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FUZZY-LYAPUNOV BASED CONTROLLER FOR A QUADROCOPTER

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Abstract

Quadrocopters are nonlinear and inherently unstable systems. To be able to account for the nonlinearities during more aggressive manoeuvres nonlinear control methods need to be utilized to obtain the desired position while at the same time guaranteeing stability. In the article, the quadrocopter dynamics is modelled using the Newton-Euler method. The propeller aerodynamics is modelled using a combination of momentum theory and blade element theory. There are two different control objectives; the 1st objective requires the quadrocopter to reach a desired attitude set point using, while the 2nd objective requires the quadrocopter to track an attitude trajectory. In both cases, Lyapunov stability criterion, in conjunction with LaSalle's invariance principle, is used to guarantee the system becomes asymptotically stable. In the case of reaching the desired attitude set point, a direct Lyapunov control method is implemented with the control constants determined empirically. For the trajectory tracking, limited knowledge is assumed on the system dynamics and the Mamdani fuzzy controller is used with a rule base that satisfy the Lyapunov stability criterion. The fuzzy membership functions developed empirically and a centre of gravity defuzzification method is used. All simulations are done in MATLAB/Simulink. The results of the numerical simulation are presented in the article.

Keywords: *quadrocopter, fuzzy control, Lyapunov stability*

1. Introduction

Vertical take-off landing (VTOL) aircraft, such as helicopters and quadrocopters, have been able to realize complex missions in which manoeuvres are required that their fixed winged counterparts are unable to do. The unique use of VTOL aircraft together with the mechanical simplicity of quadrocopters has led to the rise in their popularity with applications including search and rescue, surveillance, inspection mapping, filming, etc., which are implemented by attaching the vision system or manipulator to a flying robot [2]. The increase in applications results in the need for quadrocopters with more aggressive manoeuvres and better hovering performance.

Classical control methods, such as PID, have been implemented in quadrocopters with success around hover position [10]. Optimal control has also been applied in quadrocopters such as LQR [7] and H-infinity control [9] with good results around the linearized position.

To be able to synthesize a controller a mathematical description of the system is a prerequisite. The Newton-Euler equations, which describe the rotational and translational dynamics of a system, have been widely used in the modelling of quadrocopters. While this method uses Euler angles, which the disadvantage of "gimbal lock" the quadrocopter would avoid manoeuvres that place it in such a situation. The aerodynamics of the propellers is approximated using a combination of momentum theory and blade element momentum theory called blade element momentum theory (BEMT) [5, 8, 12].

The quadrocopter dynamics can be further split into two separate parts: the attitude dynamics and the translation dynamics. Due to the under actuated nature of the quadrocopter the translation dynamics are dependent on the attitude dynamics. The scope of this article only covers the control of the attitude dynamics, in which two different control objectives are set. In the first case, a desired attitude set point is to be reached while in the second case an attitude trajectory is to be tracked. For the first case, a direct Lyapunov control is proposed, while in the second case a fuzzy controller with rules based on Lyapunov stability is proposed.

The rest of the paper is organized as follows. Section two provides the mathematical description of the quadrocopter dynamics and the propeller aerodynamics. The control schemes, which consist of the Lyapunov control and the fuzzy Lyapunov based approach, are discussed in Section three. Numerical results of the simulation are presented in Section four. The concluding argument is made in Section five.

2. Mathematical model of a quadrocopter

The quadrotor dynamics will be based on the Newton-Euler method [11], the assumptions made for this model is that the earth is flat and non-rotating so that we can place the fixed frame on the surface of the earth, this is a reasonable assumption since the quadrotor flies at low altitude and distances. Another assumption made is that the quadrotor body can be treated as a completely rigid structure. Fig. 1 shows the body fixed coordinates.

