Omega$; $L = 0.08$ H; $C = 0.03$ F; $E = 10$ V

For the needs of this problem, dynamic model of fuzzy relational cognitive map considering 4 concepts: E, i_L, u_C and i, was chosen. These concepts values were marked: X_1, X_2, X_3 and X_4, and the cognitive map based on them is schematically presented in Fig. 5.

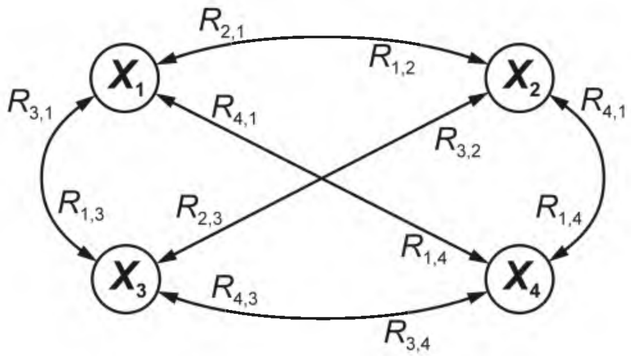

Fig. 5. General diagram of the relational fuzzy cognitive map chosen for the method testing, where: $R_{i,j}$ – fuzzy relation between concepts i and j

The purpose of modeling is to find the time courses of all the concepts after the step change one of them – between two consecutive steady states of the system (for certain value T of discrete time t).

4. Obtaining the reference courses

For purposes of a cognitive map supervised learning the access to reference data is necessary. It can be obtained through measurement or gathering statistical data. For the purpose of this example the method consisting in a building precise mathematical model of tested system and then simulating its work was chosen. Such an approach is especially advantageous for testing the created method because it makes easier the representation of different states of the system work.

The circuit from Fig. 4 can be modeled using a set of equations (5):

$$\begin{cases} \dfrac{di_L(t)}{dt} = \dfrac{1}{L}\left[E(t) - Ri(t) - R_L i(t)\right] \\ \dfrac{du_C(t)}{dt} = \dfrac{1}{CR_C}\left[E(t) - Ri(t) - u_C(t)\right] \\ ic = \dfrac{E(t) - Ri_L(t) - u_C(t)}{R + R_C} \\ i(t) = i_L(t) + i_C(t) \\ u_R(t) = Ri(t) \\ u_L(t) = E(t) - u_R(t) - R_L i_L(t) \end{cases} \tag{5}$$

The modeling subject are time courses of selected currents and voltages in a situation when, in earlier stabilized circuit, the disturbances, consisting in step lowering the voltage E value from 10 V to 5 V, arises. Exemplary course of $i_L(t)$ current in time of 1 second is presented in Fig. 6.

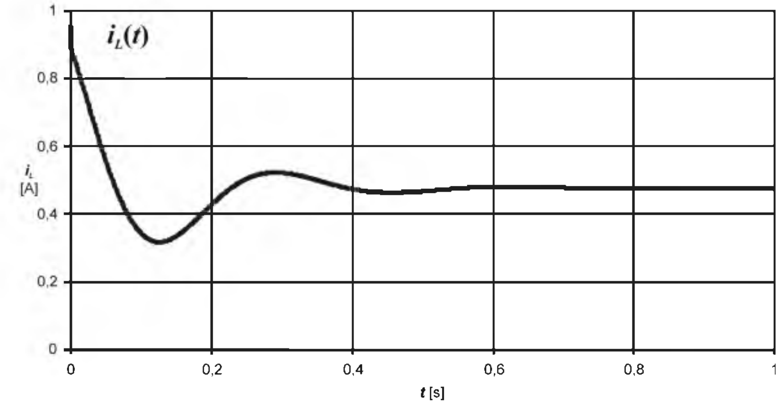

Fig. 6. Exemplary course of $i_L(t)$ current value obtained from classic differential model

The course from Fig. 6 can't be yet a reference for the RFCM learning. Firstly, it was made in a simulation process with time step equal to 1 ms, which means that to obtain it 1000 steps were made, which is too many from cognitive modeling point of view. Secondly, similar courses were obtained also for other quantities and there were calibrated in different units, which makes them poorly useful to build dimensionless relations. To solve these problems discrete time was introduced, where one step corresponds to 50 ms of a real time. Moreover, dimensionless normalization [7] was applied in which, for each considered quantity, value 0 corresponds to real value of stationary state (from before the appearance of the disturbance) and values 1 and –1 correspond to limitary real values anticipated for given quantities. In such a depiction exemplary course from Fig. 6 get the form like in Fig. 7.

