

## ELASTIC MODULI OF CARBON NANOTUBES WITH NEW GEOMETRY BASED ON FEM

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In this paper, the elastic moduli of elliptic single walled carbon nanotubes (ESWCNTs) are described. A three-dimensional finite element (FE) model for such carbon nanotubes is proposed. The covalent bonds are simulated by beam elements in the FE model. The elastic moduli of beam elements are ascertained from a linkage between molecular and continuum mechanics. The deformations of the FE model are subsequently used to predict the elastic moduli of ESWCNTs. In order to demonstrate the FE performance, the influence of length, chirality, diameter and cross sectional aspect ratios on the elastic moduli (Young's modulus and shear modulus) of ESWCNTs is investigated. It is found that the cross sectional aspect ratio of ESWCNTs significantly affects the elastic moduli. With increasing cross sectional aspect ratio, the Young's modulus and shear modulus decrease. As a result, every change in geometry operates as a defect and decreases the elastic moduli. With increasing the length, Young's modulus increases and the shear modulus decreases.

*Key words:* elastic moduli, elliptic single walled carbon nanotubes, length, chirality, cross sectional aspect ratio, finite element model

### 1. Introduction

Since discovery of carbon nanotubes (CNTs) they have attracted considerable attention in scientific communities. This is due to their remarkable mechanical, electrical and thermal properties. In particular, composite materials such as carbon nanotube, nanoparticle-reinforced polymers and metals have shown potentially wide applications (Meo and Rossi, 2007).

The novelty in the field of mechanics includes their greatest Young's modulus and tensile strength among all known materials (Thostenson *et al.*, 2001). As a consequence, they may provide the ideal reinforcing material for a new class of nanocomposites (Lau and Hui, 2002a,b). Their practical application, however, poses a great challenge to nanotechnology due to their nanoscale size (Zaeri *et al.*, 2010). Recent investigations (Fereidoon *et al.*, 2012b, 2013; Hemmatian *et al.*, 2011; Rajabpour *et al.*, 2012) have shown that CNTs when aligned perpendicular to cracks are able to slow down the crack growth by bridging up the crack path. Although CNTs can be used as conventional carbon fibers to reinforce polymer matrix in order to form advanced nanocomposites, they may be also used to improve the out-of-plane and interlaminar properties of current advanced composite structures (Lau and Hui, 2002a). These composites have attracted a lot of attention due to their high strength-to-weight ratio (Thostenson *et al.*, 2001).

Qian *et al.* (2002) reported on one of the early investigations on nanotube-based composites; it was observed that when nanotubes of about 1% weight are dispersed homogeneously into

polystyrene matrices, the elastic modulus increases by 36-42% and the tensile strength improves by about 25%. Parl *et al.* (2002) and Smith *et al.* (2002) developed and characterized nanotube polymer composite films for spacecraft applications.

Two main classes of theoretical methods are the atomistic based methods (Yakobson *et al.*, 1996; Lu, 1997; Sanchez-Portal *et al.*, 1999) and the continuum mechanics based ones (Li and Chou, 2003a; Chang and Gao, 2003; Odegard *et al.*, 2002a). A few recent publications have proposed the development of structure-property relationships for nanotubes and nanostructured materials through the substitution of the discrete molecular structure with equivalent continuum models (Odegard *et al.*, 2001, 2002b). Li and Chou (2003a, 2004) suggested a linkage between molecular mechanics and structural mechanics in terms of geometric parameters of frame structures.

Some researchers have attempted to characterize CNTs based on the effective bending moduli. Wang *et al.* (2004) studied the relation between bending moment and bending curvature of CNTs to describe the variation of effective bending modulus to Young's modulus ratio with ripples in nanotubes. Another investigation by Wang *et al.* (2004) studied the non-linear bending moment-curvature relationship of CNTs with rippling deformation. Various theoretical and experimental studies have been carried out to predict and characterize the mechanical properties of CNTs and related materials, all of which generally confirm their superiority. As an example, although the results cover a wide range of 270-5500 GPa for Young's modulus and 290-2300 GPa for shear modulus of different kinds of CNT, which are several times greater than of steel (Lau *et al.*, 2004; Meo and Rossi, 2006; Qian *et al.*, 2002; Thostenson *et al.*, 2005; Yu, 2004).

Fundamental to Li and Chou (2003a,b) approach was the notion that CNTs are geometrical space-frame structures and therefore, can be analyzed by classical structural mechanics. In this research, based on the concept of Li *et al.* (2003a), three-dimensional (3D) FE models are proposed for investigating ESWCNTs.

## 2. FE modeling

CNTs atoms are bonded together with covalent bonds forming a hexagonal lattice. An elliptic single walled carbon nanotube (ESWCNT) is a kind of nanotube that has oval cross sectional area (Fereidoon *et al.*, 2012a). This kind of CNT may be produced in special states and conditions. The ESWCNT with  $a/b = 1$  is known as SWCNT, where  $a$  and  $b$  are major and minor diameters of ellipse, respectively.

