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Jacek MICHALSKI*, Piotr KOZIERSKI*, Joanna ZIĘTKIEWICZ*

THE NEW APPROACH TO HYBRID KALMAN FILTERING, BASED ON THE CHANGED ORDER OF FILTERS FOR STATE ESTIMATION OF DYNAMICAL SYSTEMS

The paper presents a new approach to Hybrid Kalman filtering, composed of Extended Kalman Filter and Unscented Kalman Filter. In known algorithms, the Unscented Kalman Filter algorithm is used as first and the result of this is given as an input to the Extended Kalman Filter. The authors checked modified Hybrid Kalman Filter with changed order of filters using theoretical object, which was created on the basis of power system. Besides traditional method, the modification of Hybrid Kalman Particle Filter was evaluated too. Results were compared with Extended Kalman Filter, Unscented Kalman Filter and Bootstrap Particle Filter. For particle filters the authors compared method estimation qualities for a different number of particles. The estimation quality was evaluated by three quality indices. Based on the obtained results, one can see that the changed order of methods in Hybrid Kalman filter can improve estimation quality.

KEYWORDS: state estimation, Kalman filtering, particle filtering, power system, quality indices.

1. INTRODUCTION

State estimation is very important part of calculations, because it is possible to reduce of negative effect from measurement noises, however, on condition that measurements number is higher than number of state variables. The state estimation is used e.g. in robotics for mobile robot localization [1], but also for tracking of hands [2] and people [3]. In [4] particle filter (one of the estimation methods) was used for tracking of an atmospheric release of radiation.

State estimation is used also in power systems. Almost half a century ago, in 1970 Schweppe pointed out that such estimation in power systems is needed. Many articles was created since then, e.g. in [5, 6] the division of whole system into smaller parts was under consideration (distributed state estimation).

Despite so many years, still the most used in practice is Weighted Least Squares method, which was already proposed almost 50 years ago. However, in

* Poznan University of Technology

research more and more often one can see also other estimation methods, e.g. in [7] for power system state estimation the Mixed Kalman Particle Filter was used, and in [8] authors used Unscented Kalman Filter. In the paper [9] linear exponential smoothing technique of forecasting (LESF) was presented, which is robust onto measurement errors, and in [10] a neural network was used to pre-estimation and to detect bad measurements.

In this paper the authors also are interested in more sophisticated methods, i.e. in hybrid filters (composed of two or more standard filters). The network, which was used in simulations is not exactly the power system, however, measurement functions are similar to these from power system.

In Section 2 the main goal of state estimation is presented. In Section 3 the authors presented algorithms of used estimation methods. In Section 4 one can find details about performed simulations. Also in this section obtained results were shown. Conclusions are presented in the last section.

2. IDEA OF STATE ESTIMATION

The main task of this research is to calculate estimated values of state variables, which are not given from sensors very often. The goal of state estimation is to estimate object's outputs in such a way that they are more accurate than the measured values. Discrete system is given by state equations

$$\begin{cases} \mathbf{x}^{(k+1)} = \mathbf{f}\left(\mathbf{x}^{(k)}, \mathbf{u}^{(k)}\right) + \mathbf{v}^{(k)} \\ \mathbf{y}^{(k)} = \mathbf{h}\left(\mathbf{x}^{(k)}\right) + \mathbf{n}^{(k)} \end{cases} \tag{1}$$

where $\boldsymbol{x}^{(k)}$ is a state vector, $\boldsymbol{u}^{(k)}$ is an input vector, $\boldsymbol{y}^{(k)}$ is output (measurement) vector, $\boldsymbol{v}^{(k)}$ and $\boldsymbol{n}^{(k)}$ are process and measurement noises, respectively, and k means current time step.

3. EXAMINED ALGORITHMS OF STATE ESTIMATION

3.1. Extended Kalman Filter

The first Kalman Filter algorithm was proposed in [11] and there was method only for linear systems. Further research into the development of state estimation has led to the creation of many algorithms of the Kalman filter for non-linear objects, such as Extended Kalman Filter or Unscented Kalman Filter. Kalman filter methods are parametric state estimation methods based on the object linearization.

