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VARIABLELOADING CONDITIONS AND CONSEQUENCES FOR WAVE PHENOMENA

Summary

In land transport, we observe an increase of loads as well as of speeds of travel, which leads to considerable stresses and strains in constructions like tracks, bridges and tunnels. The wave propagation caused by constant and harmonic loads moving at constant speed along straight lines has been well covered in the recent literature. Problems including other types of variable loading conditions are much less understood. In the article, some problems of this type are posed and discussed together with applications in vehicle mechanics. In particular, motion of out-of-the-round wheels on rails and some problems of brake disk dynamics are presented.

INTRODUCTION

Moving forces on constructions like beams and plates lead to fourth order partial differential equations in one or two space dimensions. In the 1d case, the Euler-Lagrange beam model (1) is the most frequently used approach to track dynamics, but also the Timoshenko model may be considered if the beam under consideration is not slender and shear effects are essential-

$$(EJu'' + Pu)'' + \rho\ddot{u} = f(x,t,u,\dot{u}) \qquad (1)$$

Here, obviously, u denotes the deflection, x the position, $t$means time. By a superposed dot, time derivatives and by an apostrophe space derivatives are indicated.

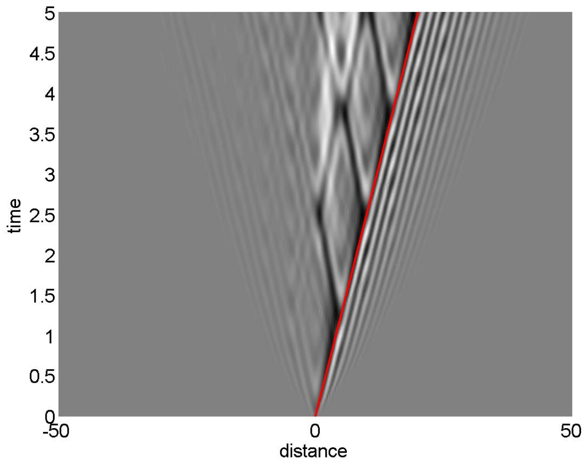


Fig. 1.Waves emanating from a moving source in space-time domain.The red line indicates the trajectory of the load.

In 2d, a plate equation has to be considered, its main part being given in terms of the bi-Laplacian operator, see (2).

$$\Delta(ED\Delta u + Pu) + \rho\ddot{u} = f(x,t,u,\dot{u}) \qquad (2)$$

Here the Laplacian operator is denoted by Δ. In both cases, E is Young's modulus, P compression, ρ mass density and f the force density.

Although linear, both equations are nontrivial to solve, whether analytically or numerically. In particular, boundary conditions and radiation conditions at infinity cause some trouble. Waves are generated even by constant forces, speeds of propagation depend on the frequency – dispersion effects have to be taken into account. There are characteristic speeds for the load motion, at which instabilities occur, and which divide regions of different types of solution behaviour.

Recently, it was observed by the authors [3, 4], that corrugations may cause considerable variations of the speed of the load's motion, even at constant traveling speed of a vehicle. Other cases of variable velocity of a load are obtained when the load has a curved trajectory. Even if the tangential speed is constant, the velocity vector will be variable in such a case.

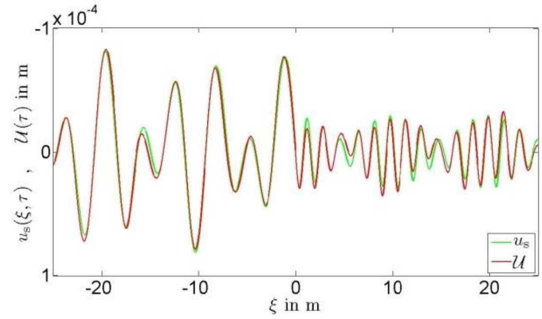


Fig. 2.Wave packages running in front and behind the current vehicle position. Numerical vs. analytical solution.

Little is known about analytical solutions to equations (1) or (2) with such types of traveling forces, regardless of the type of boundary conditions assumed.

1. APPLICATIONS

In Figs. 1 and 2, waves are presented, which are caused by a force moving at constant speed along a straight track, modeled as an Euler beam. Analytical solutions to this problem are known, the first fully correct description of solutions on the whole speed/frequency range was contributed by the first author already in [1, 2, 5].

More recently, the first two authors observed that corrugations on railway wheel lead in a natural way to contact problems with variable forward speed of the load's position. Further, there are considerable oscillations around the average axle load. At the same time, the third author studied forces with curved trajectories on plates, in continuation of a study presented in [3, 6]. Fig. 3 shows a sketch of a typical situation. The applied forces may be due to tools, or, after some adaptation, to brake shoes. In any case, wave effects may influence the considered process in an essential way, hence numerical studies are required.

