The Use of Computer Simulations in the Analysis of Physical Phenomena Models

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Abstract

Tests of the technical condition of machines sometimes involve the analysis of dynamic phenomena associated with their work. The simplest methods are based on basic measurements, e.g. effective velocities and body vibration accelerations as well as sound pressure levels recorded in close vicinity of the tested object. For analysis, more advanced techniques use signals recorded in a certain period of time representing the studied phenomena. The development of computer techniques has enabled a relatively convenient modelling of technical objects. A well-identified model allows to extend research on real objects to include comprehensive computer analyses, thanks to which we can reduce time-consuming measurements or perform analyses for excitations which are difficult to perform at a test stand. Another application of a model is to generate signals, on the basis of which we can choose parameters of analysis methods to increase the effectiveness of diagnosis or test new analysis and inference algorithms. Several presented examples were implemented in the Matlab Simulink environment. Because the presented methods are universal, they can be used to analyze any physical phenomena described with more or less complicated models.

Keywords: dynamic models, simulation, signal analysis

1. Introduction

Processes related to designing and physical system tests should be supported by computer calculations. In addition to designing and manufacturing support methods, all types of simulation methods, including those related to dynamic phenomena, are becoming more and more popular. They complement dynamic properties tests carried out during active experiments. In many cases, it is impossible to conduct an experiment, e.g. due to technological reasons or because of the cost of carrying it out. An important factor is also the duration of the test which can be very long (lasting up to several years).

Regardless of the field of technology, the computer simulation process should be preceded by modelling of a real system. This is most often done on the basis of the physical description of the examined object. It should take into account basic characteristics of the system and the medium (cooperating systems) with which the examined system interacts (cooperates). Based on the physical model, we create a mathematical model, which is an abstract description that can be used in computer modelling. Its degree of simplification should not introduce significant errors in the reproduction of basic physical phenomena.

If the created model is to support diagnostics of the technical condition of tested machines, it should be correctly identified and verified [1]. Signal processing and analysis methods are often used for this purpose. Models underlying simulation can also be used to test and examine the effectiveness of applied or new inference algorithms.

2. Application of signal analysis to simulation studies

The combination of computer simulation with the interpretation of obtained results, based on the methods of signal analysis, may create some technical problems. Algorithms for iterative solving of ordinary differential equations are usually based on various modifications of the Runge-Kutta method. In these methods, step sizes of calculations are important [2]. The step size will vary depending on the dynamics of the model. With rapid changes in the state of the model, the step size is reduced in order to maintain the accuracy of calculations. When the states of the model change slowly, the step size increases.

Calculations performed using digital signal analysis methods require the adoption of a constant step Δt between individual points of time series [3]. This is due to the lack of explicit representation of t time in computer calculations, which is replaced by the numbers of consecutive n points (time $t_n = (n-1)\cdot\Delta t$). With a variable step, in extreme cases, there can be a significant difference in the reproduction of the function. As a result, there will be analysis errors obtained from the simulation of signals. An example of the influence of step variability on signal reproduction is shown in Figure 1.

