ARCHIVES OF ELECTRICAL ENGINEERING VOL. 63(1), pp. 107-114 (2014)

DOI 10.2478/aee-2014-0008

Calculation of eddy current losses using the electrodynamic similarity laws

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(Received: 14.11.2013, revised: 25.01.2014)

Abstract: The results of the eddy currents losses calculations with using electrodynamics scaling were presented in this paper. Scaling rules were used for obtain the values of the eddy currents losses. For the calculations Finite Element Method was used. Numerical calculations were verified by measurements and a good agreement was obtained.

Key words: field analysis, scaling in electrodynamics, eddy current losses

1. Introduction

Big dimensions of some electrical devices (for example power transformers) in present days cause difficulties in measurement verification of their field analysis. On the other hand side, many electrical appliances have small dimensions, so that the testing probes cannot be placed inside their construction. Taking into account the difficulties, one wants to build the scaled physical models. The manufactured scaled physical objects can be built with some similarity rules. Their testing allows to determine the full scaled object characteristics [3]. The similarity rule depends on quantity, which must be hold for scaled model according to the original one. In our work we have modeled the eddy current losses in a frame-shape magnetic core. Therefore, the same value of the scaled model eddy current losses should govern the losses of the original object.

2. Similarity rules used in this work

2.1. General assumptions

$$\begin{cases} \nabla_{org} \times \vec{H}_{org} = \gamma_{org}\vec{E}_{org} + \dfrac{\partial \vec{D}_{org}}{\partial t_{org}}, \\ \nabla_{org} \times \vec{E}_{org} = -\dfrac{\partial \vec{B}_{org}}{\partial t_{org}}. \end{cases} \tag{1}$$

We assumed that the quantities in Maxwell's equations which describe the electromagnetic fields for the original, have subscript "*org*" in the paper. Similarly, the quantities in the equations above concern the scaled models and have subscript "*scl*". A scale factor, as a ratio of a scale-model quantity (X_{scl}) to the original quantity (X_{org}), has been defined in this work. The electromagnetic scale factors, for the most important quantities are given in Equation 2

$$m_H = \frac{\vec{H}_{scl}}{\vec{H}_{org}}, \quad m_E = \frac{\vec{E}_{scl}}{\vec{E}_{org}},$$
$$m_f = \frac{f_{scl}}{f_{org}} = \frac{t_{scl}}{t_{org}}, \quad m_\gamma = \frac{\gamma_{scl}}{\gamma_{org}}, \tag{2}$$
$$m_\mu = \frac{\mu_{scl}}{\mu_{org}}, \quad m_\varepsilon = \frac{\varepsilon_{scl}}{\varepsilon_{org}}, \quad m_l = \frac{l_{scl}}{l_{org}}.$$

The Maxwell's equations for original and scaled model [2, 3, 5] force the expressions 3a and 3b:

$$m_f m_\gamma m_\mu m_l^2 = 1, \tag{3a}$$

$$m_\varepsilon m_\mu m_f m_l^2 = 1. \tag{3b}$$

For materials with linear characteristics, which have been assumed in this work, we have obtained the expression below:

$$m_\gamma = m_\varepsilon = m_\mu = 1. \tag{4}$$

Thus, the magnetic field strength ***H*** and the current density ***J*** can be scaled according to the expression:

$$\frac{m_J m_l}{m_H} = \frac{m_I}{m_H m_l} = 1. \tag{5}$$

2.2. The eddy current losses balance (ECLB)

Losses which are generating inside the conductors and magnetically soft iron elements influence heating of all electromagnetic device. Thus, the computer aided design (CAD) should allow us to calculate the losses as exactly as possible. Using the field analysis, we have computed the losses in original magnetic circuit, and inside its scaled duplicate. We have determined the similarity rules to define scaled object eddy currents losses.

