Improvement of the Vibration Diagnostics of Rotation Shaft Damage Based on Fractal Analysis

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Abstract

This work is devoted to further research and improvement of the vibration diagnostics of the initial crack-like damage of rotation shaft in aviation gas-turbine engines (GTE). We propose to use fractal analysis of the accelerating shaft response in order to increase the damage detection efficiency. Responses of the accelerating shaft are derived by using simulation in absence and in presence of the initial traverse crack. The responses of the cracked shaft have sub-critical peaks; the increase in size of a crack leads to the increase in peak values of the vibration amplitude in the range of sub-harmonic resonances. The Hurst exponent is obtained for the time series in the range of sub-harmonic resonances. The research shows that a small change in the crack size results in considerable change of the Hurst exponent, which allows to detect the mentioned sub-harmonic resonances of the measured signal in order to identify the initial crack-like damage of the rotation shaft.

Keywords: gas-turbine engine, cracked shaft, vibration diagnosis, fractal analysis, Hurst exponent

1. Introduction

This paper is a continuation of the previous researches [1,2] dedicated to development of the multilevel vibration control system of aviation gas-turbine engines (GTE) and its practical implementation. The system mentioned above comprises the following three levels: (i) the first (main) level - for current control and awareness of the actual levels of vibration at the harmonics of the rotor rotation, (ii) the second (auxiliary) level - for diagnostics of the initial crack-like damages of the engine blades and (iii) the third (auxiliary) level - for diagnostics of the initial crack-like damage of the rotor's shaft during startup at the acceleration to operating speed. In order to diagnose the damage of the rotor's shaft, the peak values of vibration amplitude in the range of sub-harmonic resonances of accelerating cracked shaft response are used as fault features. Therefore, the narrow-band digital tracking filter was developed in order to extract the main rotor harmonic vibration at the non-steady-state mode, as presented in [2]. The peak values of vibration amplitudes are determined after filtration in the field of sub-harmonic resonances. The received values are compared with the threshold and the decision on the presence or absence of a crack in the shaft is made.

We propose to improve the diagnostics of the initial crack-like damage of the rotating shaft by using fractal analysis of the accelerating shaft response in order to increase the efficiency of the damage detection. It is very important for detection of the initial crack-like damage and especially in case of the low signal-to-noise ratio.

Fractal analysis is a promising signal processing method used for the noise-like signals [3]. The analysis of fractal and multifractal properties of time series allows obtaining simple and suitable characteristics of the investigated signals, such as the fractal dimension, Hurst exponent, and other characteristics (correlation dimension, embedding dimension), if necessary. Changes of the mentioned characteristics can be used to detect the local changes in the measured signal which are generated by the initial crack-like damage of the rotation shaft.

2. Estimation the Hurst exponent

We propose to use the Hurst exponent of the accelerating shaft response as a faul feature. The oldest and best-known method to estimate the Hurst exponent is *R*/*S* analysis [4]. Ratio *R*/*S* indicates ratio of the range *R* to the standard deviation S of the analyzed time series. The procedure of estimation of the Hurst exponent presented in [4] is as follows:

1. It is necessary to find the mean *E* and the standard deviation *S* of the analyzed time series Z_i ($i = 1, \dots, n$).
2. The data of the series Z_i has to be normalized by subtracting the sample mean

$$X_i = Z_i - E$$

3. Create the cumulative time series for $i = 1, \dots, n$:

$$Y_i = \sum_{j=1}^{i} X_j$$

4. Find the range

$$R = max(Y_1,...,Y_n) - min(Y_1,...,Y_n)$$

5. Calculate the mean value (*R*/*S*) of the series of length *n*.
6. Obtain the value of Hurst exponent *H*, taking into consideration that the *R*/*S* statistic asymptotically follows the relation

$$R/S \approx \tau^H ,$$

where τ is a time interval of the analyzed time series Z_i.

The value of Hurst exponent allows to recognize a persistent process ($H > 0{,}5$) and anti-persistent process ($H < 0{,}5$), for a Gaussian noise $H = 0{,}5$.