Algorithm 1: Extended Kalman Filter

– prediction step (time update)

$$\hat{\mathbf{x}}^{(k|k-1)} = \mathbf{f}\left(\hat{\mathbf{x}}^{(k-1)}, \mathbf{u}^{(k-1)}\right), \tag{2}$$

$$\mathbf{P}^{(k|k-1)} = \mathbf{F}^{(k-1)}\mathbf{P}^{(k-1|k-1)}\mathbf{F}^{(k-1)T} + \mathbf{Q}, \tag{3}$$

– filtration step (measurement update)

$$\mathbf{K}^{(k)} = \mathbf{P}^{(k|k-1)}\mathbf{H}^{(k)^T}\left[\mathbf{H}^{(k)}\mathbf{P}^{(k|k-1)}\mathbf{H}^{(k)^T} + \mathbf{R}\right]^{-1}, \tag{4}$$

$$\hat{\mathbf{x}}^{(k|k)} = \hat{\mathbf{x}}^{(k|k-1)} + \mathbf{K}^{(k)}\left[\mathbf{y}^{(k)} - \mathbf{h}\left(\hat{\mathbf{x}}^{(k|k-1)}\right)\right], \tag{5}$$

$$\mathbf{P}^{(k|k)} = \left[\mathbf{I} - \mathbf{K}^{(k)}\mathbf{H}^{(k)}\right]\mathbf{P}^{(k|k-1)} \tag{6}$$

where $\mathbf{F}^{(k-1)} = \nabla\mathbf{f}\,|_{\hat{\mathbf{x}}^{(k-1|k-1)}}$, $\mathbf{H}^{(k)} = \nabla\mathbf{h}\,|_{\hat{\mathbf{x}}^{(k|k-1)}}$ are Jacobian matrices. Subscripts (k|k-1) denote values predicted at k-th time step using the information from previous time step.

3.2. Unscented Kalman Filter

This algorithm, besides prediction and filtration step, contains also sigma points choice and the unscented transformation. It makes linearization of nonlinear plants too.

<u>Algorithm 2: Unscented Kalman Filter</u>

– choice of the sigma points

$$\hat{\mathbf{x}}_0^{(k-1)} = \hat{\mathbf{x}}^{(k-1|k-1)}, \tag{7}$$

$$\hat{\mathbf{x}}_i^{(k-1)} = \hat{\mathbf{x}}^{(k-1|k-1)} + \left(\sqrt{\frac{N_x}{1-W_0}\mathbf{P}^{(k-1|k-1)}}\right)_i, i = 1,\ldots,N_x, \tag{8}$$

$$\hat{\mathbf{x}}_i^{(k-1)} = \hat{\mathbf{x}}^{(k-1|k-1)} - \left(\sqrt{\frac{N_x}{1-W_0}\mathbf{P}^{(k-1|k-1)}}\right)_i, i = 1,\ldots,N_x, \tag{9}$$

– prediction step (time update)

$$\hat{\mathbf{x}}_j^{(k|k-1)} = \mathbf{f}\left(\hat{\mathbf{x}}_j^{(k-1)}, \mathbf{u}^{(k-1)}\right), \hat{\mathbf{x}}^{(k|k-1)} = \sum_{j=0}^{2N_x} W_j \hat{\mathbf{x}}_j^{(k|k-1)}, \tag{10}$$

$$P^{(k|k-1)} = \sum_{j=0}^{2N_x} W_j\left(\hat{\mathbf{x}}_j^{(k|k-1)} - \hat{\mathbf{x}}^{(k|k-1)}\right)\left(\hat{\mathbf{x}}_j^{(k|k-1)} - \hat{\mathbf{x}}^{(k|k-1)}\right)^T + \mathbf{Q}, \tag{11}$$

$$\hat{\mathbf{y}}_j^{(k|k-1)} = \mathbf{h}\left(\hat{\mathbf{x}}_j^{(k|k-1)}\right), \hat{\mathbf{y}}^{(k|k-1)} = \sum_{j=0}^{2N_x} W_j \mathbf{y}_j^{(k|k-1)}, \tag{12}$$