As shown in Fig. 1, these bonds have a characteristic bond length  $\alpha_{C-C}$  and bond angle in the 3D space. The displacement of individual atoms under an external force is constrained by the bonds. Therefore, the total deformation of the nanotube is a result of the interactions between the bonds. By considering the bonds as connecting load-carrying elements, and the atoms as joints of the connecting elements, CNTs may be simulated as space-frame structures. In this work, a 3D FE model of ESWCNTs is proposed to assess the mechanical properties. The 3D FE model is developed using the ANSYS commercial FE code. For the modeling of the bonds, the 3D elastic BEAM4 element is used. The properties of these elements are obtained by linking the potential energy of bonds (from a chemical point of view) and the strain energy of mechanical elements (from a mechanical point of view). Considering a circular beam of length  $l$ , diameter  $d$ , Young's modulus  $E$ , and shear modulus  $G$ , the model represents the covalent bond between carbon atoms. The properties of this element are given in Table 1 (Li and Chou, 2003a).

The simulation relates the bond length  $\alpha_{C-C}$  with the element length  $L$  as well as the wall thickness  $t$  with the element diameter  $d$ , as shown in Fig. 1.

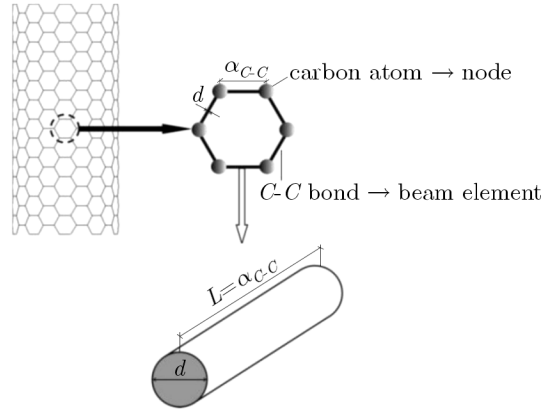


Fig. 1. Simulation of a SWCNT as a space-frame structure

**Table 1.** The properties of beam elements for real carbon nanotube (Li and Chou, 2003a)

Parameters	Value
Nanotube diameter $d$	1.466 Å
The length of carbon-carbon bond $L$	1.421 Å
Cross-sectional area $A$	1.68794 Å <sup>2</sup>
Polar inertia momentum $I_{xx}$	0.453456 Å <sup>4</sup>
Inertia momentum $I_{zz} = I_{yy} = I$	0.22682 Å <sup>4</sup>
Young modulus $E$	5.188 · 10 <sup>-8</sup> N/Å <sup>2</sup>
Shear modulus $G$	8.711 · 10 <sup>-9</sup> N/Å <sup>2</sup>

### 3. Calculation of elastic moduli

#### 3.1. Young's modulus

Young's modulus of a material is the ratio of normal stress to normal strain as obtained from a uni-axial tension test ( $\sigma = E\varepsilon$ ), and the strain energy of ESWCNTs is

$$U = \frac{1}{2}\sigma\varepsilon AL \quad U = \frac{1}{2}E\varepsilon^2 A_0 L \quad (3.1)$$

Therefore, Young's modulus of ESWCNT is calculated using the following equation

$$E = \frac{2U}{A_0 L \varepsilon^2} \quad (3.2)$$

where  $U$  is total strain energy of ESWCNT obtained from summing individual strain energy of all elements of the nanotube,  $\varepsilon$  – total applied strain,  $A_0$  – cross sectional area and  $L$  – initial length.  $A_0$  is

$$A_0 = \pi \left[ \left( a + \frac{t}{2} \right) \left( b + \frac{t}{2} \right) - \left( a - \frac{t}{2} \right) \left( b - \frac{t}{2} \right) \right] \quad (3.3)$$

In order to apply the conditions of tension, the nodes of the bottom end of the ESWCNT have been fully built-in (zero displacement and rotation conditions), while the nodes of the upper end are subjected to tension (Jalalahmadi and Naghdabadi, 2007) (Fig. 2a). In this paper, it has been assumed that the wall thickness of the ESWCNT is 0.34 nm.

#### 3.2. Shear modulus

For calculating the shear modulus of ESWCNTs, the following relation is used

$$G = \frac{TL}{J\theta} \quad (3.4)$$

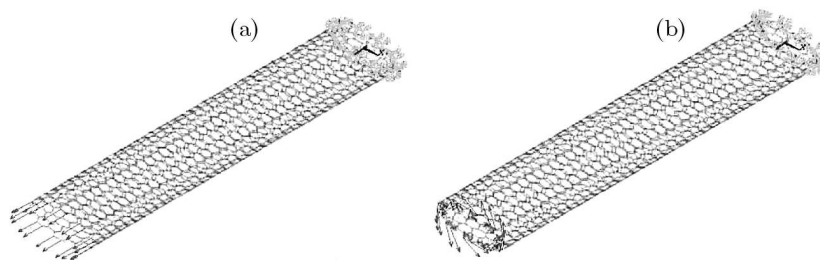


Fig. 2. Isometric view of FE meshes of the (10, 10) elliptic CNT along with the applied boundary conditions: (a) tension, (b) torsion

where  $T$  stands for the torque acting at one end of the ESWCNT,  $\theta$  for the torsional angle of the tube and  $J$  for the polar moment of inertia of the cross-sectional area. For calculating  $J$ , the ESWCNT is considered as a hollow tube with elliptic cross section. In this case  $J$  is

$$J = \frac{\pi}{4} \left\{ \left( a + \frac{t}{2} \right) \left( b + \frac{t}{2} \right) \left[ \left( a + \frac{t}{2} \right)^2 + \left( b + \frac{t}{2} \right)^2 \right] - \left( a - \frac{t}{2} \right) \left( b - \frac{t}{2} \right) \left[ \left( a - \frac{t}{2} \right)^2 + \left( b - \frac{t}{2} \right)^2 \right] \right\} \quad (3.5)$$

The torsional angle  $\theta$  is calculated by the FE model. In order to apply the conditions of torsion, the nodes of the bottom end of the ESWCNT have been fully built-in, while the nodes of the upper end, were constrained from moving in the radial direction ( $U_R = 0$ ) and subjected to tangential forces (Jalalahmadi and Naghdabadi, 2007) (Fig. 2b).