3. Simulation and analysis of accelerating shaft response

The equations of motion for a Jeffcott rotor with a cracked shaft in presence of the gravity forces and unbalance excitation, and subject to constant acceleration, were presented and investigated in [5]. The following from among the equations of motion mentioned above are used for simulating of the accelerating shaft response:

- in inertial coordinate system (*xyz*):

$$\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}\begin{Bmatrix} \ddot{z} \\ \ddot{y} \end{Bmatrix}+\begin{pmatrix} F & 0 \\ 0 & F \end{pmatrix}\begin{Bmatrix} \dot{z} \\ \dot{y} \end{Bmatrix}+\begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{21} \end{pmatrix}\begin{Bmatrix} z \\ y \end{Bmatrix}=\begin{Bmatrix} Mg \\ 0 \end{Bmatrix}+M\varepsilon\begin{Bmatrix} \dot{\theta}^2 \cos\theta+\ddot{\theta}\sin\theta \\ \dot{\theta}^2 \sin\theta-\ddot{\theta}\cos\theta \end{Bmatrix}, \tag{1}$$

where **M** is the mass matrix; **F** is the damping matrix; **K** is the stiffness matrix; z and y are the displacements; θ is the angle of orientation of unbalance mass ε relative to the axes z;

- in body-fixed rotating coordinate frame ($\xi\eta\varsigma$):

$$\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}\begin{Bmatrix} \ddot{\xi} \\ \ddot{\eta} \end{Bmatrix}+\begin{pmatrix} F & -2M\omega(t) \\ 2M\omega(t) & F \end{pmatrix}\begin{Bmatrix} \dot{\xi} \\ \dot{\eta} \end{Bmatrix}+\begin{pmatrix} K-f(\psi)\Delta K_{\xi}-M\omega^2(t) & -F\omega(t) \\ F\omega(t) & K-M\omega^2(t) \end{pmatrix}\begin{Bmatrix} \xi \\ \eta \end{Bmatrix}=$$

$$=Mg\begin{Bmatrix} \cos\Phi \\ -\sin\Phi \end{Bmatrix}+M\varepsilon\omega^2(t)\begin{Bmatrix} \cos\beta \\ \sin\beta \end{Bmatrix}+M\varepsilon a\begin{Bmatrix} \sin\beta \\ -\cos\beta \end{Bmatrix}, \tag{2}$$

where $\omega(t)$ is the instantaneous speed of rotation; a is the constant acceleration of rotation; Φ is the angle of position of the rotating coordinate frame ($\xi\eta\varsigma$) relative to the inertial coordinate frame (*xyz*); β is the angle of orientation of unbalance mass ε with respect to crack; ΔK_{ξ} is the shaft rigidity decrease at the crack presence; $f(\psi)$ is the function for crack accounting to the shaft stiffness according to the crack angular position ψ.

The transformation between the inertial and rotating coordinate frames is carried out according to the following dependence:

$$\begin{pmatrix} z \\ y \end{pmatrix}=\begin{pmatrix} \cos\Phi & -\sin\Phi \\ \sin\Phi & \cos\Phi \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix}. \tag{3}$$

The model of the transverse crack is a function of “breathing”, the relative rigidity changing of the shaft $\Delta K = \Delta K_{\xi}/K$ depends on the cross location of crack section and stress-strain area of the shaft.

The computer simulation of the accelerating shaft response in absence ($\Delta K = 0$) and in presence of a small crack (ΔK = 0,005,...,0,1) is carried out by using the transformed equations (2) to non-dimensional form and dependence (3). The time plots of non-steady-state vibration of the rotating shaft are shown in Figure 1 for the following data: ΔK = (0; 0,01; 0,05; 0,1) and $\psi = \beta = 0^0$. These plots are represented in the relative scale on the ordinate axis (non-dimensional vibration amplitude z) and on the abscissa (non-dimensional time τ). Value τ = 1000 corresponds to transition through critical frequency of rotation. It can be seen that the initial transverse crack results in presence of 1/2 order sub-critical peak, and the increase of the crack parameter ΔK leads to the increase in sub-critical peak values of vibration amplitude.

Simulated signals were processed using the above presented procedure of estimation of the Hurst exponent. We used two separate parts of each signal for the analysis: a) a sample of 500 values of non-dimensional vibration amplitude z in the range of sub-harmonic resonances and b) a sample of 500 values of non-dimensional vibration amplitude z in the range of main resonance. Figure 2 represents the dependence of obtained values of Hurst exponent H on the relative rigidity changing ΔK of the shaft for the mentioned samples.