$$P_y^{(k|k-1)} = \sum_{j=0}^{2N_x} W_j\left(\hat{\mathbf{y}}_j^{(k|k-1)} - \hat{\mathbf{y}}^{(k|k-1)}\right)\left(\hat{\mathbf{y}}_j^{(k|k-1)} - \hat{\mathbf{y}}^{(k|k-1)}\right)^T + \mathbf{R}, \tag{13}$$

$$P_{xy}^{(k|k-1)} = \sum_{j=0}^{2N_x} W_j \left(\hat{\mathbf{x}}_j^{(k|k-1)} - \hat{\mathbf{x}}^{(k|k-1)}\right)\left(\hat{\mathbf{y}}_j^{(k|k-1)} - \hat{\mathbf{y}}^{(k|k-1)}\right)^T , \tag{14}$$

– filtration step (measurement update)

$$\mathbf{K}^{(k)} = \mathbf{P}_{xy}^{(k|k-1)} \left(\mathbf{P}_y^{(k|k-1)}\right)^{-1} , \tag{15}$$

$$\hat{\mathbf{x}}^{(k|k)} = \hat{\mathbf{x}}^{(k|k-1)} + \mathbf{K}^{(k)} \left[\mathbf{y}^{(k)} - \hat{\mathbf{y}}^{(k|k-1)}\right], \tag{16}$$

$$P^{(k|k)} = \mathbf{P}^{(k|k-1)} - \mathbf{K}^{(k)} \mathbf{P}_y^{(k|k-1)} \mathbf{K}^{(k)^T} \tag{17}$$

In the prediction step, it is necessary to choice weights values in such a way that $W_j = \frac{1-W_0}{2N_x}$ More information about Kalman filtering one can find in [11, 12].

3.3 Bootstrap Particle Filter

Bootstrap Particle Filter (BPF) was proposed in [13] by Gordon, Salmond and Smith and this is the simplified version of Sampling Importance Resampling (SIR) Particle Filter [14]. IN BPF it is assumed that particles in the prediction step are drawing from the measurement model and also that resampling is executed in every iteration, not depends on the suitable quality index in every iteration loop.

The Particle Filter algorithm works definitely better than Kalman filters for nonlinear objects, but by the need to repeat calculations for each particle during each iteration computation time is longer than in Kalman filters – it was shown in [12].

Algorithm 3: Bootstrap Particle Filter

1. Initialization. Draw N_p particles from initial PDF $\boldsymbol{x}^{i,(0)} \sim p(\boldsymbol{x}^{(0)})$. Set time step $k=1$.

2. Prediction. Draw N_p particles from the transition model $\boldsymbol{x}^{i(k)} \sim p(\boldsymbol{x}^{(k)} | \boldsymbol{x}^{i(k-1)})$.

3. Update. Calculate weights of the particles from the measurement model

$$q^{i,(k)} = p\left(\mathbf{y}^{(k)} \mid \mathbf{x}^{i,(k)}\right). \tag{18}$$

4. Normalization. Scale the weights in such a way that their sum be equal to 1.

5. Resampling re-drawing of particles (systematic resampling has been used [15]).

6. End of the iteration. Calculate $\boldsymbol{x}^{(k)}$ estimate, update time step $k := k+1$, go to Step 2.

More information about particle filtering and Particle Filters types one can find in [14, 16–17].

3.4 Modified Hybrid Kalman filters

Hybrid Kalman filters [18] are algorithms which combine Extended Kalman Filter and Unscented Kalman Filter distinguishing features. In the article [19] authors studied traditional Hybrid Kalman Filter and Hybrid Kalman Particle Filter for different number of particles. It was proven that for relatively simple object and small number of particles Hybrid Kalman filters could work better than traditional Bootstrap Particle Filter. In this paper authors proposed modifications of these methods Modified Hybrid Kalman Filter and Modified Hybrid Kalman Particle Filter. These are analogous algorithm with other order of Kalman filters inside. The influence of changing this order on the operation of algorithms for different number of particles was checked.