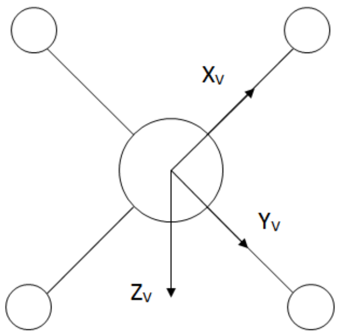

Fig. 1. Body-fixed coordinates

The equations of motion of the quadrotor are obtained and are:

$$
\begin{aligned}
\dot{P} &= \frac{(I_{yy}-I_{zz})QR + QJ_p\omega_p + C_T\, l(\omega_4^2-\omega_2^2)}{I_{xx}},\\
\dot{P} &= \frac{(I_{zz}-I_{xx})PR + PJ_p\omega_p + C_T\, l(\omega_1^2-\omega_3^2)}{I_{yy}},\\
\dot{R} &= \frac{(I_{xx}-I_{yy})PQ + J_p\,\dot{\omega}_p + C_Q\, l(\omega_1^2-\omega_2^2+\omega_3^2-\omega_4^2)}{I_{zz}},\\
\dot{U} &= VR - QW - g\sin\theta,\\
\dot{V} &= VR - QW - g\sin\theta,\\
\dot{W} &= QU - PV + g\cos\theta - \frac{k_T(\omega_1^2+\omega_2^2+\omega_3^2+\omega_4^2)}{m},\\
\dot{\varphi} &= Q\cos\varphi - R\sin\varphi,\\
\dot{\psi} &= (Q\sin\varphi + R\cos\varphi)\sec\varphi,
\end{aligned}
\tag{1}
$$

where P, Q, R are the pitch, roll and yaw rates respectively, J_p is the propeller inertia, ω_i is the angular velocity of the i-th propeller, C_T and C_Q are the thrust coefficient and torque coefficient respectively. The kinematic angles ϕ, θ, and ψ are the pitch, roll and yaw angles, respectively.

The motors used to drive the quadrotor propellers are brushless direct current (DC) motors, their use in quadrotors is widespread due to their many advantages such as high reliability and longer lifetime due to the absence of mechanical commutators. The equation used to describe the motor dynamics is:

$$\left(\frac{J_p}{\eta r^2}+J_m\right)\dot{\omega}_m=\frac{k_t}{R_{mot}}(u-k_\omega\,\omega_m)-\frac{d}{\eta r^3}\omega_m^2\,. \tag{2}$$

The aerodynamics of the propeller blades are modelled using a combination of Momentum Theory and Blade Element Theory called Blade Element Momentum Theory (BEMT) [5]. This method assumes two dimensional blade elements, which are then integrated over the span of the propeller [3]. Assumptions made are that blade flapping is neglected, the relative inflow angle is small, the drag is much smaller than lift and that the perpendicular component of velocity is significantly smaller than the tangential velocity. From this the equations for the thrust and torque coefficients are given as:

$$\begin{aligned} C_T&=\sigma\alpha\left[\theta_0\left(\frac{1}{6}+\frac{\mu^2}{4}\right)-\frac{\theta_{tw}}{8}(1+\mu^2)-\frac{\lambda}{4}\right],\\ C_Q&=\sigma\alpha\left[\frac{C_D(1+\mu^2)}{8\alpha}+\lambda\left(\frac{1}{6}\theta_0-\frac{1}{8}\theta_{tw}-\frac{1}{4}\lambda\right)\right]. \end{aligned} \tag{3}$$

3. Control schemes

3.1. Lyapunov stability

Lyapunov's direct method, which is also known as Lyapunov's second method, gives us the possibility to determine the stability of the system without having to integrate the full differential equation. This method uses the concept of the "measure of energy" of the system. If it is possible to find such a measure, then one can study the rate of change of energy of the system to deduce if the system is stable. For this case, we will assume that the system is time invariant, if the system has an energy function, $V(x)$, and a lie derivative $\dot{V}(x)$, which is given as:

$$\dot{V}(x)=\nabla V(x)f(x)=\frac{\partial V(x)}{\partial x_1}f_1(x)+\ldots+\frac{\partial V(x)}{\partial x_n}f_n(x)\,. \tag{4}$$

Then the sufficient conditions for stability are:

i) if $V(x)$ is locally positive definite and $\dot{V}(x)\le 0$ locally in x then the origin of the system is locally stable in the sense of Lyapunov.
ii) if $V(x)$ is locally positive semi-definite and decrescent, and $\dot{V}(x)\le 0$ locally in x, then the origin of the system is uniformly locally stable in the sense of Lyapunov.
iii) if $V(x)$ is locally positive definite and decrescent, and $-\dot{V}(x)$ is locally positive definite, then the origin of the system is uniformly locally asymptotically stable.