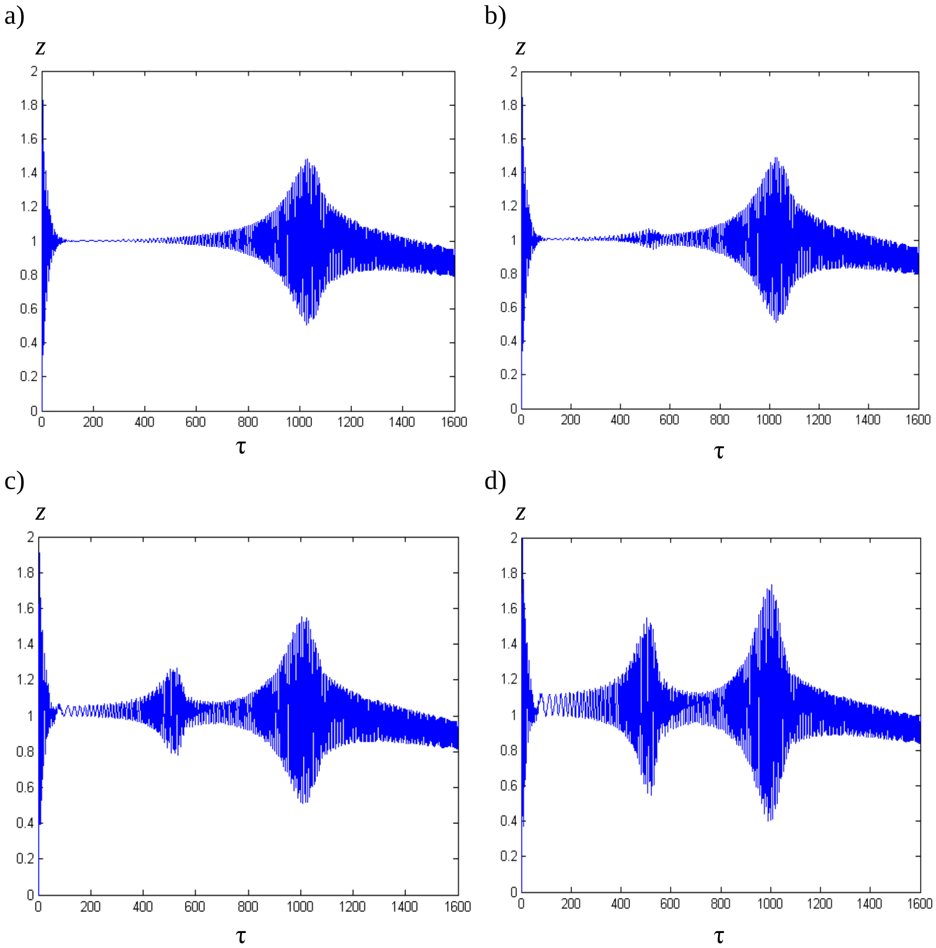

Figure 1. Non-dimensional vibration amplitude of accelerated rotor at the $\Delta K = 0$ (a), $\Delta K = 0{,}01$ (b), $\Delta K = 0{,}05$ (c) and $\Delta K = 0{,}1$ (d)

In general, the Hurst exponent is decreasing at the increasing of a crack parameter ΔK for both analyzed parts of simulated signal. It can be seen in Figure 2b that the initial transverse crack results in small changing of Hurst exponent of signal in the range of main resonance (decreasing is about 19%). In the range of sub-harmonic resonances (Fig. 2a) the value of Hurst exponent is decreasing to a considerable extent, this decreasing is more than 3 times at the interval of relative rigidity changing $\Delta K = 0{,}005, \ldots, 0{,}1$. In the case of $\Delta K < 0{,}005$, the Hurst exponent dependence on ΔK is not informative for crack detection.

