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Influence of some speed parameters on the dynamics of nonlinear flexural vibrations of a drill column

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Abstract. We investigate the influence of the motion of fluid flushing the cutter of a well drilling column, and the angular rotational velocity upon dynamic characteristics of its flexural vibrations. We take into account the nonlinear elastic features of column material. As a base of the research we took the Galerkin method and the Van der Pol method. Combining those two methods made possible to obtain the relations describing the main parameters of the dynamical process in both nonresonance and resonance case.

Key words: nonlinear elastic properties, mathematical model, Galerkin method, Van der Pol method, resonance.

THE URGENCY OF THE TOPIC AND THE PROBLEM STATE

The investigation of dynamical processes in various media and oscillating systems in the applications where one not always can use classical methods of integrating partial differential equations, is an urgent technical-engineering problem. This applies first and foremost to problems describing dynamical processes of longitudinally moving media. It is a matter of longitudinal and flexural vibrations of belt, rope or chain transmissions; pipelines with moving fluid inside; auger machines with viscous or granulated medium moving along; the process of vibroseparation (to a certain extent) etc. As shown in [1-3], the longitudinal component of the media motion velocity affects not only quantitative characteristics of the systems mentioned above, but could also significantly affect the qualitative side of the process – lead to an oscillation stop or stability loss. The fluids being transported by the pipelines or used in technological processes, e.g. in well drilling columns, cause the changes of quantitative, and in some cases qualitative, characteristics of dynamical process. This applies first and foremost to the amplitude-frequency characteristic and the stability of dynamical process. The issues on the influence of constant velocity of the motion of one- and two-dimensional media upon the main characteristics and the stability of its nonlinear vibrations have been studied e.g. in [4–7]. In the case of well drilling columns, the problem gets more complicated because the column performs a rotary motion as well. It is a matter of such plants, in which a combined drill actuator (rotary and hydraulic) is used. In such plants, the fluid moves under a certain pressure with a high velocity. Moreover, interacting with a rock, the drill permanently perturbs the column vibration. All the facts mentioned above imply the urgency of studying dynamics of a drill column allowing for the fluid motion and the angular rotational velocity of the column. *The aim of the paper* is developing the methodology of estimating the influence of entire complex of factors (external and internal) upon the dynamic process of the drilling column; obtaining the calculating relations useful for engineering research, which determine the influence of main physico-mechanical, kinematical, geometrical characteristics upon the main oscillation parameters.

PROBLEM STATEMENT

As a mathematical model of flexural vibrations of the well drilling column, rotating with an angular velocity Ω and with incompressible fluid moving along it with constant relative linear velocity V, we consider the equation:

$$L(u)=(\rho_1+\rho_2)\frac{\partial^2 u(x,t)}{\partial t^2}+\rho_2 V\frac{\partial^2 u(x,t)}{\partial t\partial x}-$$
$$-\left(S(x)-\rho_2 V^2\right)\frac{\partial^2 u(x,t)}{\partial x^2}-\frac{\partial S(x)}{\partial x}\frac{\partial u(x,t)}{\partial x}+$$
$$+EI\frac{\partial^4 u(x,t)}{\partial x^4}-(\rho_1+\rho_2)\Omega^2 u(x,t)=$$
$$=k_1 EI\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)^3-k_2\frac{\partial u(x,t)}{\partial t}. \quad (1)$$

In equation (1), $u(x,t)$ is a transverse deviation of the column section with x coordinate at arbitrary instant of time t, ρ_1, ρ_2 are respectively masses of length unit of the column and the fluid, moving inside, $S(x)$ is an axial thrust in any column section made by special loads for the pressure on the drill, and the force of column weight, EI is a flexural rigidity of the column, k_1 and k_2 are the coefficients that define deviation of elastic properties of the drilling column material from a linear law and the resistance force respectively. Here the resistance force is assumed proportional to relative velocity of column motion.

Taking into account that the upper part of the column is placed into a bearing with a fixed upper clip, and the lower one gets small horizontal displacement (external perturbations) caused by the interaction of the drill and a rock, we can write down the boundary conditions in the form:

$$u(0,t)=\frac{\partial^2 u(0,t)}{\partial x^2}=0,$$
$$u(l,t)=k_3\sin(pt+\theta),\quad \frac{\partial^2 u(l,t)}{\partial x^2}=0. \quad (2)$$

In relations (2) k_3, p, θ are constants (amplitude, frequency and initial phase of external periodic perturbation respectively).