In this paper, the conductivity and magnetic permeability factors $m_\gamma = m_\mu = 1$ were assumed. Using the expression (3a) the frequency scale-factor can be given as $m_f = 1/m_l^2$. Thus, the relationship between scale the factors of magnetic field strength m_H and geometric dimensions m_l is equal to

$$m_H = \frac{1}{\sqrt{m_l}}. \tag{6}$$

In our mathematical models we assumed the same number of turns N in the original and scaled model. Thus, the values of the excitation currents for scaled models are given below

$$I_{scl} = m_H m_l I_{org} = \sqrt{m_l} I_{org}. \tag{7}$$

2.3. The current density balance (CDB)

The limits in the supplying of electrical devices determine the range of prototype testing. Thus, in this work the second similarity rules were used where the current density balance has been included ($m_J = J_{scl}/J_{org} = 1$). The conductivity and magnetic permeability relation $m_\gamma = m_\mu = 1$ has been assumed. Using the expression (3a) the frequency scale-factor can be determined $m_f = 1/m_l^2$. In this case, the relation between factors of the excitation current and linear dimensions is $m_I = m_l^2$. The same values of the excitation current has been assumed in our mathematical models. Thus, the number of turns N_{scl} approximated by the nearest turn integer number, and the winding cross-sections S_{scl}^{Cu} are expressed below

$$N_{scl} = m_l^2 N_{org}, \tag{8a}$$

$$S_{scl}^{Cu} = m_l^2 S_{org}^{Cu}. \tag{8b}$$

The eddy current losses P_{EDscl} calculations must be scaled according the relation

$$P_{EDscl}^{org} = \frac{P_{EDscl}}{m_l^3}. \tag{9}$$

3. Analysed object and its mathematical model

3.1. Analysed object

We have studied the modular construction of the amorphous, 1-phase transformer. Outline and cross-section along YZ plane of the analysed object are presented in Figures 1a) and 1b). Moreover, in this figures the main dimensions and Cartesian system are given.

Owing to modular construction of the amorphous transformer, we determined the eddy current losses in the toroidal solid element. It has been placed in the magnetic circuit as one part of its core. The core part was made from casting steel and situated in the right side leg of the amorphous magnetic circuit (Fig. 1b). Eddy current losses inside the solid toroid are several times higher than the ones inside the laminated amorphous toroid which create a part of the magnetic leg. Due to the solid part, the magnetic flux density in all magnetic circuit is relatively low. Thus, the circuit can be assumed as magnetically linear. We assumed the relative permeability of the magnetic parts from the amorphous ribbon: μ_r = 20173 (for the yokes) and μ_r = 2067 (for the legs). Because the flux in the legs is perpendicular to wounding direction of the amorphous ribbon, the small permeability value is assumed. For the solid hollow cylinder, made from casting steel, the relative permeability μ_r = 150 is assumed, and electrical conductivity $\gamma = 2.5 \cdot 10^6$ S/m have been included.

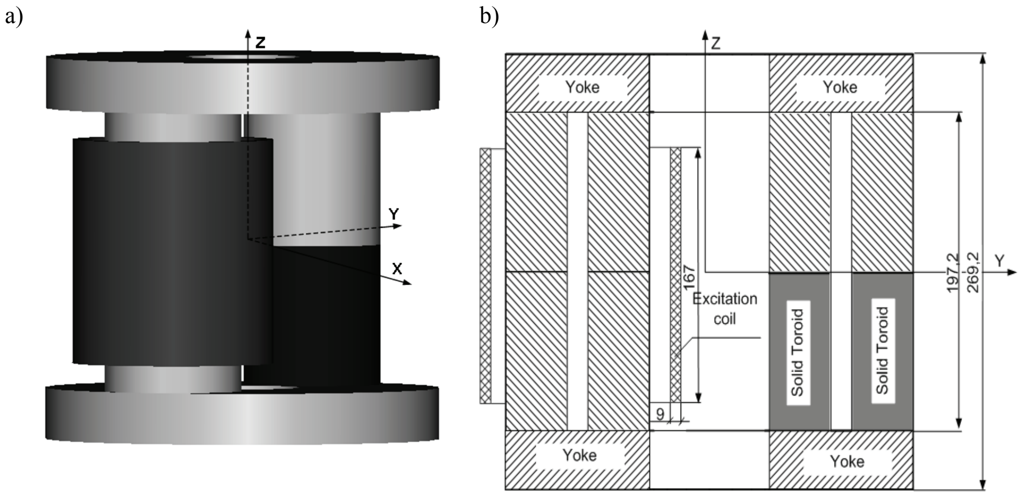

Fig. 1. Amorphous 1-phase transformer (original): a) Outline of the transformer, b) Cross-section along YZ plane