The values of Young's and shear modulus calculated for SWCNTs ( $a/b = 1$ ) have been compared with the corresponding values from the literature. Table 2 summarizes all calculations and the comparison. As may be seen in Table 2, elastic moduli computed by the present FE model agrees very well with each elastic moduli obtained by the corresponding methods, and the present method is verified.

**Table 2.** Comparison of elastic moduli by different researchers

Researcher	Element	$E$ [TPa]	$G$ [TPa]
Li and Chou (2003a)	beam	1.010	0.475
Tserpes and Papanikos (2005)	beam	1.028	0.410
Kalamkarov <i>et al.</i> (2006)	beam	0.980	0.350
Giannopoulos <i>et al.</i> (2008)	spring	1.307	0.383
Shokrieh and Rafiee (2010)	beam	1.033-1.042	0.417
Present work	beam	1.033-1.035	0.46-0.49

#### 4. Results and discussion

The mechanical properties of ESWCNTs depend on their length, chirality, diameter and cross sectional aspect ratio ( $a/b$ ). Several works investigating the dependence of elastic moduli of SWCNTs on their diameter and chirality have been reported. In the majority of them, armchair and zig-zag SWCNTs have been only included. In the present work, the FE model is applied to investigate the effect of length, diameter, chirality and cross sectional aspect ratio ( $a/b$ ) on the elastic moduli of ESWCNTs. All three types of SWCNTs, namely, armchair, zig-zag and chiral are included in the investigation.

The boundary conditions for tension and torsion are shown in Figs. 2a and 2b, respectively. Cross sections of ESWCNTs for different  $a/b$  from 1 to 2 are represented in Fig. 3.

Young's and shear modulus of ESWCNTs for some kind of armchair and zig-zag nanotubes with different lengths and cross sectional aspect ratios are calculated. Young's and shear modulus

of ESWCNTs with different lengths of 25, 50, 75, 100 Å for armchair nanotubes with chiralities of (5, 5), (10, 10), (15, 15), (20, 20) are plotted against the cross sectional aspect ratio ( $a/b$ ) in Figs. 4-7.

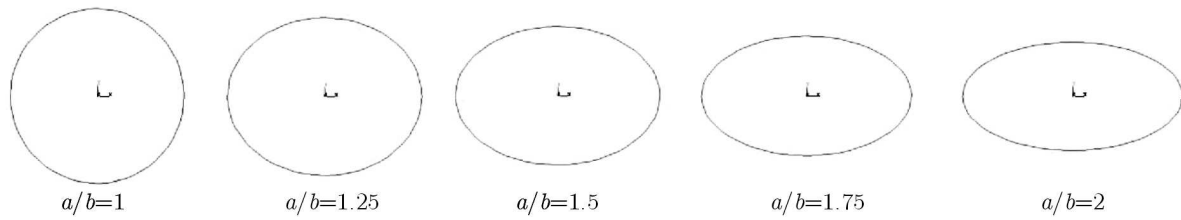


Fig. 3. Cross sections of ESWCNTs for different  $a/b$

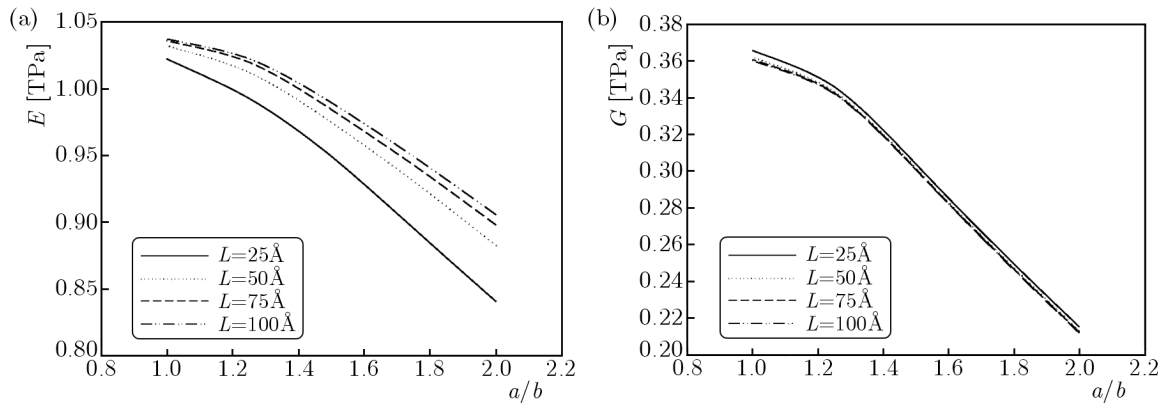


Fig. 4. Variation of Young's modulus (a) and shear modulus (b) of (5, 5) with the cross sectional aspect ratio ( $a/b$ ) for different lengths