Due to the rectangular structure of the domain, in 2d examples as on Fig. 3, a Finite Difference (FD) scheme is the method of choice, as it is also in the 1d case.

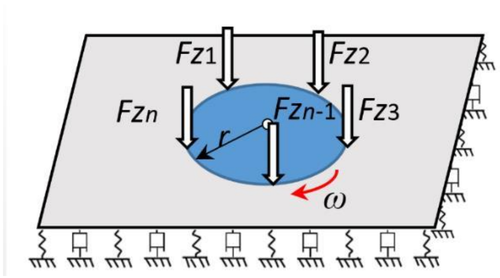
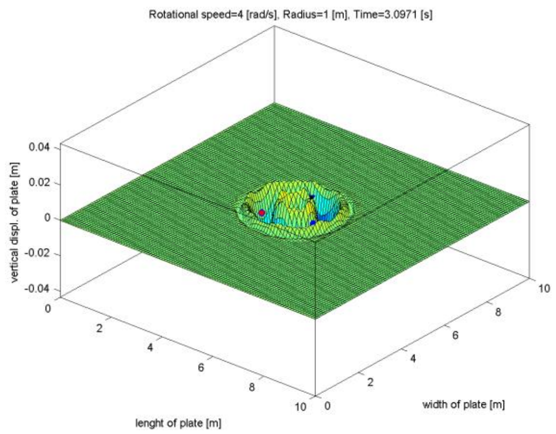


Fig. 3.*Wave generation by forces applied in a rotating formation on a rectangular plate*

The method was previously applied, among others, to taxiing airplanes on an airfield composed of coupled plates.



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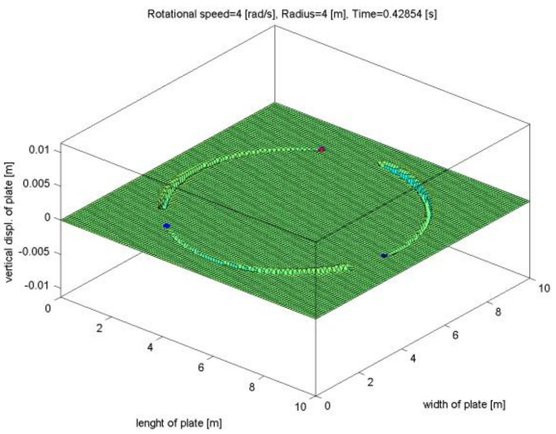

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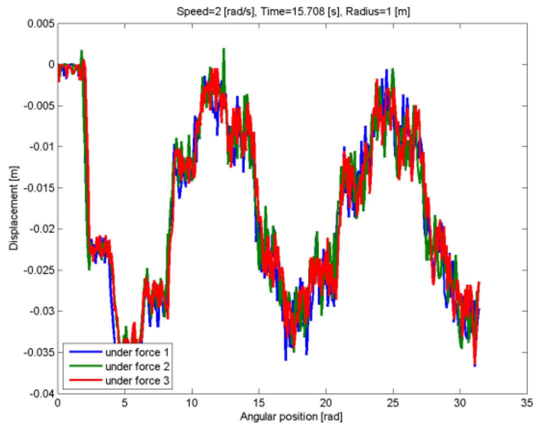
Fig. 4.*Different stages of waves running towards the boundary*



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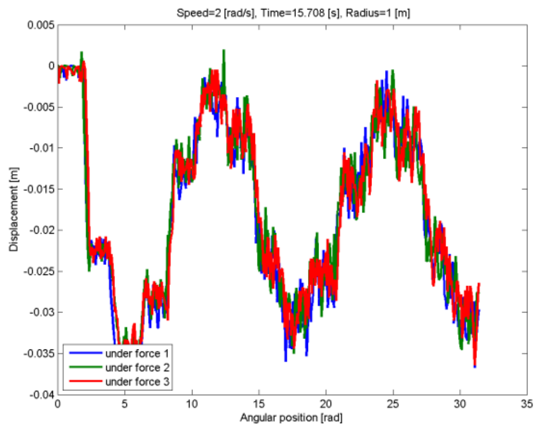

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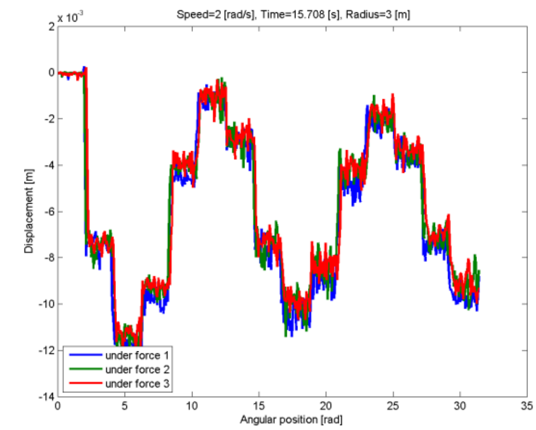