The excitation winding has been wounded with N = 116 turns and placed on the laminated (left) leg, Figure 1b.

3.2. Mathematical model

For numerical calculations the Finite Element Method (FEM) has been implemented in module Elektra of the commercial package OPERA 3D [1]. This module enables us to analyse the electromagnetic fields taking into account the effects of eddy currents. It can be done using total $\boldsymbol{A_t}$ and reduced $\boldsymbol{A_r}$ magnetic vector potentials [1, 4].

$$\vec{B} = \nabla \times \vec{A}_t, \tag{10}$$

$$\vec{B} - \mu_0 \vec{H}_S = \nabla \times \vec{A}_r. \tag{11}$$

In our mathematical models, we assumed several values of rms values for the sinusoidal waves of the excitation current: (1.42 to 9.50) A. The frequency of the supplying signal was assumed to be f = 50 Hz. Due to significant current losses inside the steel solid toroid the losses in the amorphous parts, can be neglected. Moreover, in our calculations we neglected the losses appeared in the excitation coil. It can be possible due to small cross section of its wires.

4. Calculation results and measurement verification

In this paper two variants of the field analysis have been carried out. The first variant W_1 concerns the model two times smaller than the original object (m_l = 0.5). The second variant W_2 relates to model which is two times bigger (m_l = 2) than the original. The assumed frequency for W_1 variant was f_{scl} = 200 Hz. Thus, the scale factor for the frequency amounts m_f = 4. In W_2 model the frequency f_{scl} = 12.5 Hz and its scale factor (m_f = 0.25) were arranged. For original object, in figure 2 are presented the eddy current density distribution (on the solid toroid surface) and magnetic flux density on plane *YZ*, under the 5.25 A rms value of the excited current.

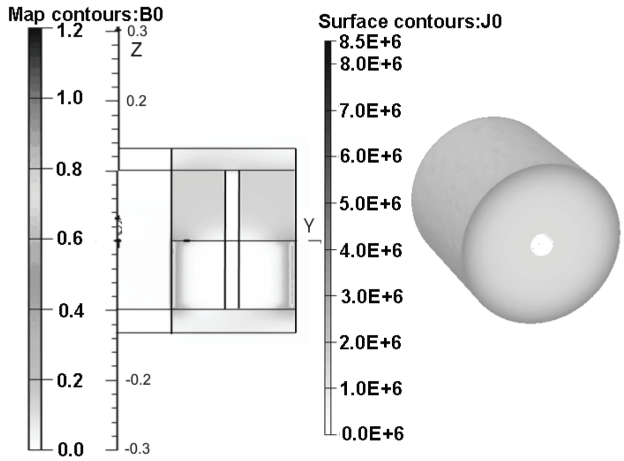

Fig. 2. Magnetic flux density (on the plane YZ) and eddy current density distributions (on the solid toroid surface) for original object supplied with the current (I_{org} = 5.25 A)

4.1. Calculation results for the eddy current losses balance

The rms values of the excitation current, has be changed (I_{scl} = 1.01÷6.74) A for considered cases. Using Equation 7 the rms current values for sinusoidal waves were changed from I_{scl} = 2.0 to I_{scl} = 13.3 A, for the modeled objects.