The Lyapunov based controller was done based on work of Bouabdallah [1], the Lyapunov candidate function is chosen to be positive definite about an equilibrium point, denoted X_e. The control inputs are then chosen in a way that the lie derivative is negative semi-definite. Since the system is autonomous, we can use LaSalle's invariance principle [4] to prove asymptotic stability.

$$f(X,U)=\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{pmatrix}=\begin{pmatrix}a_1 x_3 x_5 + a_2 x_3 \omega_p + b_1 U_1\\ x_1\\ a_3 x_1 x_5 - a_4 x_1 \omega_p + b_2 U_2\\ x_3\\ a_5 x_1 x_3 - b_3 U_3\\ x_5\end{pmatrix}, \tag{5}$$

$$\dot{V}(X_e)=b_1 x_1 U_1 + x_1(x_2 - x_2^d) + b_2 x_3 U_2 + x_3(x_4 - x_4^d) - b_3 x_5 U_3 + x_5(x_6 - x_6^d),$$

where $x_1 = P$, $x_3 = Q$ and $x_5 = R$.

The control inputs can then be chosen as:

$$\begin{aligned} U_1 &= -\frac{1}{b_1}(x_2 - x_2^d) - k_1 x_1,\\ U_2 &= -\frac{1}{b_2}(x_4 - x_4^d) - k_2 x_3,\\ U_3 &= \frac{1}{b_3}(x_6 - x_6^d) - k_3 x_5. \end{aligned} \tag{6}$$

where values of k_1, k_2 and k_3 should always be positive constants to keep the function negative semi-definite.

3.2. Fuzzy-Lyapunov based controller

Fuzzy controllers are an implementation of common sense or that of knowledge of a human operator. The implementation of this "common sense" though, is not such a trivial matter. As seen in the previous section, designing a system based on knowledge of a Lyapunov function creates a stable controller for nonlinear autonomous systems, as is the case with the quadrotor. We can base the fuzzy rules on the Lyapunov approach [6] to devise an adaptive controller that exhibits asymptotic stability. We will assume that we know very little about the system dynamics with this limitation we will try to synthesize a tracking controller. The partial knowledge we have of the plant is:

a) the roll angular acceleration, $\dot{x}_1$, is proportional to the control input U_1,
b) the pitch angular acceleration, $\dot{x}_3$, is proportional to the control input U_2,
c) the yaw angular acceleration, $\dot{x}_5$, is proportional to the negation of the control input U3,
d) $\dot{x}_2 = x_1, \dot{x}_4 = x_3, \dot{x}_6 = x_5$.

Since it is a tracking problem, we will want to minimize the error as well as create a stable system. We then take the tracking error and derivative of the tracking error as:

$$\begin{aligned} e_2 &= x_2 - x_2^d,\\ \dot{e}_2 &= \dot{x}_2 - \dot{x}_2^d,\\ \ddot{e}_2 &= \ddot{x}_2 - \ddot{x}_2^d = \dot{x}_1 - \ddot{x}_2^d. \end{aligned} \tag{7}$$

We then take a candidate Lyapunov function and its respective lie derivative as:

$$\begin{aligned} V &= \frac{1}{2}(e_2^2 + \dot{e}_2^2),\\ V &= e_2 \dot{e}_2 + \dot{e}_2 \ddot{e}_2. \end{aligned} \tag{8}$$

For our instance, we can neglect the term created by the 2nd derivative of the desired function since it would only slightly increase accuracy while requiring more computational cost. The same is done for pitch and yaw and we obtain the final derivatives:

$$\begin{aligned} \dot{V}_R &= \dot{e}_2(e_2 + \dot{x}_1), \\ \dot{V}_P &= \dot{e}_4(e_4 + \dot{x}_3), \\ \dot{V}_Y &= \dot{e}_6(e_6 + \dot{x}_5). \end{aligned} \tag{9}$$

To obtain asymptotic stability the fuzzy rules are proposed so that $\dot{V} \le 0$:

R1: **IF** $\dot{e}_2$ is *positive* **AND** e_2 is *positive* **THEN** U_1 is *negative,*

R2: **IF** $\dot{e}_2$ is *positive* **AND** e_2 is *zero* **THEN** U_1 is *negative,*