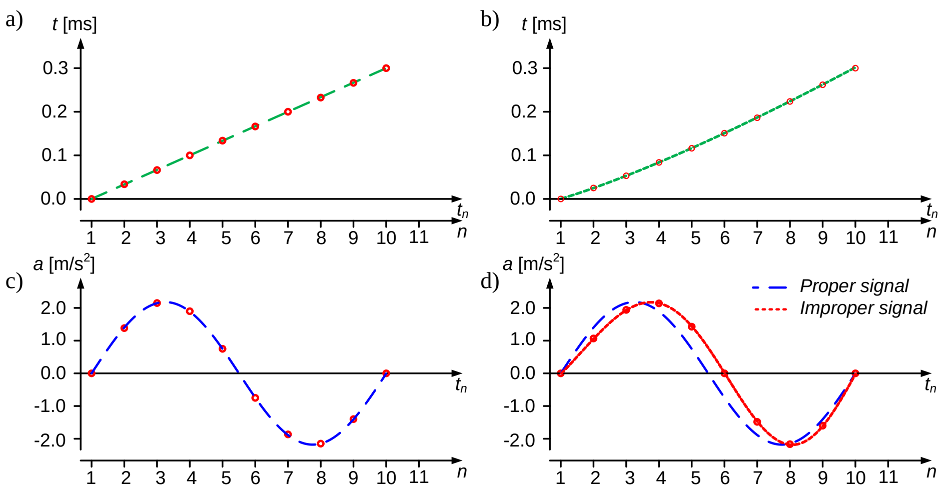

Figure 1. Example of the influence of the variable time resolution Δt:
a) constant value Δt, b) variable value Δt, c) properly sampled signal (Δt = const),
d) comparison of proper and improper signals

However, the use of constant-step simulation methods suffers from several disadvantages, the most important of which is a decrease in accuracy and increase in calculation time. However, a variable step is recommended for models in which states change rapidly or which contain discontinuities. In these cases, the variable step simulation method requires fewer time steps than the constant-step method to achieve a comparable level of accuracy. This can significantly reduce simulation time.

The right way seems to be to use simulation with a variable step, but with a limited maximum value to the value of constant time resolution Δt, required in digital signal analysis, equal to the inverse of sampling frequency f_s. Figure 2 presents a modification of the parameters of solution methods in Matlab Simulink.

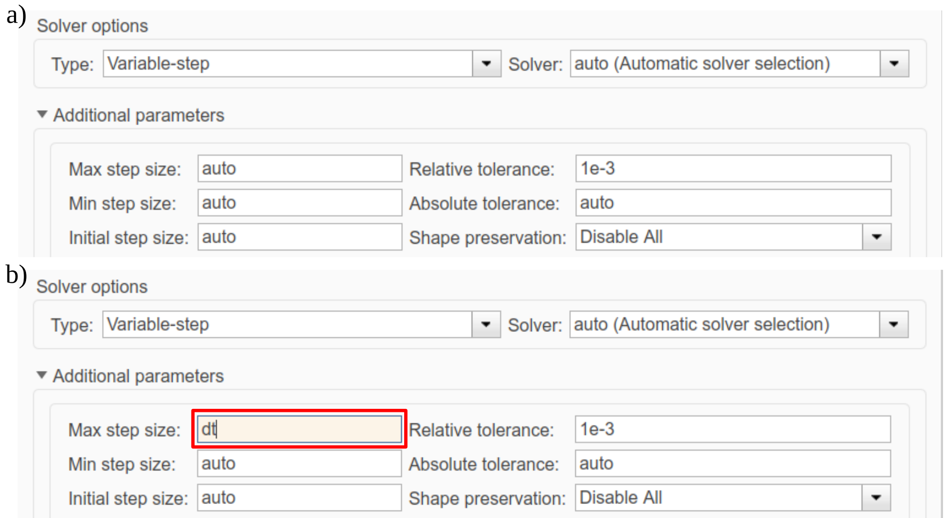

Figure 2. Limitation of the maximum integration step of methods for solving differential equations: a) parameters automatically selected, b) to obtain results for the adopted step Δt

The analyses of complicated systems with many degrees of freedom confirm the effectiveness of this approach. However, one should remember to supplement the simulation and analysis of the results with checking the variability of solution steps. Figure 3 shows examples of the course of changes in the integration step with the maximum step set as *auto* and equal to Δt. A simple system was analyzed, where a rectangular pulse with the minimum possible duration was introduced at the input at t = 0.2 s. With automatic step determination, the simulation only takes place after the system is activated and quickly stabilizes with a large, little variable step of the order of Δt_n = 25-30 ms. When setting the maximum step to the expected value, the calculations will unfortunately take longer, but in addition to cases where Δt_n is not equal to Δt , we get relatively long sections for which signals resulting from the simulation were generated with the required constant step.