In Figures 3a÷3b the eddy current densities distributions on the surface of the casting steel i.e. solid toroid, for both original and scaled (two variants) numerical models, are presented. The calculations have been carried out for rms value of the excitation current I_{org}=5.25 A. The presented distributions are similar to each other. Despite different dimensions and excitation current frequencies, the magnetic fields inside the analyzed magnetic circuits are not significantly different. For original object the maximal value of the eddy current density is slightly lower than 3 MA/m^2, whereas for the scaled model with dimension factor m_l = 0.5 (variant W_1) the maximum values reach almost 8 MA/m^2. In the case of two times bigger object (m_l = 2), this value is about 1.28 MA/m^2.

Eddy currents significantly influence the core losses, naturally. In this paper the losses of the solid steel toroid were calculated using the expression below

$$P_{ED} = \int_V \left(\frac{\vec{J}^2}{\gamma} \right) dV. \tag{12}$$

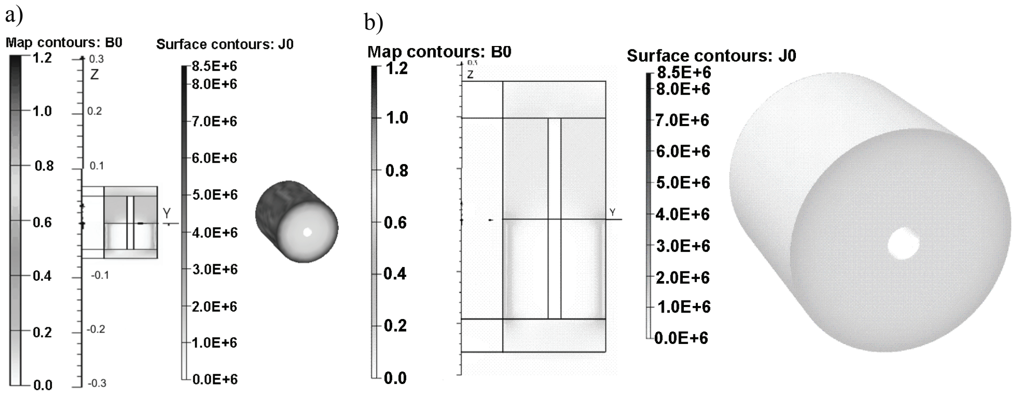

Fig. 3. Magnetic flux density on plane YZ and eddy current density distributions on the solid toroid surface: a) model with scaled factor $m_l = 0.5$ ($I_{scl} = 3.71$ A), b) model with scaled factor $m_l = 2$ ($I_{scl} = 7.42$ A)

Calculated values of the eddy current losses for different excitations in both scaled variants and original object are placed in the Table 1.

Table 1. Calculated values of the eddy current losses inside the solid toroid

I_{org}	Calculations		
	$m_l = 0.5$	original	$m_l = 2$
[A]	[W]	[W]	[W]
1.42	5.44	5.85	6.00
3.06	24.90	27.29	28.00
5.25	73.05	80.07	82.16
7.23	138.40	151.70	155.66
9.50	239.2	262.56	269.42

The relative differences between obtained values for both scaled (variants W_1 and W_2) and original models are lower than 10%. Thus the calculations carried out in this work confirm correctness of the scaling rules.

4.2. Calculation results for the current density balance

The rms values of the excitation current for the current density balance, has be changed (I_{scl} = 1.42÷9.50) A for considered cases. Obtained the eddy current densities distributions on the surface of the casting steel i.e. solid toroid for both scaled variants numerical models are presented in Figures 4a and 4b. Below distributions concern rms value of the excitation current I_{org} = 5.25 A.