In what follows, we assume that the gyroscopic moment is small and neglect one in the motion equations. Also assume that for the drilling column the flat cross-section hypothesis holds, and the reference frame, in which the deflection is registered, is attached to movable vertical plane and coincides with the maximum deflections plane. Besides, assume from now on that the coefficients $k_i, i=1,2,3$ are small in comparison with flexural rigidity.

Note that the issue of substantiation of well-posedness of certain weakly and strongly nonlinear mathematical models of nonlinear oscillating systems has been considered in the works [8–19]. In particular, in those works there have been developed a methodology of investigating the well-posedness (existence and uniqueness of solutions) of mixed problems for quasi-linear and strongly nonlinear evolutional equations of beam vibration type (in the case of presence of dissipative forces in the system) in bounded and unbounded domains.Thus, the problem on investigating flexural vibrations of a well drilling column has been reduced to constructing and investigating the solution of boundary value problem (1), (2).

SOLVING METHODOLOGY

First of all, we shall reduce the problem with nonhomogeneous boundary conditions to simpler one – problem with homogeneous boundary conditions. For this purpose, in equation (1) we shall perform a change of variables:

$$u(x,t)=v(x,t)+k_3 w(x,t). \quad (3)$$

In representation (3), the function $v(x,t)$ is a solution of homogeneous boundary value problem:

$$(\rho_1+\rho_2)\frac{\partial^2 v}{\partial t^2}+\rho_2 V\frac{\partial^2 v}{\partial t\partial x}-\left(S(x)-\rho_2 V^2\right)\frac{\partial^2 v}{\partial x^2}+$$
$$+EI\frac{\partial^4 v}{\partial x^4}-\frac{\partial S(x)}{\partial x}\frac{\partial v}{\partial x}-(\rho_1+\rho_2)\Omega^2 v=$$
$$=k_1 EI\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 v}{\partial x^2}\right)^3-k_2\frac{\partial v}{\partial t}-L(w), \quad (4)$$
$$v(0,t)=\frac{\partial^2 v(0,t)}{\partial x^2}=0,$$
$$v(l,t)=\frac{\partial^2 v(l,t)}{\partial x^2}=0. \quad (5)$$

And the function $w(x,t)$ is a solution of the differential equation:

$$\frac{\partial^4 w}{\partial x^4}=0,$$

under the boundary conditions:

$$(\rho_1+\rho_2)\ w(0,t)=\frac{\partial^2 w(0,t)}{\partial x^2}=0,$$
$$w(l,t)=k_3\mathbf{sin}(pt+\theta),$$
$$\frac{\partial^2 w(l,t)}{\partial x^2}=0. \quad (6)$$

Considering (6), the solution of the boundary value problem could be found quite easily. Directly we make sure that:

$$w(x,t)=\frac{k_3}{l}x\sin(pt+\theta). \quad (7)$$

Considering (4) and the obtained solution (7), to find the function $v(x,t)$ we use the autonomic differential equation:

$$(\rho_1+\rho_2)\frac{\partial^2 v}{\partial t^2}+\rho_2 V\frac{\partial^2 v}{\partial t\partial x}-\left(S(x)-\rho_2 V^2\right)\frac{\partial^2 v}{\partial x^2}+$$

$$+EI\frac{\partial^4 v}{\partial x^4}-\frac{\partial S(x)}{\partial x}\frac{\partial v}{\partial x}-(\rho_1+\rho_2)\Omega^2 v=$$
$$=k_1 EI\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 v}{\partial x^2}\right)^3-k_2\frac{\partial v}{\partial t}+$$
$$+\frac{k_3}{l}\left(p^2+\Omega^2\right)\mathbf{sin}\left(pt+\theta\right)-$$
$$-2V\rho_2\frac{k_3 p}{l}\cos\left(pt+\theta\right) \qquad (8)$$

in which the function $v(x,t)$ should satisfy the homogeneous boundary conditions (5). It is easy to make sure that the system of functions $\left\{X_k(x)\right\}=\left\{\sin\frac{k\pi}{l}x\right\}$ satisfies the condition:

$$X_k(0)=X_k(l)=X_k''(0)=X_k''(l)=0 .$$

This allows expressing the solution of boundary value problem (8), (5) according to Galerkin method in the form:

$$v(x,t)=\sum_k X_k(x)T_k(t). \qquad (9)$$

To find unknown functions $T_\kappa(t)$ in expression (9), we obtain the system of ordinary nonlinear differential equations:

$$\frac{d^2T_k(t)}{dt^2}+\frac{\left(S_0+\frac{\rho_1 gl}{2}-\rho_2V^2\right)\left(\frac{k\pi}{l}\right)^2}{\rho_1+\rho_2}T_k(t)+$$
$$+\frac{EI\left(\frac{k\pi}{l}\right)^4-(\rho_1+\rho_2)\Omega^2}{\rho_1+\rho_2}T_k(t)=$$
$$=\frac{2k_3}{(\rho_1+\rho_2)l}\left\{\frac{k_1EIl}{k_3}\left(\frac{k\pi}{l}\right)^8T_k^3(t)-\frac{k_2l}{k_3}\frac{dT_k(t)}{dt}+\right.$$
$$\left.+\frac{(\rho_1+\rho_2)l}{k\pi}\left(p^2+\Omega^2\right)\sin\left(pt+\theta\right)\right\}. \qquad (10)$$

In the relation above, we took into account that the axial thrust $S(x)$ changes according to the linear law:

$$S(x)=S_0+\rho_1 g(l-x),$$

where: S_0 is a constant component of the axial thrust, made by special loads placed in the lower part of the column for the pessure of the drill onto a rock, and $\rho_1 g(l-x)$ is the force in the column section caused directly by its weight.

Differential equation (10) allows to determine directly the proper frequency ω of linear vibrations of the column (without considering the nonlinearly elastic properties of the column material):

$$\omega=\sqrt{\frac{\left(S_0+\frac{\rho_1 gl}{2}-\rho_2V^2\right)\left(\frac{k\pi}{l}\right)^2+EI\left(\frac{k\pi}{l}\right)^4-(\rho_1+\rho_2)\Omega^2}{\rho_1+\rho_2}}. \qquad (11)$$

Note that in formula (11) and below for the sake of more compact expression of the results, we omit the "k" index, which specifies the form of a “dynamical balance”.Not less important problem of operation of wells for drilling is studying the influence periodical forces upon nonlinear column vibrations and their stability. It is a matter, first of all, of resonant phenomena prevention. Those problems could be solved mainly basing on constructing a solution of the perturbed equation (10).As it was emphasized before, the coefficients $k_i, i=1,2,3$ are small quantities in comparison with flexural rigidity and other coefficients of the right-hand side of equation (1). This allows for searching for solution of equation (10) to use general approaches for constructing asymptotical solutions of ordinary quasi-linear equations. Below we shall use relatively simple, useful for engineering research, Van der Pol method [20]. According to it, the solution of unperturbed ($k_3\to 0$) equation, which corresponds to equation (10), i.e. $T(t)=a\cos(\omega t+\varphi)$, could be considered as a solution of perturbed one (with such a difference that parameters a and φ would be functions of time). For finding those parameters a and φ we obtain the system of ordinary differential equations:

$$\frac{da}{dt}=\frac{-k_3}{(\rho_1+\rho_2)l}\left\{\frac{k_1EIl}{k_3}\left(\frac{k\pi}{l}\right)^8a^3\cos^3\phi+\right.$$
$$\left.+\frac{k_2l}{k_3}a\omega\sin\phi+\frac{(\rho_1+\rho_2)l}{k\pi}\left(p^2+\Omega^2\right)\sin\left(pt+\theta\right)\right\}\sin\phi$$
$$\frac{d\varphi}{dt}=\frac{-k_3}{(\rho_1+\rho_2)al}\left\{\frac{k_1EIl}{k_3}\left(\frac{k\pi}{l}\right)^8a^3\cos^3\phi+\right.$$
$$\left.+\frac{k_2l}{k_3}a\omega\sin\phi+\frac{(\rho_1+\rho_2)l}{k\pi}\left(p^2+\Omega^2\right)\sin\left(pt+\theta\right)\right\}\cos\phi \qquad (12)$$

where: $\phi=\omega t+\varphi$.

For differential equations (12), we shall consider two cases: nonresonant case $r\omega\neq sp$ and resonant case $r\omega\approx sp$.