Algorithm 4: Modified Hybrid Kalman Filter

1. Calculate estimates $\hat{\mathbf{x}}_{EKF}^{(k|k)}$ and $\mathbf{P}_{EKF}^{(k|k)}$ according to the EKF algorithm with results (33-34) taken from previous time step.

$$\left[\hat{\mathbf{x}}_{EKF}^{(k|k)}, \mathbf{P}_{EKF}^{(k|k)}\right] = \mathrm{EKF}\left[\hat{\mathbf{x}}^{(k-1)}, \mathbf{P}^{(k-1)}, \mathbf{y}^{(k)}\right],$$

– prediction step (time update)

$$\hat{\mathbf{x}}^{(k|k-1)} = \mathbf{f}\left(\hat{\mathbf{x}}^{(k-1)}, \mathbf{u}^{(k-1)}\right), \tag{19}$$

$$\mathbf{P}^{(k|k-1)} = \mathbf{F}^{(k-1)}\mathbf{P}^{(k-1)}\mathbf{F}^{(k-1)T} + \mathbf{Q}, \tag{20}$$

– filtration step (measurement update)

$$\mathbf{K}^{(k)} = \mathbf{P}^{(k|k-1)}\mathbf{H}^{(k)^T}\left[\mathbf{H}^{(k)}\mathbf{P}^{(k|k-1)}\mathbf{H}^{(k)^T} + \mathbf{R}\right]^{-1}, \tag{21}$$

$$\hat{\mathbf{x}}_{EKF}^{(k|k)} = \hat{\mathbf{x}}^{(k|k-1)} + \mathbf{K}^{(k)}\left[\mathbf{y}^{(k)} - \mathbf{h}\left(\hat{\mathbf{x}}^{(k|k-1)}\right)\right], \tag{22}$$

$$P_{EKF}^{(k|k)} = \left[\mathbf{I} - \mathbf{K}^{(k)}\mathbf{H}^{(k)}\right]\mathbf{P}^{(k|k-1)}. \tag{23}$$

2. Calculate estimates $\hat{\mathbf{x}}_{UKF}^{(k|k)}$ and $\mathbf{P}_{UKF}^{(k|k)}$ according to the UKF algorithm with results taken from Step 1.

$$\left[\hat{\mathbf{x}}_{UKF}^{(k|k)}, \mathbf{P}_{UKF}^{(k|k)}\right] = \mathrm{UKF}\left[\hat{\mathbf{x}}_{EKF}^{(k|k)}, \mathbf{P}_{EKF}^{(k|k)}, \mathbf{y}^{(k)}\right],$$

– choice of the sigma points

$$\hat{\mathbf{x}}_0^{(k-1)} = \hat{\mathbf{x}}_{EKF}^{(k|k)}, \tag{24}$$

$$\hat{\mathbf{x}}_i^{(k-1)} = \hat{\mathbf{x}}_{EKF}^{(k|k)} + \left(\sqrt{\frac{N_x}{1-W_0}\mathbf{P}_{EKF}^{(k|k)}}\right)_i, i = 1,\ldots,N_x, \tag{25}$$

$$\hat{\mathbf{x}}_i^{(k-1)} = \hat{\mathbf{x}}_{EKF}^{(k|k)} - \left(\sqrt{\frac{N_x}{1-W_0}\mathbf{P}_{EKF}^{(k|k)}}\right)_i, i = 1,\ldots,N_x, \tag{26}$$

– prediction step (time update)

$$\hat{\mathbf{x}}_j^{(k|k-1)} = \mathbf{f}\left(\hat{\mathbf{x}}_j^{(k-1)}, \mathbf{u}^{(k-1)}\right), \quad \hat{\mathbf{x}}^{(k|k-1)} = \sum_{j=0}^{2N_x} W_j \hat{\mathbf{x}}_j^{(k|k-1)}, \tag{27}$$

$$P^{(k|k-1)} = \sum_{j=0}^{2N_x} W_j \left(\hat{\mathbf{x}}_j^{(k|k-1)} - \hat{\mathbf{x}}^{(k|k-1)}\right)\left(\hat{\mathbf{x}}_j^{(k|k-1)} - \hat{\mathbf{x}}^{(k|k-1)}\right)^T + \mathbf{Q}, \tag{28}$$