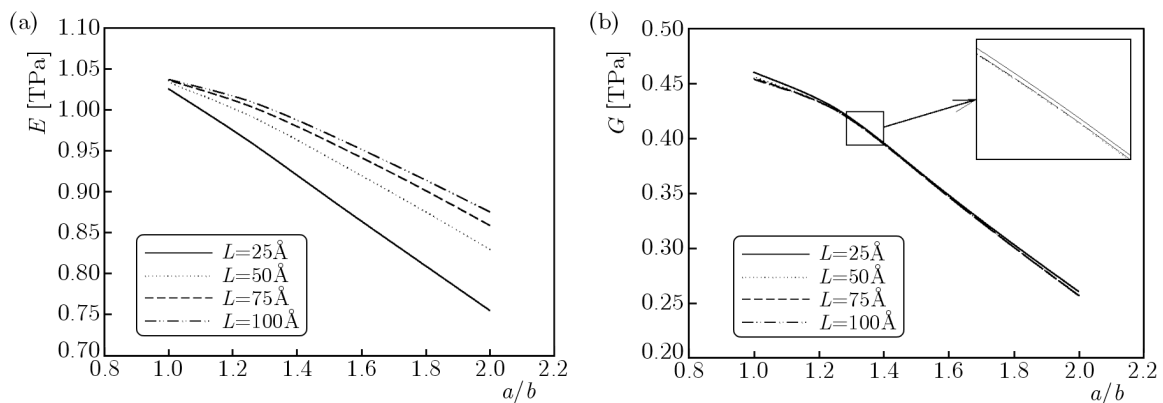


Fig. 5. Variation of Young's modulus (a) and shear modulus (b) of (10, 10) with the cross sectional aspect ratio ( $a/b$ ) for different lengths

Similarly, Figs. 8-11 show the variation of Young's and shear modulus with the cross sectional aspect ratio ( $a/b$ ) for different lengths of 25, 50, 75, 100 Å for zig-zag nanotubes with chirality of (5, 0), (10, 0), (15, 0), (20, 0).

#### 4.1. Effects of length and cross sectional aspect ratio ( $a/b$ ) on the elastic moduli of ESWCNTs

Figures 4-11 show the variation of elastic moduli of armchair, zig-zag ESWCNTs with different length and cross sectional aspect ratio ( $a/b$ ). It can be observed that there is an evident effect of these parameters on Young's and shear modulus of the armchair and zig-zag ESWCNTs.

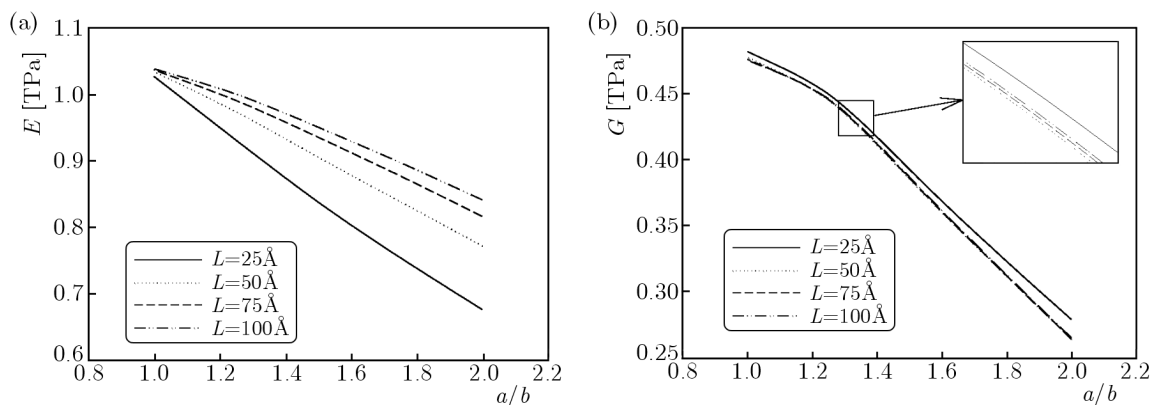


Fig. 6. Variation of Young's modulus (a) and shear modulus (b) of (15,15) with the cross sectional aspect ratio ( $a/b$ ) for different lengths

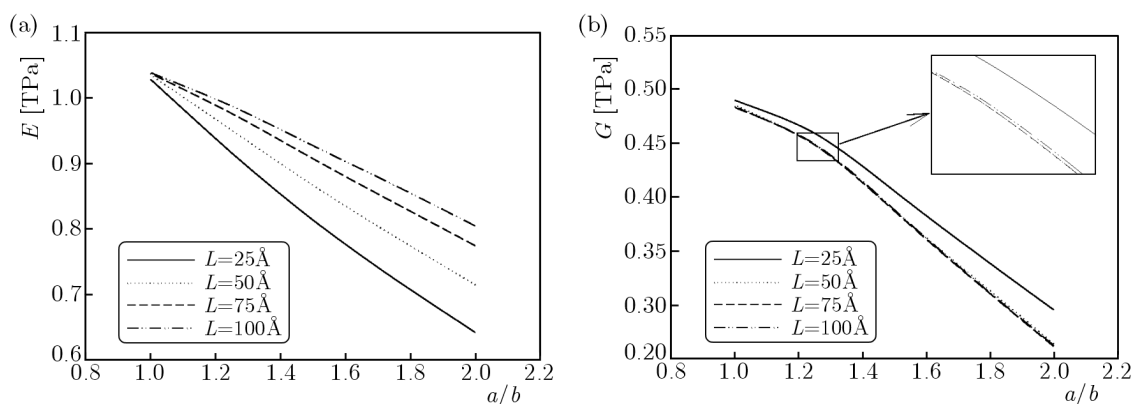


Fig. 7. Variation of Young's modulus (a) and shear modulus (b) of (20,20) with cross sectional aspect ratio ( $a/b$ ) for different lengths