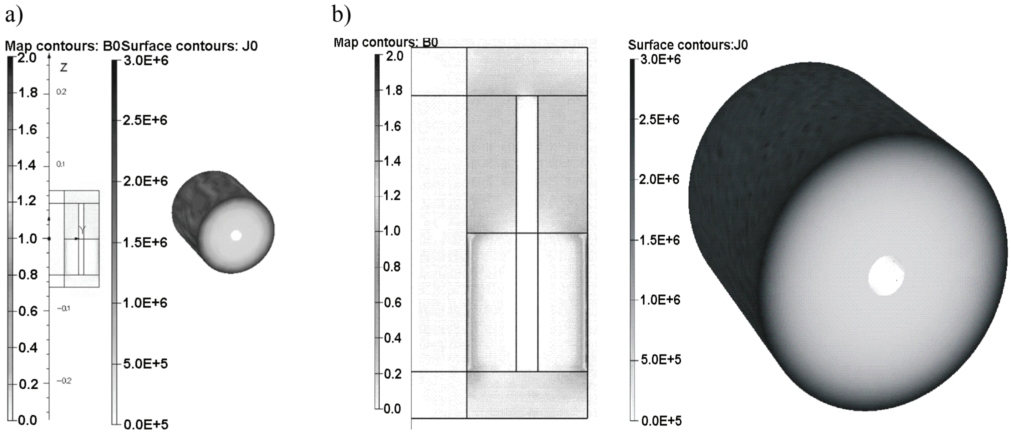

Fig. 4. Magnetic flux density on plane YZ and eddy current density distributions on the solid toroid surface for excited current $I = 5.25$ A: a) model with scaled factor $m_l = 0.5$, b) model with scaled factor $m_l = 2$

Maximal values of the eddy current density are similar. Whereas the maximal value of the magnetic flux density inside of the solid toroid for W_2 variant ($m_l = 2$) is circa 4 times higher than for W_1 variant ($m_l = 0.5$) and reach about 1.9 T. Calculated, using Equations 9 and 12, values of the core losses for both variants and original object are given in Table 2.

Table 2. Calculated values of the eddy current losses inside the solid toroid for the current density balance

I_{org}	Calculations		
	$m_l = 0.5$	original	$m_l = 2$
[A]	[W]	[W]	[W]
1.42	5.44	5.85	6.00
3.06	24.90	27.29	28.00
5.25	73.05	80.07	82.16
7.23	138.40	151.70	155.66
9.50	239.52	262.56	269.42

Good agreement between the results of the eddy current losses for scaled models and the original has been obtained. Thus, that presented scaling rules for the models with the current density balance are correct.

4.3. Measurement verification

In Figure 5 the calculated and measured values of the core losses inside the solid toroid, for original and scaled objects, were given. The measurements of the core losses were made by using oscilloscope. Due to relatively low value of the magnetic flux density inside magnetic circuit, the core losses of the amorphous parts were neglected. As we can see, the presented values for the two scaling rules (ECLB and CDB) come to near.

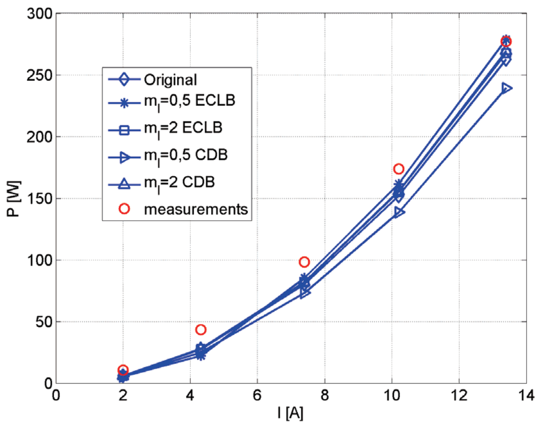

Fig. 5. The calculated and measured values of the eddy current losses inside the solid toroid for the original and scaled objects

5. Conclusions

In this paper the eddy current losses inside the solid toroid, placed in the modular 1-phase magnetic circuit from amorphous ribbon, were calculated. The electromagnetic similarity rules have been analysed and used. The similarity rules were assumed to hold the same values of the eddy current losses. Objects for two scaled factors m_l = 0.5 and m_l = 2 have been modeled. Obtained values of the losses (Tabs. 1-2) confirm the correctness of the similarity rules. Thus, by combining the scaled physical model and the mathematical field problem solution, the designer can create, without expensive prototyping, the accurate geometry of an electromagnetic device.

Acknowledgements

This paper is partially supported by the National Science Centre under grant no. N N510 533739.

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