In the nonresonant case, the amplitude and the phase of the dynamical process in the first approximation does not depend on a harmonic perturbation. This allows, without loss of accuracy of approximation, to average the equation (12) by the phases of proper vibrations ϕ and forced ones $\vartheta=pt+\theta$. Therefore, in nonresonant case, the dynamical process is described by the relation as follows:

$$\frac{da}{dt}=-\frac{k_2\omega}{(\rho_1+\rho_2)\pi}a,$$
$$\frac{d\phi}{dt}=\frac{-\bar{k}_1EI}{(\rho_1+\rho_2)}\left(\frac{k\pi}{l}\right)^8a^2+...$$

As to the case of main resonance, introducing in (12) the phase difference $\gamma = \phi - \vartheta$ of proper and forced vibrations, i.e. $\phi = \gamma + \vartheta,\ \vartheta = pt + \theta$, we obtain:

$$\frac{da}{dt} = \frac{-k_3}{(\rho_1+\rho_2)l}\left\{\frac{k_1 EIl}{k_3}\left(\frac{k\pi}{l}\right)^8 a^3 \mathbf{cos}^3(\gamma+\vartheta) + \right.$$
$$+\frac{k_2 l}{k_3} a\omega\, \mathbf{sin}(\gamma+\vartheta) +$$
$$\left. +\frac{(\rho_1+\rho_2)l}{k\pi}(p^2+\Omega^2)\, \mathbf{sin}\,\vartheta\right\} \mathbf{sin}(\gamma+\vartheta)$$

$$\frac{d\gamma}{dt} = \omega - p - \frac{k_3}{(\rho_1+\rho_2)al}\left\{\frac{k_1 EIl}{k_3}\left(\frac{k\pi}{l}\right)^8 a^3 \mathbf{cos}^3(\gamma+\vartheta) + \right.$$
$$+\frac{k_2 l}{k_3} a\omega\, \mathbf{sin}(\gamma+\vartheta) +$$
$$\left. +\frac{(\rho_1+\rho_2)l}{k\pi}(p^2+\Omega^2)\, \mathbf{sin}\,\vartheta\right\} \mathbf{cos}(\gamma+\vartheta). \qquad (13)$$

The fact that the resonant process largely depends on the phase difference of proper and forced vibrations, allows to simplify relations (13) slightly. Actually, the averaging of system of differential equations (13) by the phase of forced vibrations would not change the approximation accuracy. This allows to replace that system by the following one:

$$\frac{da}{dt} = -\frac{k_2\omega}{(\rho_1+\rho_2)\pi} a + \frac{k_3}{k\pi}(p^2+\Omega^2)\cos\gamma,$$

$$\frac{d\gamma}{dt} = \omega - p - \frac{k_1 EI}{(\rho_1+\rho_2)}\left(\frac{k\pi}{l}\right)^8 a^2 -$$
$$-\frac{k_3}{k\pi a}(p^2+\Omega^2)\sin\gamma.$$

The last equations determine the resonant curve:

$$-\frac{k_2\omega}{(\rho_1+\rho_2)\pi} a + \frac{k_3}{k\pi}(p^2+\Omega^2)\cos\gamma = 0,$$

$$\omega - p - \frac{k_1 EI}{(\rho_1+\rho_2)}\left(\frac{k\pi}{l}\right)^8 a^2 -$$
$$-\frac{k_3}{k\pi a}(p^2+\Omega^2)\sin\gamma = 0.$$

Below there is given a graphical representation of dependence of the proper frequency ω of linear column vibrations on other parameters of the oscillating system. On Fig. 1 we give a graphical dependence $\omega = \omega(\Omega,\ V)$ under: $\rho_1 = 35\frac{kg}{m}$, $\rho_2 = 35\frac{kg}{m}$, $l = 50\ m$, $EI = 2{,}85\cdot 10^6\ Nm^2$, $S_0 = 1000\ N$, $k = 1$, $g = 9{,}8\ \frac{m}{s^2}$.

On Fig. 2 we give a graphical dependence $\omega = \omega(\Omega,\ l)$ under $\rho_1 = 35\frac{kg}{m}$, $\rho_2 = 35\frac{kg}{m}$, $V = 0$, $EI = 2{,}85\cdot 10^6\ Nm^2$, $S_0 = 1000\ N$, $k = 1$, $g = 9{,}8\ \frac{m}{s^2}$.

On Fig. 3 we give a graphical dependence $\omega = \omega(V,\ l)$ under $\rho_1 = 35\frac{kg}{m}$, $\rho_2 = 35\frac{kg}{m}$, $\Omega = 10\ s^{-1}$, $EI = 2{,}85\cdot 10^6\ Nm^2$, $S_0 = 1000\ N$, $k = 1$, $g = 9{,}8\ \frac{m}{s^2}$.