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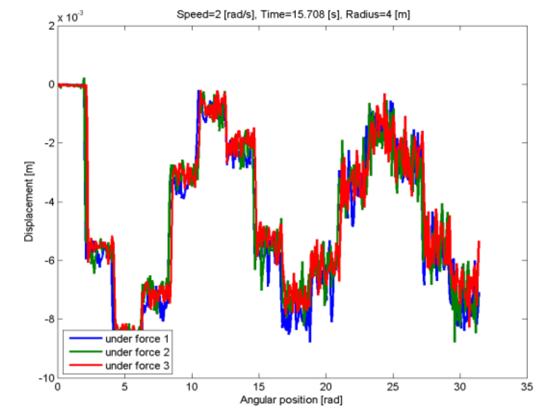

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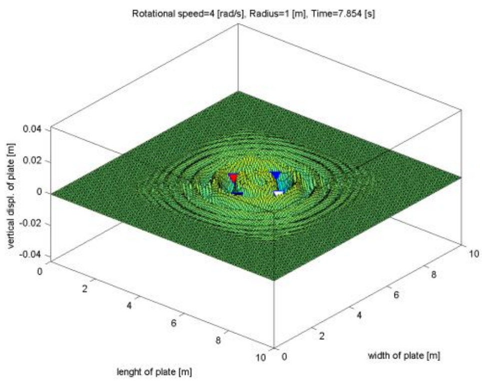
Fig. 5.*Oscillations of deflections under constant load*



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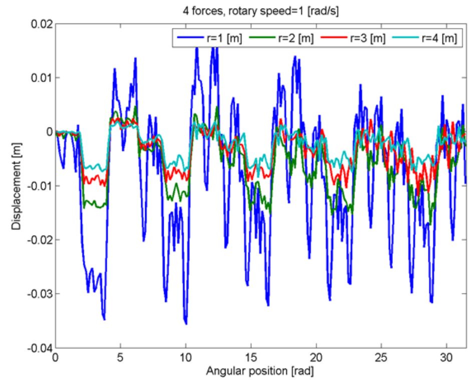

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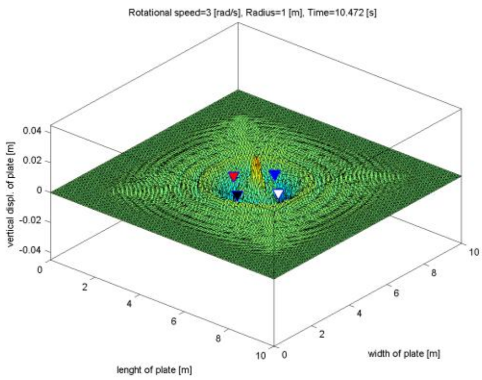

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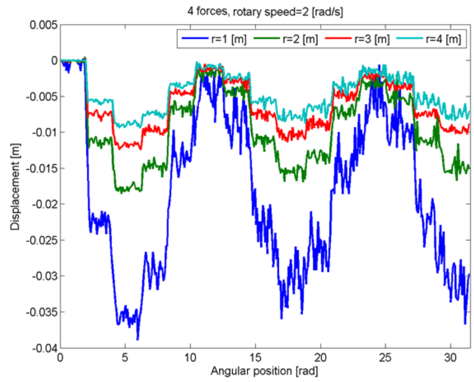

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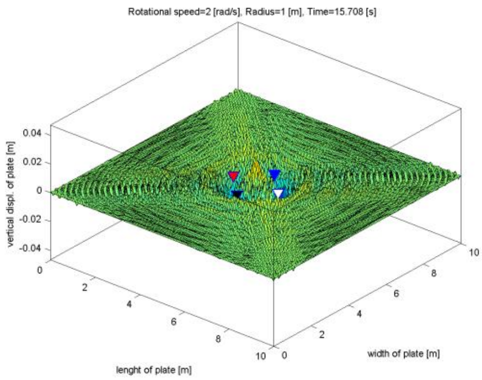

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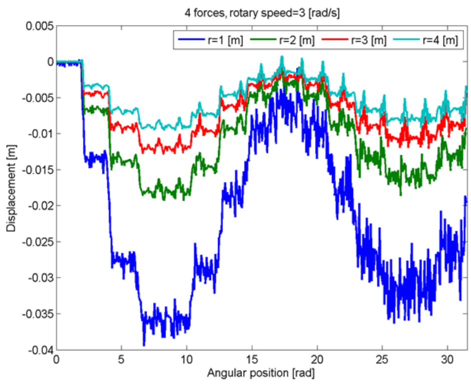