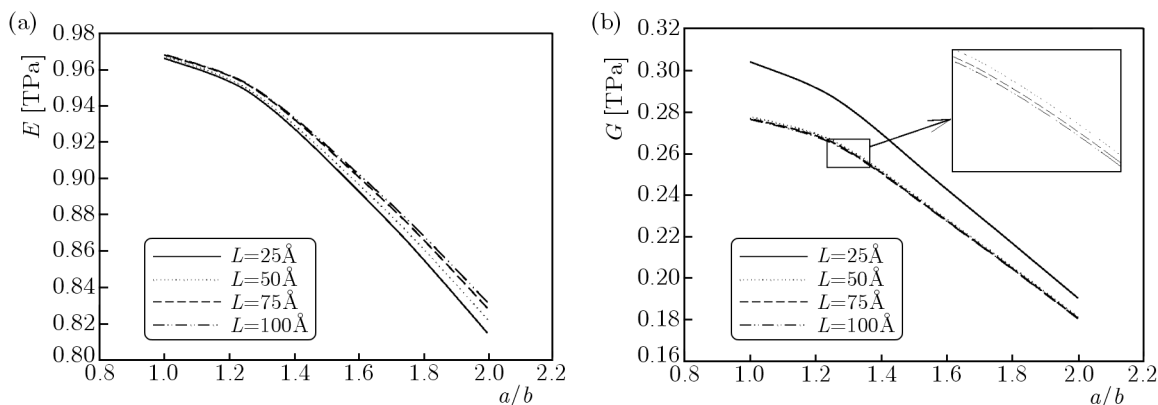


Fig. 8. Variation of Young's modulus (a) and shear modulus (b) of (5,0) with the cross sectional aspect ratio ( $a/b$ ) for different lengths

As may be seen for all kinds of armchair and zig-zag nanotubes, with increasing the length, Young's modulus increases and the shear modulus decreases. Furthermore, for each length with the increasing cross sectional aspect ratio ( $a/b$ ), elastic moduli decrease. The figures suggest that the elastic moduli values are inversely proportional to the cross sectional aspect ratio. On the other hand, elastic moduli of ESWCNTs are smaller than SWCNTs ( $a/b=1$ ). So, by changing geometry of SWCNTs, their elastic moduli decrease. As a result, every change in geometry operates as a defect and decreases the elastic moduli.

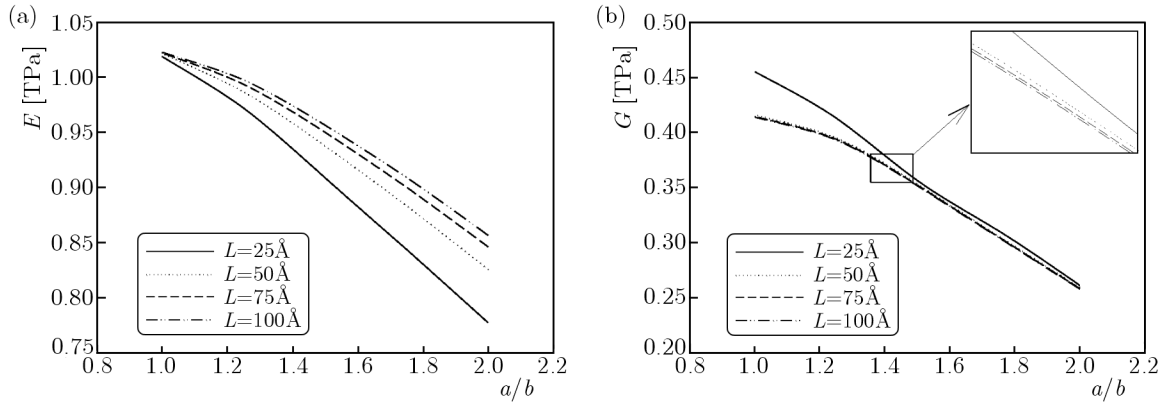


Fig. 9. Variation of Young's modulus (a) and shear modulus (b) of (10,0) with the cross sectional aspect ratio ( $a/b$ ) for different lengths