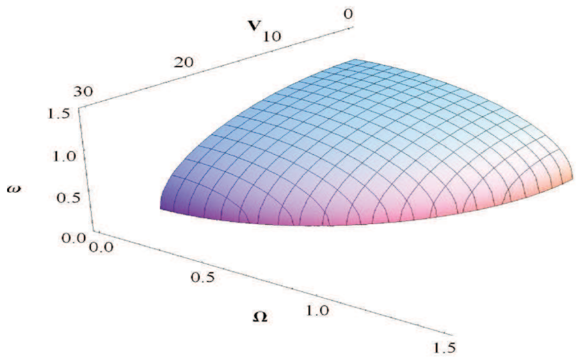

Fig. 1. Graphical dependence $\omega = \omega(\Omega,\ V)$

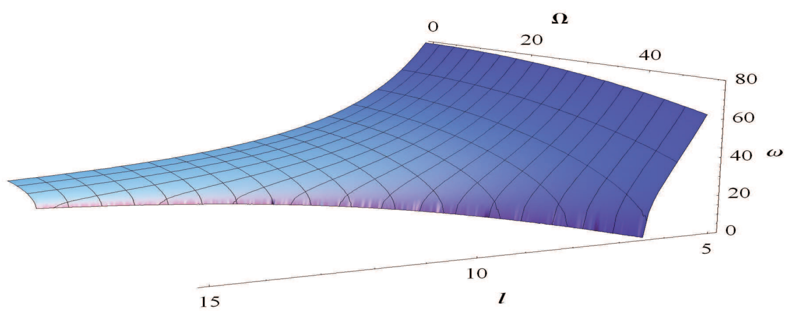

Fig. 2. Graphical dependence $\omega = \omega(\Omega,\ l)$

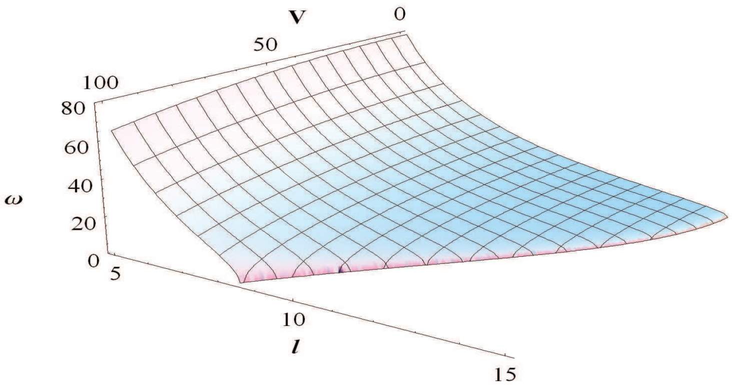

Fig. 3. Graphical dependence $\omega = \omega(V,\ l)$

CONCLUSIONS

From the obtained results we conclude that:

a) for greater values of the angular rotational velocity of the column and the fluid motion velocity, the proper vibrations frequency of the column becomes less,

b) under the constant angular rotational velocity of the drilling column Ω_1, the oscillation stop comes when the fluid longitudinal motion velocity equals to:

$$V_{cr} = \sqrt{\frac{S_0 + \frac{\rho_1 g l}{2} + EI\left(\frac{k\pi}{l}\right)^2 - (\rho_1 + \rho_2)\Omega_1^2\left(\frac{l}{k\pi}\right)^2}{\rho_2}},$$

c) under the constant fluid motion velocity V_1 along the tube of the drilling column the oscillation stop comes when the angular rotational velocity equals to:

$$\Omega_{cr} = \left(\frac{k\pi}{l}\right)\sqrt{\frac{S_0 + \frac{\rho_1 g l}{2} - \rho_2 V_1^2 + EI\left(\frac{k\pi}{l}\right)^2}{\rho_1 + \rho_2}}.$$

The obtained results should be considered in the drilling technological processes, because the oscillation stop is closely connected with such negative phenomenon as the loss of stability of the process. Moreover, the relations obtained show the ways of preventing the oscillation stop: if the technological process allows the column rotation with angular velocity near Ω_{cr} then the fluid should be delivered with linear velocity different from (less than) V_1 and vice versa, if the fluid motion velocity in the tube equals V_{cr} then the angular rotational velocity of the drilling column should be less than Ω_1.

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