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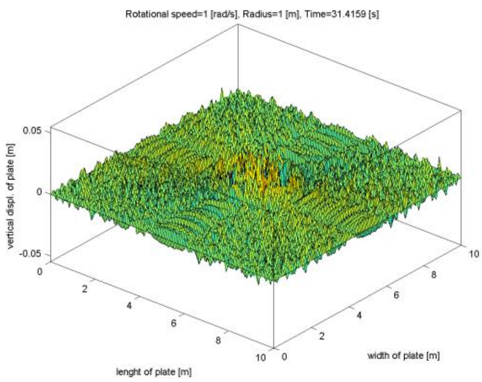

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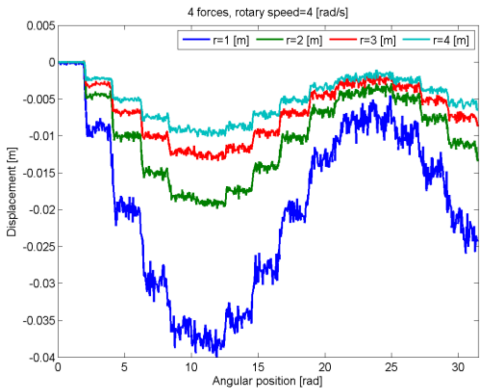

d)

Fig. 6.*Vertical displacement of a plate under four forces moving along a circular path ofofradius r=1m at rotational speed ω=1÷4 [rad/s] (after five turns)*

Fig. 7*.Vertical displacement under the first of four moving forcesfor various radii of a circular path and at constant angular speeds ω=1÷4 [rad/s] (after five turns)*

Most essentially, the partial differential equation (1), respectively (2), is reformulated as a system of ordinary differential equations for the nodal deflections at all grid points via semi-discretisation.

The presented method allows to obtain numerical solutions with a very high accuracy corresponding to analytical solutions known for special cases. In Fig. 2, this is demonstrated for the case of an infinite one-dimensional domain. Here the analytical solution fulfils a Sommerfeld radiation condition, while the numerical solution is restricted to a finite grid, where at the boundary nodes suitable conditions have to be imposed in order to prevent unphysical reflections.

In the remaining Figures 3 – 7, the problem of a system of equally spaced moving forces along a circular path is considered. In that case, the domain is finite, and we discuss only the numerical approach. Due to waves propagating in the plate and reflections from the boundary, the indentation under each of the constant and identical loads exhibits oscillations. The interaction in the contact points becomes more and more dominated by the wake of previous turns.

Further studies of visco-elastic contact partners moving in circular formations are under preparation. In general, an important goal is the control the normal load in such a way, that the observed variation of the indentation becomes as small as possible.

Obviously, for the considered case of a bunch of forces moving fast around a small plate, also the in-plane forces are of great interest. For now, we neglected the heat produced due to energy dissipation and the resulting thermal stresses. The modeling of friction forces under variable thermal conditions, together with permanent changes of the material properties, is one more target of further analysis.

CONCLUSIONS

Analytical methods, used in the past to study wave propagation in engineering constructions, are not sufficient to obtain solutions to problems of modern railway mechanics. On the other hand, classical techniques still give good guidance as to general effects and may hence serve to choose proper parameters for numerical procedures necessary to handle problems with variable loading characteristics.

Waves resulting from moving load positions in problems with non-negligible bending stiffness – such as the Bernoulli-Euler beam model or the Kirchhoff plate equation, differ qualitatively from waves in hyperbolic problems [7]. The latter, e.g. in the classical second order wave equation or the Helmholtz equation, are much more amiable for numerical analysis.

In the fourth order case, of particular interest are dispersion and the fact, that there may be waves that seemingly do not run from their source – the traveling load – but towards it, from infinity. It is most important to study the flow of the energy connected with this type of solutions.

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ZMIENNOŚĆ OBCIĄŻEŃ I SKUTKI W ZAKRESIE PROPAGACJI FAL

Streszczenie

W transporcie lądowym, zwłaszcza szynowym, obserwuje się tendencję do coraz wyższych prędkości i większych obciążeń, skąd wynikają między innymi znaczne naprężenia wtakich konstrukcjach jak szyny, mosty i tunele.Efekty falowe z tym związane dobrze są zbadane w przypadku stałych prędkości ruchu prostoliniowego i harmonicznych obciążeń. Zagadnienia uwzględniające innego typu zmienności sił czy warunków brzegowych dotąd nie znalazły odpowiednio dużo uwagi w literaturze. W obecnej pracy kilka zadań tej klasy zostało postawionych i przedyskutowanych, między innymi problem związany z kołami skorugowanymi i problem dynamiki dysku hamulcowego jako przykłady z zakresu mechaniki pojazdów szynowych.

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