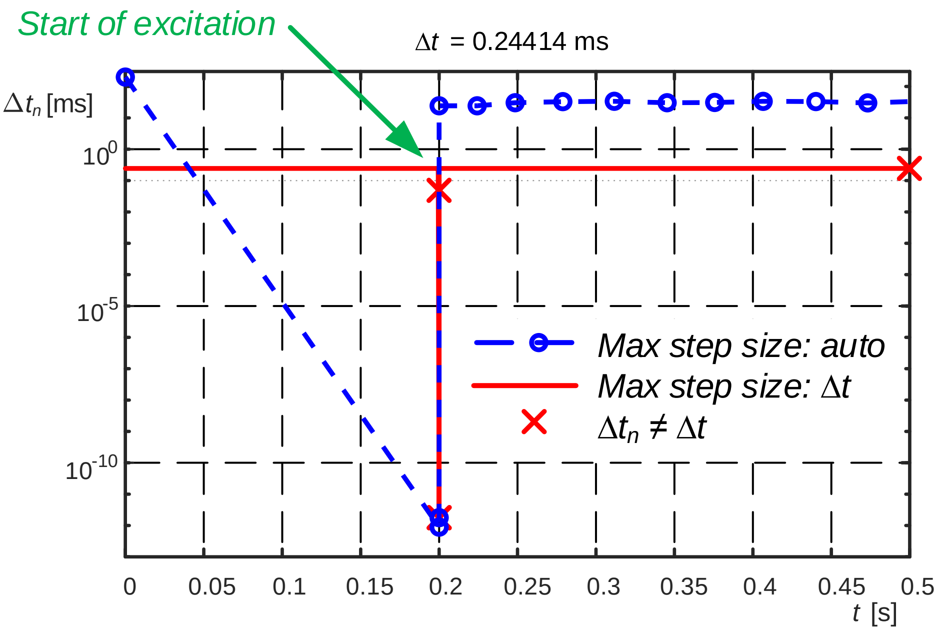

Figure 3. Comparison of integration step changes for different *Max step size* settings

3. Different ways to describe the models

It often happens that the dynamic properties of the same system can be modelled with a differential equation (system of equations), transfer function or spectral transmittance. In this case, an interesting issue may be the analysis of connections of several, theoretically independent, fields of knowledge such as automatic control, vibration theory or signal analysis.

Let us consider, for example, a linear oscillating system with one degree of freedom with dampening and excitation with force varying in time $F(t)$:

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{1}$$

where: m – body weight, c – system dampening, k – system stiffness.

The same system can be presented using the concept of a transfer function using the Laplace transform:

$$mX(s)s^2 + cX(s)s + kX(s) = F(s) \tag{2}$$

where: $X(s)$ – Laplace transform of the output signal $x(t)$ (system response), and $F(s)$is the Laplace transform of the excitation force $F(t)$.

These two quantities are associated with a transfer function, which characterizes dynamic properties of the system in the s domain:

$$G(s) = \frac{X(s)}{F(s)} \tag{3}$$

In the case of a simple system, dynamic properties can be determined by solving equations (1) or (2). Of course, this description will be used in other areas.

In the case under consideration, the solution to equation (1) is, among others, a formula for the vibration amplitude of the system at a given frequency ω of a force harmonically variable in time:

$$A(\omega) = \frac{F(\omega)}{m} \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4h^2\omega^2}} \tag{4}$$

where: ω_0 – natural frequency of the undamped system, $h = \frac{c}{2m}$.

The problem appears when we analyze systems with a greater degree of complexity for which it is difficult to obtain a solution analogous to the one described by equations (1) and (4). Searching for a solution consisting in determining system parameters for different excitation frequencies may be reduced in simulation tests to repeated repetition of calculations. Such an approach significantly extends the solution time. An alternative solution is to use links with spectral transmittance $H(f)$ [4]:

$$X(f) = H(f) \cdot F(f) \tag{5}$$

$$H(f) = |H(f)| e^{-i\varphi(f)} \tag{6}$$

where: $X(f)$ – spectrum of the signal at output $x(t)$, $F(f)$ – spectrum of the signal at output $F(t)$, $|H(f)|$ – gain factor, $\varphi(f)$ – phase factor.