R3: **IF** $\dot{e}_2$ is *positive* **AND** e_2 is *negative* **THEN** U_1 is *zero,*

R4: **IF** $\dot{e}_2$ is *negative* **AND** e_2 is *positive* **THEN** U_1 is *zero,*

R5: **IF** $\dot{e}_2$ is *negative* **AND** e_2 is *zero* **THEN** U_1 is *positive,*

R6: **IF** $\dot{e}_2$ is *negative* **AND** e_2 is *negative* **THEN** U_1 is *positive,*

R7: **IF** $\dot{e}_4$ is *positive* **AND** e_4 is *positive* **THEN** U_2 is *negative,*

R8: **IF** $\dot{e}_4$ is *positive* **AND** e_4 is *zero* **THEN** U_2 is *negative,*

R9: **IF** $\dot{e}_4$ is *positive* **AND** e_4 is *negative* **THEN** U_2 is *zero,*

R10: **IF** $\dot{e}_4$ is *negative* **AND** e_4 is *positive* **THEN** U_2 is *zero,*

R11: **IF** $\dot{e}_4$ is *negative* **AND** e_4 is *zero* **THEN** U_2 is *positive,*

R12: **IF** $\dot{e}_4$ is *negative* **AND** e_4 is *negative* **THEN** U_2 is *positive,*

R13: **IF** $\dot{e}_6$ is *positive* **AND** e_6 is *positive* **THEN** U_3 is *positive big,*

R14: **IF** $\dot{e}_6$ is *positive* **AND** e_6 is *zero* **THEN** U_3 is *positive small,*

R15: **IF** $\dot{e}_6$ is *positive* **AND** e_6 is *negative* **THEN** U_3 is *zero,*

R16: **IF** $\dot{e}_6$ is *negative* **AND** e_6 is *positive* **THEN** U_3 is *zero,*

R17: **IF** $\dot{e}_6$ is *negative* **AND** e_6 is *zero* **THEN** U_3 is *negative small,*

R18: **IF** $\dot{e}_6$ is *negative* **AND** e_6 is *negative* **THEN** U_3 is *negative big,*

where e_2, e_4, e_6 and their time derivatives are the controller inputs, U_1, U_2 and U_3 are the controller outputs, the *negative* and *positive* specified for input variables are fuzzy sets with trapezoidal-shaped membership function (10), the *negative small*, *zero* and *positive small* are fuzzy sets with Gaussian-shaped membership function (11), and the *negative*, *negative big*, *positive* and *positive big* specified for the outputs are the fuzzy sets with Z-shaped (12) and S-shaped (13) membership functions, respectively.

$$f(x,a,b,c,d) = \begin{cases} 0, & x \le a \vee x \ge d \\ \dfrac{x-a}{b-a}, & a \le x \le b, \\ 1, & b \le x \le c, \\ \dfrac{d-x}{d-c}, & c \le x \le d, \end{cases} \tag{10}$$

$$f(x,\sigma,\mu) = \exp\left(\frac{-(x-\mu)}{2\sigma^2}\right), \tag{11}$$

$$f(x,a,b)=\begin{cases}1, & x\le a,\\ 1-2\left(\dfrac{x-a}{b-a}\right)^2, & a\le x\le \dfrac{a+b}{2},\\ 2\left(\dfrac{x-b}{b-a}\right)^2, & \dfrac{a+b}{2}\le x\le b,\\ 0, & x\ge b,\end{cases} \tag{12}$$

$$f(x,a,b)=\begin{cases}0, & x\le a,\\ 2\left(\dfrac{x-a}{b-a}\right)^2, & a\le x\le \dfrac{a+b}{2},\\ 1-2\left(\dfrac{x-b}{b-a}\right)^2, & \dfrac{a+b}{2}\le x\le b,\\ 0, & x\ge b.\end{cases} \tag{13}$$

4. Results of simulations

The simulations were carried out for the parameters of the quadrocopter's model presented in [1]. The Lyapunov controller was tested for $k_1 = k_2 = 0.05$, $k_3 = 0.5$. The parameters of fuzzy-Lyapunov controller were empirically selected during simulations. The membership functions parameters are presented in Tab. 1. The implication and conjunction operations are performed using minimum method, while the defuzzification is carried out using centre of gravity method [13].