$$\hat{\mathbf{y}}_j^{(k|k-1)} = \mathbf{h}\left(\hat{\mathbf{x}}_j^{(k|k-1)}\right), \quad \hat{\mathbf{y}}^{(k|k-1)} = \sum_{j=0}^{2N_x} W_j \mathbf{y}_j^{(k|k-1)}, \tag{29}$$

$$P_y^{(k|k-1)} = \sum_{j=0}^{2N_x} W_j \left(\hat{\mathbf{y}}_j^{(k|k-1)} - \hat{\mathbf{y}}^{(k|k-1)}\right)\left(\hat{\mathbf{y}}_j^{(k|k-1)} - \hat{\mathbf{y}}^{(k|k-1)}\right)^T + \mathbf{R}, \tag{30}$$

$$P_{xy}^{(k|k-1)} = \sum_{j=0}^{2N_x} W_j \left(\hat{\mathbf{x}}_j^{(k|k-1)} - \hat{\mathbf{x}}^{(k|k-1)}\right)\left(\hat{\mathbf{y}}_j^{(k|k-1)} - \hat{\mathbf{y}}^{(k|k-1)}\right)^T, \tag{31}$$

– filtration step (measurement update)

$$\mathbf{K}^{(k)} = \mathbf{P}_{xy}^{(k|k-1)} \left(\mathbf{P}_y^{(k|k-1)}\right)^{-1}, \tag{32}$$

$$\hat{\mathbf{x}}_{UKF}^{(k|k)} = \hat{\mathbf{x}}^{(k|k-1)} + \mathbf{K}^{(k)}\left[\mathbf{y}^{(k)} - \hat{\mathbf{y}}^{(k|k-1)}\right], \tag{33}$$

$$P_{UKF}^{(k|k)} = \mathbf{P}^{(k|k-1)} - \mathbf{K}^{(k)}\mathbf{P}_y^{(k|k-1)}\mathbf{K}^{(k)^T}. \tag{34}$$

3. Set $\hat{\mathbf{x}}^{(k)} = \hat{\mathbf{x}}_{UKF}^{(k|k)}$ and $\mathbf{P}^{(k)} = \mathbf{P}_{UKF}^{(k|k)}$, update time step $k := k+1$, go to Step 1.

Algorithm 5: Modified Hybrid Kalman Particle Filter

1. Initialization. Draw N_p particles from initial PDF $\boldsymbol{x}^{i(0)} \sim p(\boldsymbol{x}^{(0)})$

2. Calculate estimates $\hat{\mathbf{x}}_{EKF}^{i,(k|k)}$ and $\mathbf{P}_{EKF}^{i,(k|k)}$ according to the EKF algorithm with results taken from previous time step.

$$\left[\hat{\mathbf{x}}_{EKF}^{i,(k|k)}, \mathbf{P}_{EKF}^{i,(k|k)}\right] = \mathrm{EKF}\left[\hat{\mathbf{x}}^{i,(k-1)}, \mathbf{P}^{i,(k-1)}, \mathbf{y}^{(k)}\right].$$

3. Calculate estimates of $\hat{\mathbf{x}}_{UKF}^{i,(k|k)}$ and $\mathbf{P}_{UKF}^{i,(k|k)}$ according to the UKF algorithm with results taken from Step 2.

$$\left[\hat{\mathbf{x}}_{UKF}^{i,(k|k)}, \mathbf{P}_{UKF}^{i,(k|k)}\right] = \mathrm{UKF}\left[\hat{\mathbf{x}}_{EKF}^{i,(k|k)}, \mathbf{P}_{EKF}^{i,(k)}, \mathbf{y}^{(k)}\right].$$

4. Draw $\hat{\mathbf{x}}^{i,(k)} \sim g\left(\mathbf{x}^{i,(k)} \mid \mathbf{x}^{i,(k-1)}, \mathbf{y}^{(k)}\right) = \mathrm{N}\left(\hat{\mathbf{x}}_{UKF}^{i,(k|k)}, \mathbf{P}_{UKF}^{i,(k|k)}\right)$.