In the example shown, for a system with one degree of freedom, a relationship is observed:

$$|H(\omega)| = \frac{A(\omega)}{F(\omega)} = \frac{1}{m} \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4h^2\omega^2}} \tag{7}$$

where: $\omega = 2\pi f$.

Instead of repeating calculations for harmonic excitations many times, in accordance with the definition of spectral transmittance, it is necessary to stimulate the system with impulse excitation and determine characteristics defined as a function of the frequency response. The calculation time will only depend on the processing speed and the adopted frequency resolution of spectral analysis. Figure 4 shows a comparison of simulation results for a system with one degree of freedom with the values obtained from formula (7).

In the models of examined physical phenomena, e.g. those being the basis for simulation in Matlab Simulink, different description methods can be used interchangeably, and elements of the system described by means of a transfer function (e.g. PID controller) can be combined with the system described by differential equations. The relationship between these systems will be time functions.

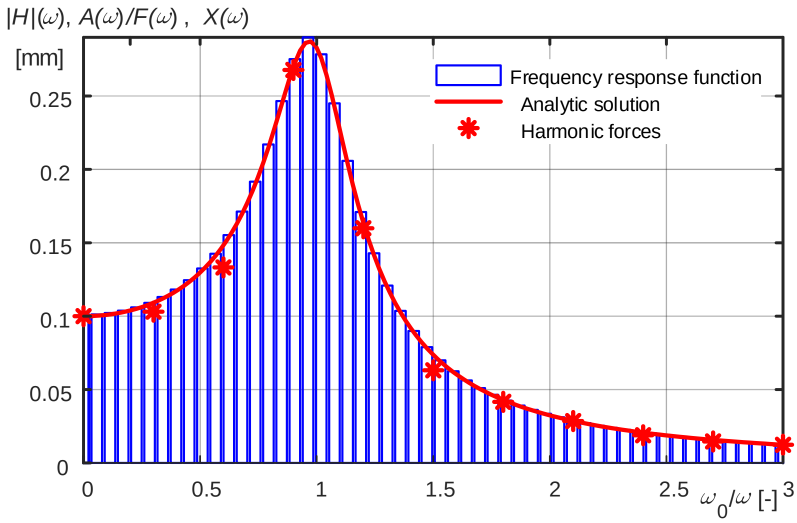

Figure 4. Comparison of different methods for determining the system characteristics

4. Analysis of system dynamics in the time-frequency domain

In many cases of dynamic system modelling, the type of excitation is an important issue. Response characteristics depend on it. This is especially important in systems with more degrees of freedom than one and in nonlinear systems.

Let us consider, for example, a linear vibration system with one degree of freedom without damping, with force varying over time, for which an undamped dynamic vibration eliminator was used. When excited by harmonic force, the system will vibrate at a frequency equal to the excitation frequency. In theoretical considerations amplitude and phase frequency characteristics can be determined. This system has two resonant frequencies, and for the frequency to which the damper has been tuned, the vibration amplitude is zero.

In the case of analysis with excitation by force with frequency of amplitude changes increasing over time, system dynamics should be considered over time and not just in the frequency domain. In a system where vibrating elements are not damped, subsequent forms of vibrations are generated after exceeding resonance frequencies by excitation frequency. Figure 5 shows a change in the response of the vibrating system with a damper when excited by a force with increasing frequency of amplitude changes over time, and Figure 6 shows a change in the response of the system with inertia excitation.

Simultaneous analysis in the time and frequency domain is even more indispensable for a system with dampening. This method of testing can be used to analyze the effects of elimінator tuning and the impact of damping on system vibrations and in technical diagnostics it can be used to determine the impact of changes in the parameters of the vibration eleminator (damper) associated with its degradation. Figure 7 shows vibrations of the system with a dampened eliminator at inertia excitation with the frequency of amplitude changes increasing over time.