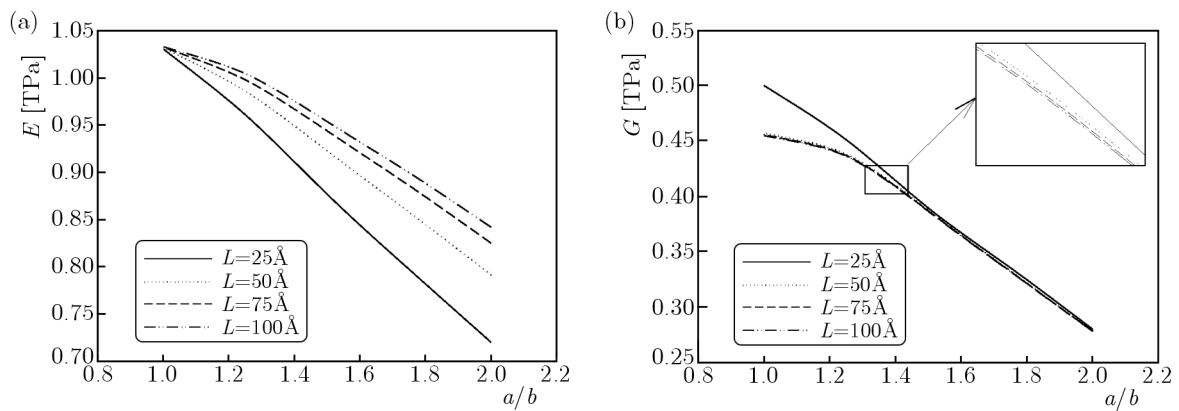


Fig. 10. Variation of Young's modulus (a) and shear modulus (b) of (15,0) with the cross sectional aspect ratio ( $a/b$ ) for different lengths

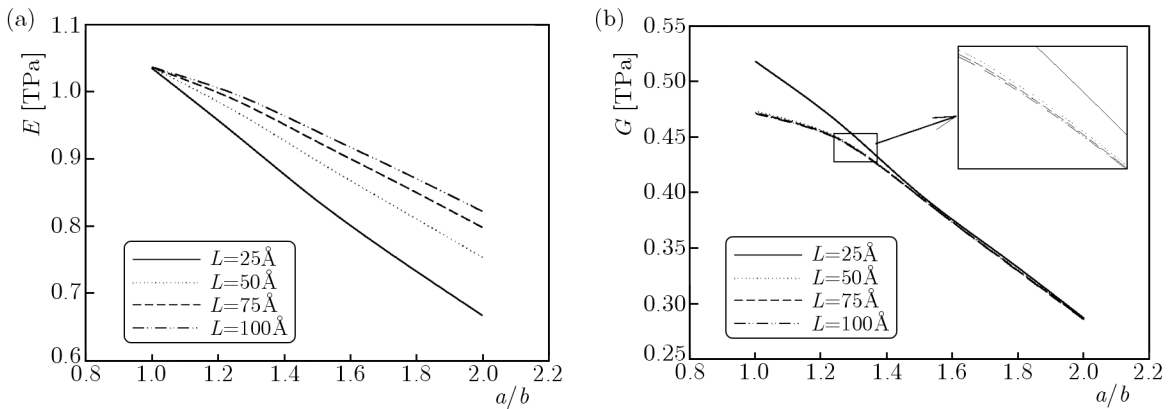


Fig. 11. Variation of Young's modulus (a) and shear modulus (b) of (20,0) with the cross sectional aspect ratio ( $a/b$ ) for different lengths

#### 4.2. Effects of chirality and cross sectional aspect ratio ( $a/b$ ) on the elastic moduli of ESWCNTs

Figures 12a and 12b indicate the variation of Young's and shear modulus of armchair and zig-zag ESWCNTs as a function of the cross sectional aspect ratio ( $a/b$ ) with length 50 Å. For armchair and zig-zag nanotubes with increasing chirality, the shear modulus increases, and with increasing cross sectional aspect ratio ( $a/b$ ), the shear modulus decreases.

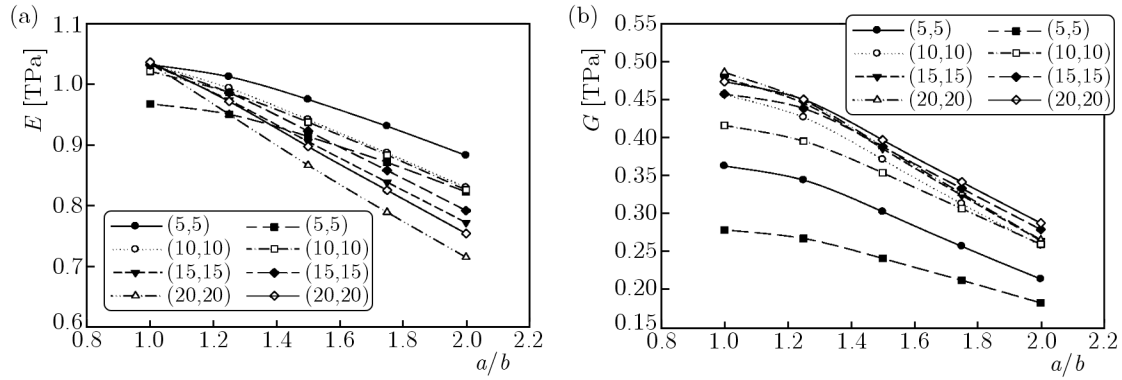


Fig. 12. Variation of Young's modulus (a) and shear modulus (b) with the cross sectional aspect ratio ( $a/b$ ) and length of 50 Å

Young's modulus of ESWCNTs with armchair and zig-zag chirality increases with the increasing chirality for  $a/b = 1$ . In the most of other cross sectional aspect ratios ( $a/b$ ) with increasing chirality, Young's modulus decreases. With the increasing cross sectional aspect ratio ( $a/b$ ) for each chirality, Young's modulus decreases.