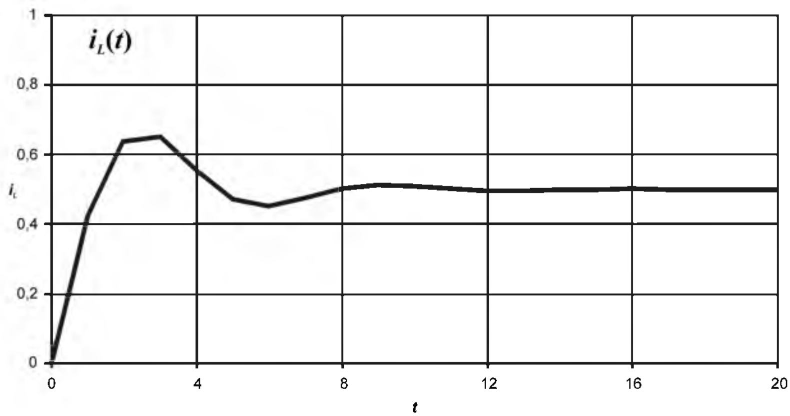

Fig. 7. Current $i_L(t)$ value course transformed do the normalized form for the needs of the model teaching, where: t – discrete time

4. Preparing the model

The RFCM model constructed on the base of a system from Fig. 4 is of fuzzy nature. All concepts were fuzzyfied basing on function type (2), whereas relations – on function type (3) (for defuzzyfication purposes the weighted average method was used) [7].

The work of a dynamic model relational fuzzy cognitive map was simulated on the basis of equation (4) [10]:

$$X_i(t+1) = X_i(t) \oplus \bigoplus_{j=1}^{4} [(X_j(t) \oplus X_j(t-1)) \circ R_{j,i}] \tag{6}$$

A learning procedure for initially designed relational fuzzy cognitive map (i.e. adaptation of power coefficients and fuzziness coefficients of fuzzy relations) was performed by a population-based difference method with variable length of the change step. Initial values of the learning process were common for all relations and they were amounted to: $r = 0$ and $\sigma = 0.4$. Fuzziness coefficients for concepts had identical value for each and were amounted to 0.6. There were also assumed universum with the range [–2, 2] and the number of linguistic variables $K = 17$.

Model learning procedure was performed separately for each successive discrete time step (for each t-th cycle of signals circulation inside the dynamic model) and it had a form of the algorithm consisting in recurrent execution of the following consecutive steps (similar algorithm was presented in [7]):

1. Take the initial value of t.
2. Select the initial values of coefficients of changes of parameters: Δr and $\Delta\sigma$.
3. Execute t cycles of circulation of signals in fuzzy model.
4. Calculate aberrations of defuzzyfied values of concepts from reference values.
5. For each tested concept (e.g. X_p) and each fuzzy relation, which introduces to it signals from other concepts (e.g. $R_{i,p}$) check out whether it's possible to reach the concept value closer to the reference value after increasing or decreasing the relation power $r_{i,p}$ with Δr. If yes, take new $r_{i,p}$ value.
6. For each tested concept (e.g. X_p) and each fuzzy relation, which introduces to it signals from other concepts (e.g. $R_{i,p}$) check out whether it's possible to reach the concept value closer to the reference value after increasing or decreasing the fuzziness coefficient $\sigma_{i,p}$ with $\Delta\sigma$. If yes, take new $\sigma_{i,p}$ value.
7. After modification of all fuzzy relations check out whether assumed model accuracy was reached. If not, go back to step 2. Also values of parameters Δr and $\Delta\sigma$ can be changed (decreased) if it's judged necessary.
8. Repeat above steps until assumed criterion of the algorithm end is reached.
9. If $t < T$ then increase the value of t by 1 and go to step 2.