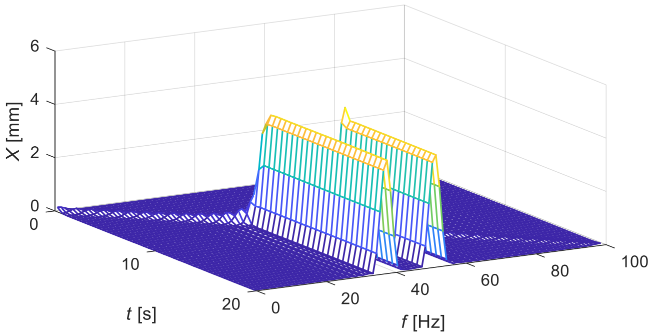

Figure 5. Vibrations of the system with an eliminator when excited by force with increasing frequency of amplitude changes over time

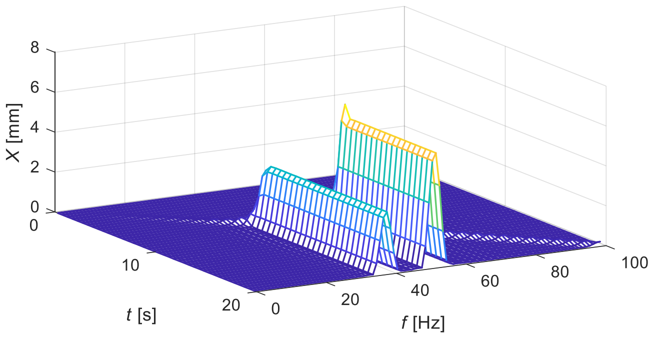

Figure 6. Vibrations of the system with an eliminator at inertia excitation with the frequency of amplitude changes increasing over time

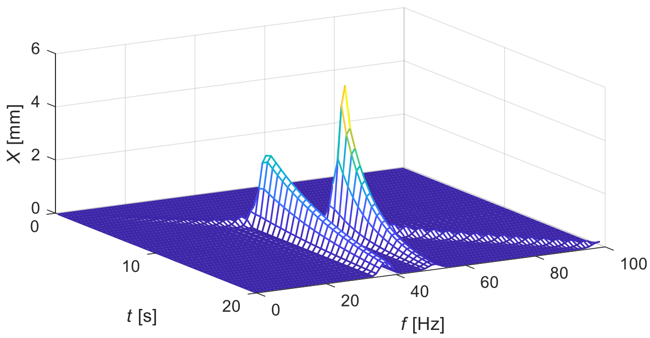

Figure 7. Vibrations of the system with a dampened eliminator at inertia excitation with the frequency of amplitude changes increasing over time

5. Conclusions

The history of simulation research is very long. Antoni Gaudi is the best known person who introduced physical models into structural analysis [5]. For this purpose, he studied laws of structural mechanics, using, among others, catenary curve properties. This method, however, does not take into account dynamic interactions. The development of electrical engineering enabled the creation of analog computers [6] that use electrical-mechanical analogies [4] to create simulation models. These methods already took into account the dynamics of systems operation. The development of digital methods and computational systems meant that the principles used in analog technology can, in effect, be applied in their entirety in discrete technology. The best example would be the development of MathWorks's Matlab Simulink program. Over 25 years, many libraries (including specialized libraries) have been created to support simulations in many fields of physics. Other simulation programs have also been created, e.g. Scilab Xcos, operating similarly to Simulink or MSC Softwares' Adams program.

The use of advanced simulations allows, among others, to analyze changes in system dynamics for various physical parameters [7] (characteristics of elements, e.g. non-linear [8]) without the need for labour- and material-intensive research. In technical diagnostics, simulations can be used to determine the impact of the degradation of mechanical system components on their dynamics and create effective methods of assessing technical condition based on the analysis of obtained signals.

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