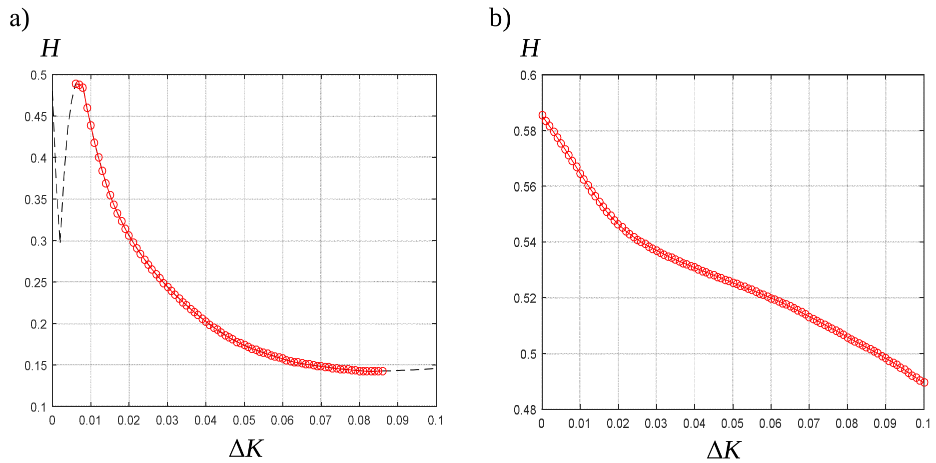

Figure 2. The Hurst exponent dependencies on relative rigidity changing ΔK in the range of sub-harmonic resonances (a) and in the range main resonance (b)

Another simulation and fractal analysis of signals are carried out taking into account of additive Gaussian noise. The value of noise standard deviation is selected 10^{-2}, in this case the value of signal to noise ratio (SNR) is different for each simulated signal. The noisy vibration amplitudes z_n in the range of sub-harmonic resonances for $\Delta K = 0$ and $\Delta K = 0{,}05$ are illustrated in Fig. 3.

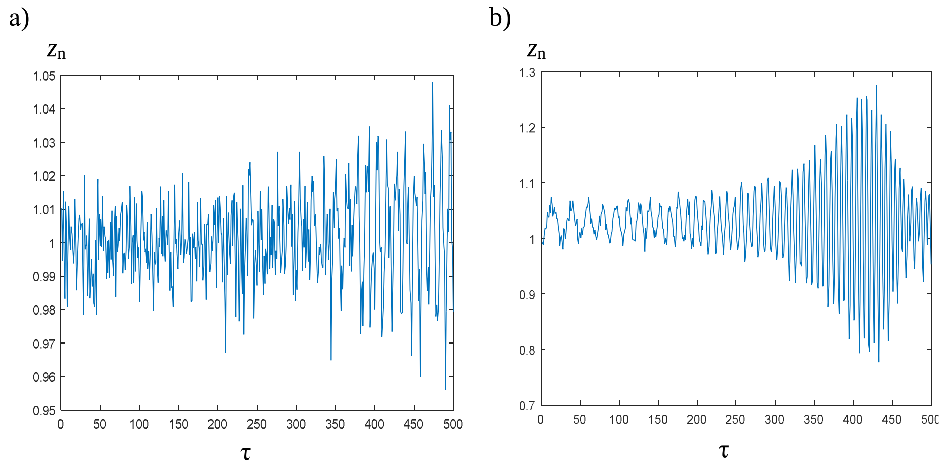

Figure 3. Non-dimensional noisy vibration amplitude in the range of sub-harmonic resonances at the $\Delta K = 0$ (a) and $\Delta K = 0{,}05$ (b)

Fig. 4 shows dependence of values of Hurst exponent H on the relative rigidity changing ΔK, which are obtained for the noisy vibration amplitudes z_n in the range of sub-harmonic resonances. The presented result show, that values of Hurst exponent is decreasing at the increasing crack parameter ΔK. The form of depengence is simiral to the graph represented in Figure 2a, the changing of Hurst exponent is more than 3 times in the presented interval of ΔK. Taking into account of additive Gaussian noise eliminates method error of Hurst exponent estimation at the $\Delta K < 0{,}005$.

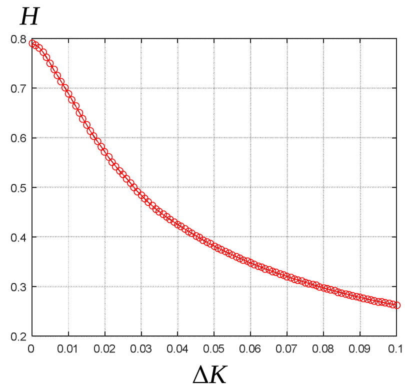

Figure 4. The Hurst exponent dependence on ΔK for the noisy vibration amplitudes in the range of sub-harmonic resonances

4. Conclusions

Research presented in this paper shows that a small change in the relative rigidity changing of shaft in presence of the initial crack-like damage results in considerable change of the Hurst exponent. This fact allows to detect the small sub-harmonic resonances of the noisy measured signal and to identify the initial crack-like damage of the rotation shaft. The usage of proposed approach to improvement of diagnostics of the crack-like damage will promote to ensure awareness of GTE.

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