5. Calculate the weights of particles according to equation

$$q^{i,(k)} = \frac{p\left(\mathbf{y}^{(k)} \mid \mathbf{x}^{i,(k)}\right) p\left(\mathbf{x}^{i,(k)} \mid \mathbf{x}^{i,(k-1)}\right)}{g\left(\mathbf{x}^{i,(k)} \mid \mathbf{x}^{i,(k-1)}, \mathbf{y}^{(k)}\right)}. \tag{35}$$

6. Normalization. Scale the weights in such a way that their sum be equal to one.

7. Resampling Resampling re-drawing of particles (systematic resampling was used [15]).

4. End of the iteration. Calculate $\hat{\mathbf{x}}^{(k)}$ estimate, set $\mathbf{P}^{i,(k)} = \mathbf{P}^{i,(k|k)}_{UKF}$, update time step $k := k+1$, go to Step 2.

4. SIMULATION RESULTS

For simulations the plant, which was proposed based on the power system model [20], was used. The choice was dedicated by the fact that in power system each additional node is associated with two additional state variables (voltage magnitude and phase angle), and in proposed network only one state is related with each node. The plant contains 4 state variables (is composed of 4 nodes) and can be written by equations

$$\begin{cases} x_i^{(k+1)} = 0.9 x_i^{(k)} + v_i^{(k)}, \quad i = 1,\ldots,4 \\ y_{ij}^{(k)} = h_{ij}\left(\mathbf{x}^{(k)}, \mu_{ij}\right) + n_{ij}^{(k)}, \quad j = 1,\ldots,4 \end{cases}$$

$$h_{ij}\left(\mathbf{x}^{(k)}, \mu_{ij}\right) = \begin{cases} \sum_{j=1}^{N_x} x_i^{(k)} x_j^{(k)} \mu_{ij} \sin\left(x_i^{(k)} - x_j^{(k)} - \mu_{ij}\right), \quad i = j \\ x_i^{(k)^2} - x_i^{(k)} x_j^{(k)} \mu_{ij} \cos\left(x_i^{(k)} - x_j^{(k)} - \mu_{ij}\right), \quad i \neq j \end{cases}$$

Three quality indices, shown below, was compared

$$aRMSE = \frac{1}{N_x} \sum_{i=1}^{N_x} \sqrt{\frac{1}{M} \sum_{k=1}^{M} \left(\hat{\mathbf{x}}_i^{(k)} - \mathbf{x}_i^{+(k)}\right)}, \tag{36}$$

$$J_y = \frac{1}{N_y} \sum_{i=1}^{N_y} J_{yi}, \quad J_{yi} = \frac{1}{M} \sum_{k=1}^{M} \left(\hat{\mathbf{y}}_i^{(k)} - \mathbf{y}_i^{+(k)}\right), \tag{37}$$

$$\varepsilon_y = \frac{\sum_{k=1}^{M} \sum_{i=1}^{N_y} \left|\hat{\mathbf{y}}_i^{(k)} - \mathbf{y}_i^{+(k)}\right|}{\sum_{k=1}^{M} \sum_{i=1}^{N_y} \left|\mathbf{y}_i^{(k)} - \mathbf{y}_i^{+(k)}\right|}, \tag{38}$$

where M is the length of the simulation, N_x is the number of state variables, N_y is the measurements number, values with hat upper are estimated values, values with "+" are real values. $y_i^{(k)}$ means the noisy measurement from k-th time step.

For each method, simulations with M = 1000 time steps were performed. Each simulation was repeated minimum 1000 times in order to decrease standard deviations (according to 68-95-99.7 rule [21]). The simulation results are presented in Fig. 1–3.