The results of simulations are presented in Fig. 2 and 3. The roll step response using the Lyapunov based control is shown in Fig. 2. Fig. 3 presents the roll and yaw responses for tracking control. Since the membership functions and rules are the same for roll and pitch, only roll and yaw are shown. For the control signal to be tracked, a sinusoidal function is used, for roll with 20º amplitude and a frequency of 0.5 rad/s, and for yaw an amplitude of 10º with a frequency of 0.1 rad/s. For the Lyapunov control the roll and pitch settling time was about 23 seconds with an overshoot of about 17%.

Tab. 1. Fuzzy membership functions parameters

input/output variable	fuzzy set	membership function	parameters of membership function					
			a	b	c	d	σ	μ
e_2, e_4, e_6	*negative*	trapezoidal	–1	–1	–0.75	0		
	zero	Gaussian					0	0.025
	positive	trapezoidal	0	0.75	1	1		
$\dot{e}_2, \dot{e}_4, \dot{e}_6$	*negative*	trapezoidal	–1	–1	–0.75	0		
	positive	trapezoidal	0	0.75	1	1		
U_1, U_2	*negative*	Z-shaped	–0.1	0				
	zero	Gaussian					0	0.005
	positive	S-shaped	0	0.1				
U_3	*negative big*	Z-shaped	–0.08	–0.05				
	negative small	Gaussian					0.01	–0.03125
	zero	Gaussian					0.025	0
	positive small	Gaussian					0.01	0.03125
	positive big	S-shaped	0.05	0.08				

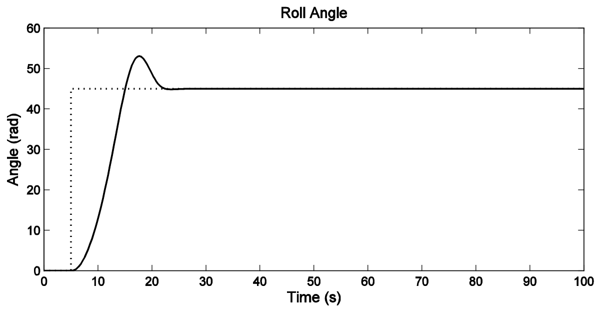

Fig. 2. Roll step response (Lyapunov control)

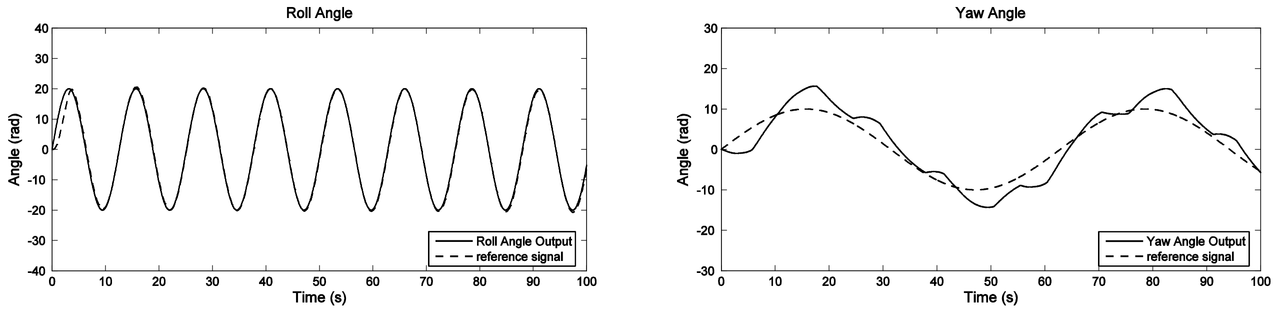

Fig. 3. Roll and yaw responses for trajectory tracking (fuzzy controller)

5. Conclusions

The idea behind Lyapunov based control was to create a stable system, even at large angles where the effects of the nonlinear dynamics are dominant. The controller was able to stabilize the quadrocopter even at high angles. The positive constants were not optimized. Further research should concentrate on optimization of the constants. The fuzzy controller based on Lyapunov stability showed promise, being able to track the desired trajectory with high accuracy for both pitch and roll even without complete knowledge of the system. The yaw angle tracking was not smooth and this was because the partial model did not account for the effects of the propeller dynamics

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