#### 4.3. Effects of chirality on the elastic moduli of ESWCNTs

In this section, elastic moduli of eight kinds of armchair, zig-zag and chiral nanotubes with similar lengths and diameters and cross sectional aspect ratio ( $a/b$ ) equal to 1.5 are calculated. The diameters and lengths of these ESWCNTs are given in Table 3.

**Table 3.** Geometry and mechanical properties of different nanotubes with the same diameter and  $a/b = 1.5$

Chirality	Radius [Å]	$L$ [Å]	$E$ [TPa]	$G$ [TPa]
(10, 10)	6.785	49.225	0.9415	0.3705
(11, 9)	6.796	49.38	0.7837	0.3087
(12, 8)	6.830	49.55	0.9017	0.3148
(13, 7)	6.886	49.50	0.7844	0.3753
(14, 5)	6.682	50.23	0.7096	0.3613
(15, 4)	6.796	49.75	0.8788	0.3569
(16, 2)	6.694	49.39	0.8314	0.3869
(17, 0)	6.659	49.489	0.9204	0.3968

As shown in Fig. 13, the armchair nanotube (10, 10) has the maximum Young's modulus, and the zig-zag nanotube (17, 0) has the maximum shear modulus. The results indicate that the chirality has evident influence on the elastic moduli. In other words, with similar length and diameter and different chirality, the elastic moduli are not equal. Therefore, the elastic moduli of ESWCNTs are sensitive to their atomic arrangement.

## 5. Conclusions

Three-dimensional finite element models for ESWCNTs have been proposed. To create FE models, nodes are placed at the locations of carbon atoms and the bonds between them are modeled using three-dimensional elastic beam elements. As the FE model comprises a small number of elements, it performs under minimal computational time by requiring minimal computational



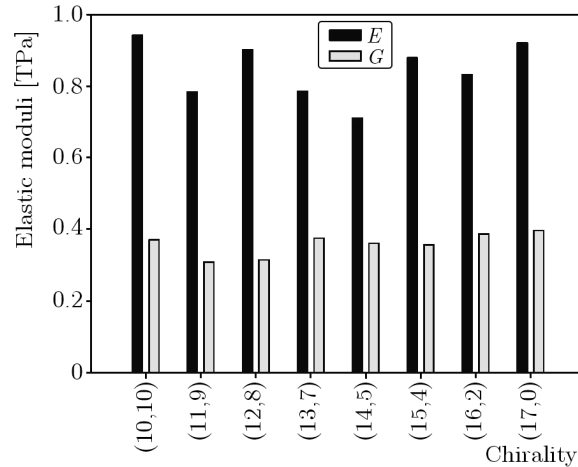


Fig. 13. The elastic moduli versus chirality for ESWCNT with the same diameter and  $a/b = 1.5$

power. This advantage, in combination with the modeling abilities of the FE method, extends the model applicability to all kinds of nanotubes with a very large number of atoms. The main conclusions drawn from this study are as follows:

- With increasing the length of ESWCNTs, Young's modulus increases and the shear modulus.
- For each length with the increasing cross sectional aspect ratio ( $a/b$ ), elastic moduli decrease.
- With increasing chirality, the shear modulus of ESWCNTs increases.
- The elastic moduli of ESWCNTs are sensitive to their atomic arrangement.

## References

1. CHANG T.C., GAO H.J., 2003, Size-dependent elastic properties of a single-walled carbon nanotube via a molecular mechanics model, *Journal of the Mechanics and Physics of Solids*, **51**, 1059
2. FERREIDON A., RAJABPOUR M., HEMMATIAN H., 2012a, Elastic moduli of elliptic carbon nanotubes based on FEM, *Mechanics of Nano, Micro and Macro Composite Structures*, Politecnico di Torino, Italy, June 18-20, Paper No. 101
3. FERREIDON A., RAJABPOUR M., HEMMATIAN H., 2012b, Fracture analysis of carbon nanotube composites using global-local models, *Mechanics of Nano, Micro and Macro Composite Structures*, Politecnico di Torino, Italy, June 18-20, Paper No. 461
4. FERREIDON A., RAJABPOUR M., HEMMATIAN H., 2013, Fracture analysis of epoxy/SWCNT nanocomposite based on global-local finite element model, *Composites Part B: Engineering*, **54**, 400-408
5. GIANNOPOULOS G., KAKAVAS P., ANIFANTIS N., 2008, Evaluation of the effective mechanical properties of single walled carbon nanotubes using a spring based finite element approach, *Computational Materials Science*, **41**, 561-569
6. HEMMATIAN H., FERREIDON A., RAJABPOUR M., 2011, Investigation of crack resistance in single walled carbon nanotube reinforced polymer composites based on FEM, *3rd International Conference on Ultrafine Grained and Nanostructured Materials*, University College of Engineering, University of Tehran, Tehran, Iran, November 2-3
7. JALALAHMADI B., NAGHDABADI R., 2007, Finite element modeling of single-walled carbon nanotubes with introducing a new wall thickness, *Journal of Physics*, **61**, 497-502