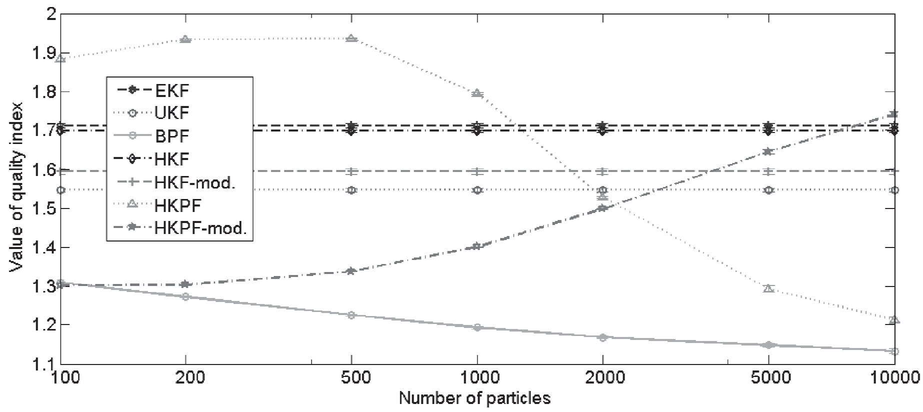

Fig. 1. Values of *aRMSE* quality index

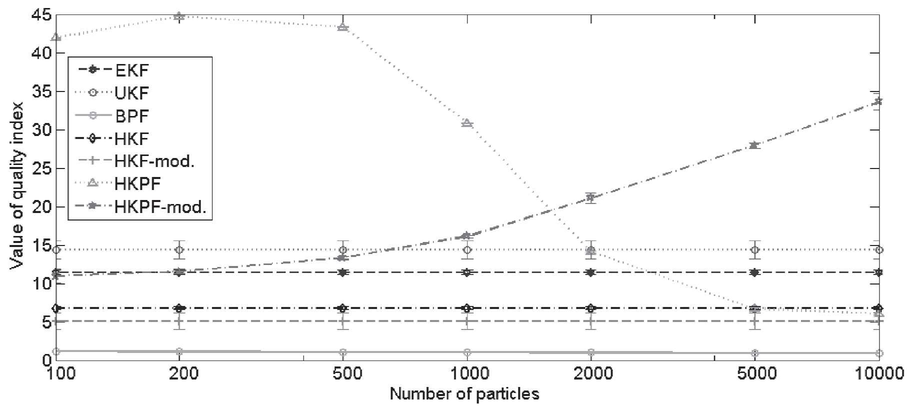

Fig. 2. Values of J_y quality index

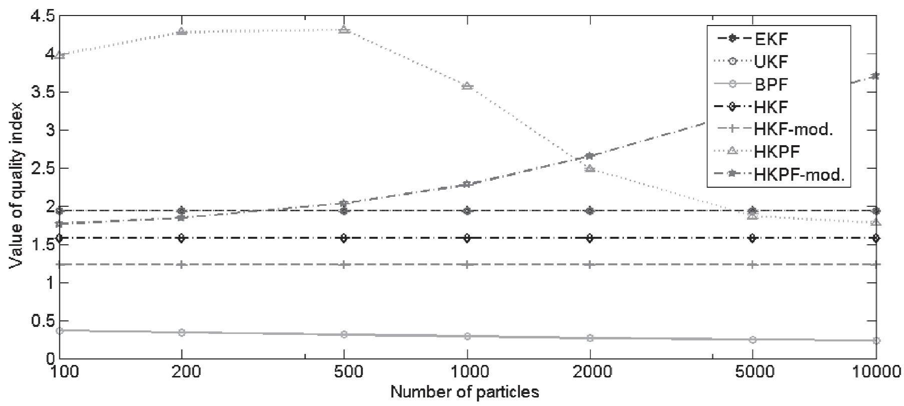

Fig. 3. Values of ε_y quality index

5. CONCLUSIONS

Based on the simulation results it can see that the best results for examined object gives traditional Bootstrap Particle Filter algorithm. In general, for small number of particles the worst operation can be observed for HKPF and for bigger particles number for EKF algorithm (big number of particles improves HKPF's operation). However, for small number of particles the better estimation quality is observed for hybrid Kalman filters after author's modifications. Quality indices for modified HKF are smaller than for normal HKF and for modified HKPF – smaller than for traditional HKPF. It was proven in [19] that for simple systems HKPF works better than BPF, so there are cases when it is better to use hybrid Kalman filters besides traditional particle filters.

In the future authors will examine modified hybrid Kalman filters for an another, different object types.

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