5. Selected simulation results

The above mentioned learning algorithm, applied into designed RFCM model, resulted in the stabilization of power and fuzziness coefficients of relations with final values like in Table 1.

Table 1. Power (a)) and fuzziness coefficients (b)) of relations in cognitive map, achieved as a result of the teaching process

a)

r	X_1	X_2	X_3	X_4
X_1	0	0.85	0.95	0.85
X_2	0	0	–0.25	0.15
X_3	0	0.25	0	0.05
X_4	0	0.10	–0.20	0

b)

σ	X_1	X_2	X_3	X_4
X_1	0.40	0.25	0.15	0.24
X_2	0.39	0.40	0.30	0.36
X_3	0.39	0.29	0.40	0.38
X_4	0.39	0.29	0.36	0.40

Remark 1. For the improvement of complete time courses representation, learning process for each consecutive step of discrete time was starting from optimal values obtained for previous step.

Then computer simulations with the use of a RFCM model from Fig. 5, where relations were built on the basis of parameters from Table 1, were performed. The time courses obtained in this way, put together with reference courses obtained from classic model, are presented in Fig. 8

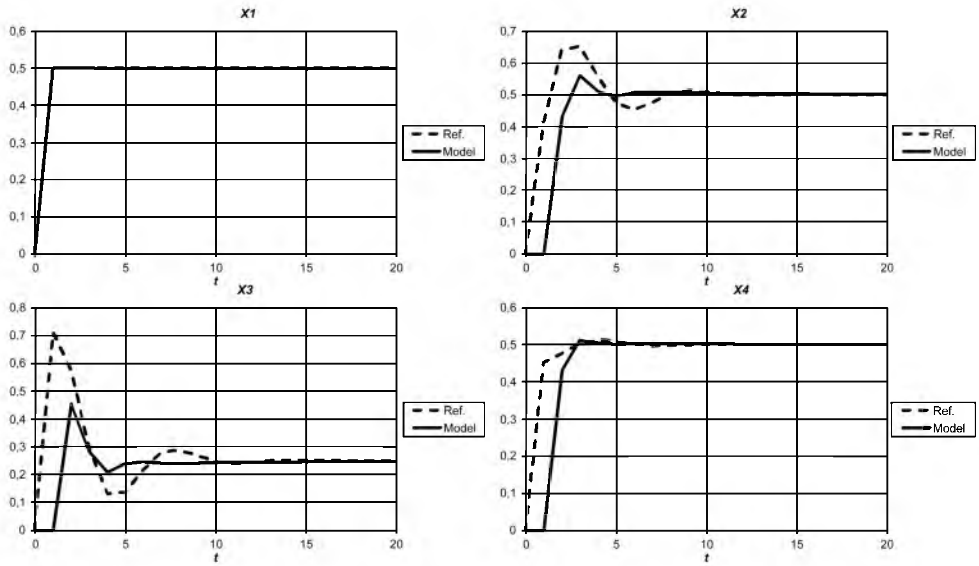

Fig. 8. Comparison of time courses (for normalized values) of selected quantities for classic (Ref.) and based on relational fuzzy cognitive map (Model) modeling

Remark 2. It should be noticed that courses from Fig. 8 are presented in normalized (dimensionless) form. Transition to dimension (physical) form is a simple arithmetic procedure, which can be performed with the use of the same coefficients as normalization process.

The model of the relational fuzzy cognitive map was built on the basis of only four concepts. From this reason, even though all concepts have their reflections in physical quantities, relations nature is purely abstract. Moreover, the number of applied linguistic variables (17 on universum [–2, 2], which means 5 linguistic variables at standard universum range [0, 1]) is relatively low. Despite these limitations, proper adaptation of parameters of fuzzy relations allowed to achieve good representation of modeled courses.

6. Conclusions

The application of relational fuzzy cognitive maps for modeling the technical systems on the example of simple RLC circuit was described. Selected results of the simulational comparison with classic model (described with differential equations) have been quoted. The analysis of the results shows that simple (with small number of concepts) relational fuzzy cognitive maps can well approximate the work of technical systems, moreover these results are comparable with ones achieved in classic model. Currently performed works on the refinement of methods of a RFCM learning should lead to continued improvement of precision of such models.

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