8. KALAMKAROV A.L., GEORGIADIS A.V., ROKKAM S.K., VEEDU V.P., GHASEMI-NEJHAD M.N., 2006, Analytical and numerical techniques to predict carbon nanotubes properties, *International Journal of Solids and Structures*, **43**, 6832-6854
9. LAU K.T., CHIPARA M., LING H.Y., HUI D., 2004, On the effective elastic moduli of carbon nanotubes for nanocomposite structures, *Composites Part B: Engineering*, **35**, 2, 95
10. LAU K.T., HUI D., 2002a, Effectiveness of using carbon nanotubes as nano-reinforcements for advanced composite structures, *Carbon*, **40**, 1605-1606
11. LAU K.T., HUI D., 2002b, The revolutionary creation of new advanced materials carbon nanotube composites, *Composites Part B: Engineering*, **33**, 263-277
12. LI C., CHOU T.W., 2003a, A structural mechanics approach for the analysis of carbon nanotubes, *International Journal of Solids and Structures*, **40**, 2487
13. LI C., CHOU T.W., 2003b, Elastic moduli of multi-walled carbon nanotubes and the effect of van der Waals forces, *Composites Science and Technology*, **63**, 1517
14. LI C., CHOU T.W., 2004, Modeling of elastic buckling of carbon nanotubes by molecular structural mechanics approach, *Mechanics of Materials*, **36**, 11, 1047
15. LU J.P., 1997, Elastic properties of carbon nanotubes and nanoropes, *Physical Review Letters*, **79**, 7, 1297
16. MEO M., ROSSI M., 2006, Prediction of Young's modulus of single wall carbon nanotubes by molecular-mechanics based finite element modelling, *Composites Science and Technology*, **66**, 11/12, 1597
17. MEO M., ROSSI M., 2007, A molecular-mechanics based finite element model for strength prediction of single wall carbon nanotubes, *Materials Science and Engineering A*, **454/455**, 170-177
18. ODEGARD G.M., GATES T.S., NICHOLSON L.M., WISE K.E., 2001, Equivalent-continuum modeling of nano-structured materials, *NASA/TM-2001-210863*, NASA Langley Research Center, Hampton, VA
19. ODEGARD G.M., GATES T.S., NICHOLSON L.M., WISE K.E., 2002a, Equivalent continuum modeling of nano-structured materials, *Composites Science and Technology*, **62**, 1869
20. ODEGARD G.M., GATES T.S., NICHOLSON L.M., WISE K.E., 2002b, Equivalent-continuum modeling with application to carbon nanotubes, *NASA/TM-2002-211454*, NASA Langley Research Center, Hampton, VA
21. PARL C., OUNAIES Z., WATSON K.A., PAWLOWSKI K., LOWTHER S.E., CONNELL J.W., SIOCHI E.J., HARRISON J.S., ST. CLAIR T.L., 2002, Polymer-single wall carbon nanotube composites for potential spacecraft applications, *ICASE Report No. 2002-36*, NASA Langley Research Center, Hampton, VA
22. QIAN D., WAGNER G.J., LIU W.K., YU M.F., RUOFF R.S., 2002, Mechanics of carbon nanotubes, *Applied Mechanics Reviews*, **55**, 6, 495
23. RAJABPOUR M., HEMMATIAN H., FEREIDOOON A., 2012, Analysis of the effect of chirality on stress intensity factor of epoxy/SWCNT based on multi-scale method, *15th Iranian Physical Chemistry Conference*, University of Tehran, Tehran, September 3-6
24. SANCHEZ-PORTAL D., ARTACHO E., SOLER J.M., 1999, Ab-initio structural, elastic, and vibrational properties of carbon nanotubes, *Physical Review Letters*, **59**, 12678
25. SHOKRIEH M., RAFIEE R., 2010, Prediction of Young's modulus of graphene sheets and carbon nanotubes using nano-scale continuum mechanics approach, *Materials and Design*, **31**, 2, 790-795
26. SMITH J.G., WATSON K.A., THOMPSON C.M., CONNELL J.W., 2002, Carbon nanotube/space durable polymer nanocomposite films for electrostatic charge dissipation, *ICASE Report No. 2002-34*, NASA Langley Research Center, Hampton, VA
27. THOSTENSON E.T., LI C., CHOU T.W., 2005, Nanocomposites in context, *Composites Science and Technology*, **65**, 3/4, 4916

28. THOSTENSON E.T., REN Z., CHOU T.W., 2001, Advances in the science and technology of carbon nanotubes and their composites: a review, *Composites Science and Technology*, **61**, 1899-1912
29. TSERPES K., PAPANIKOS P., 2005, Finite Element modeling of single-walled carbon nanotubes, *Composites Part B: Engineering*, **36**, 468-477
30. WANG X., WANG X.Y., XIAO J., 2005, A non-linear analysis of the bending modulus of carbon nanotubes with rippling deformation, *Composite Structures*, **69**, 315
31. WANG X., ZHANG Y.C., XIA X.H., HUANG C.H., 2004, Effective bending modulus of carbon nanotubes with rippling deformation, *International Journal of Solids and Structures*, **41**, 6429
32. YAKOBSON B.I., BRABEC C.J., BERNHOLC J., 1996, Nanomechanics of carbon tubes: instabilities beyond linear range, *Physical Review Letters*, **76**, 2511
33. YU M.F., 2004, Fundamental mechanical properties of carbon nanotubes: current understanding and the related experimental studies, *Journal of Engineering Materials and Technology*, **126**, 3, 271
34. ZAERI M.M., ZIAEI-RAD S., VAHEDI A., KARIMZADEH F., 2010, Mechanical modelling of carbon nanomaterials from nanotubes to buckypaper, *Carbon*, **48**, 3916-3930

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