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Long term projection of the demographic and financial evolution of the parliamentary pension scheme of Uganda

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Abstract

We study the Parliamentary Pension Scheme of Uganda, a hybrid cash balance scheme which is contributory. It has two categories of members, the staff of the Parliamentary Commission and the Members of Parliament. A long term projection of the scheme's demographic and financial evolution is done to asses its sustainability and fairness with respect to the two categories of members. The projection of the scheme's future members is done using non-linear regression. The distribution of future members by age states is done by Markov model using frequencies of state transition of the scheme members. We project the future contributions, accumulated funds, benefits, asset and liability values together with associated funding ratios. The results show that the fund is neither sustainable nor fair with respect to the two categories of members.

Keywords: *guaranteed pension, hybrid cash balance scheme, projection, demography, finance*

1. Introduction

A pension is a term for single or periodic payments to a beneficiary, which replaces the income of an employee in case of reaching a certain age, or in the case of disability or death. A pension fund is an organisation obliged with paying pensions. It has a task of making benefit payments to members who have ended their active working and earning careers. The payments are made to the retirees in accordance to a benefit formula, which prescribes the flow of payments to which each member in the fund is entitled. Pension funds may be defined as forms of institutional investors which collect, pool and invest funds contributed by sponsors and beneficiaries, to provide for the future pension entitlements of beneficiaries [19].

From a financial point of view, pension schemes are classified into Pay-As-You-Go (PAYG) and funded systems. In PAYG systems, pension for retirees is paid out of contributions from current members. According to [39], in a pure PAYG system contributions should match benefits. For the case of funded systems, there is no inter-generational redistribution. The contributions are used to purchase assets that finance benefits upon retirement. Pension funds always face demographic and financial risk.

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Several studies that have been conducted in this area include [4, 6, 7, 11, 13, 16, 22, 26, 27, 33, 36, 40]. The mandatory Tanzania pension fund, which is a final salary defined benefit fund was analysed by [3]. They projected the future contributions, benefits, asset values and liabilities. The projection shows that the fund will not be fully sustainable on a long term due to an increase in life expectancy of the members.

In several countries, mandatory public schemes are usually supplemented by occupational schemes. According to [37], access to public pension for the working population in emerging economies is limited to 10% to 25%. Occupational schemes are broadly categorised into Defined Contribution (DC) and Defined Benefit (DB) schemes. In DB schemes a benefits formula linked to salary and years of service is used, while in DC schemes the amount contributed to the fund is specified. According to [25], most DB type PAYG systems, however, have no or little linkage between annual benefits and retirement age, while funded DC plans are actuarially neutral, since conversion to an annuity takes place at actual retirement. These schemes give incentives that are of strong economic importance for their sustainability [24]. Most of the pension fund activities are restricted by policies and regulations. Whether the scheme is DC or DB, still there are restrictions on how members contribute, paid benefits as well as investment processes. According to [20], the management of a pension fund can not only manage its liabilities, but also the assets can be managed. One of the instruments to manage the assets is by means of the contribution policy, which is the system on which the level of the contribution rate is determined.

Pension systems allover the world are under stress due to demographic trends, coupled with rigidity of legislation and implied financial imbalances. According to [18], the 1980s were characterised by search for solutions to short term financial gaps and long term demographic challenges. However, the reforms were limited to interventions into parametric adjustments, such as reductions in retirement benefit levels and increase in contribution rates. A more systematic reform was implemented in 1981 by Chile, a pioneer country to introduce funded pension systems [17]. The Chilean reform was adopted by many Latin American countries and some European countries like Croatia, Hungary and Kazakhstan. In 1994, Sweden developed a reform option that structurally modified its pension system. The reform introduced the Notional Defined Contribution (NDC) pension system, which was later implemented in other countries such as Germany, Italy [23], Latvia [28], Poland [14] and Norway [15], due to un-sustainability of the DB pension systems. According to [10], properly designed NDC public pension systems contain powerful economic and political mechanisms that may facilitate pension reform. The NDC is closer to traditional DB than funded DC systems. Pension benefits are paid out of current contributions as in a conventional PAYG system, but the link between benefits and contributions are individualised and defined by the NDC accounting mechanism. This minimises the role of the normal retirement age, permitting a flexible choice between working longer and receiving a lower replacement rate.

Increase in life expectancy and decreases in fertility rates are predicted to make financing of public pension systems in the European countries hard in the coming years [5]. European union member countries, with very different pension systems have made pension reforms for sustainability. The reforms include increasing retirement age gradually, linking retirement age or benefits with changes in life expectancy. These have mitigated financial burden of pension systems, but there have been limited recent reforms aimed at increasing pre-funded pension savings such as occupational schemes. In low income countries (a low income country is defined by the World Bank as a nation with gross national income per capita of United States dollars $1,025$ or more in 2014, calculated using the World Bank Atlas method [21]), legacies of special privileges to selected groups, corrupt public sector agencies which manage the funds and mis-management of fund assets has led to un-sustainability of pension funds [29].

The pension sector in Uganda is currently composed of the Public Service Pension Scheme (PSPS), which is a government scheme that caters for the pensions of civil servants, and the National Social Security Fund (NSSF), which is responsible for the retirement benefits under private sector. The PSPS is a defined benefit non-contributory system that is guaranteed by the state and funded from taxes collected by the government, thus making it fiscally unsustainable. The results of the 2014 population census indicate that Uganda had a labour force of 16 million people. By the end of 2015, only $757,179$ Ugandans were actively saving with the NSSF, while there were $307,000$ active civil servants, and this translates into slightly over one million Ugandans having some form of retirement benefit. This means that only about 6% of Uganda's labour force is actively saving for retirement, and the rest face the risk of old age poverty. A brief summary of the different retirement benefit schemes in Uganda is given in Table 1, where PS stands for Pension Scheme and AFPS stands for Armed Forces Pension Scheme.

Table 1. Uganda's retirement benefit schemes

	PSPS/Local authorities PS	AFPS	NSSF	Occupational schemes
Legal framework	Pensions Act [Cap 286]	Armed Forces Pension Act [Cap 298]	NSSF Act [Cap 222]	UBRA Act 2011/Trust Law/ Act of Parliament, for example Parliamentary Pension Scheme
Coverage	Civil servants; local government workers; prison officers; judiciary; doctors; teachers	Military officers	Formal sector workers - employers with five or more employees	Formal employees in companies/institutions with pension plans Members of parliament
Benefits financing	Tax revenues	Tax revenues	Accumulated individual accounts (employer 10%, employee 5%)	Scheme funds - Contributory as per plan rules/enabling Act
Benefits payment	Annuities and lump sums	Annuities and lump sums	Lump sums - Provident Fund	Annuities and lump sums depending on plan rules

Some studies on Uganda's social security system include: study of asset liability management for the Parliamentary Pension Scheme of Uganda by stochastic programming [34], the status of social security in Uganda [12], proposed adoption of a twin peak mechanism in the financial sector [31], examination of social, economic and demographic risk factors [35], the World Bank PROST model was used by [8] to analyse the future liabilities that the Ugandan PSPS might accumulate under the provisions of Cap 286, unless it is reformed. The latter recommended a hybrid reform option composed by a small DB scheme and a complementary DC scheme. In 2011, the parliament of Uganda passed a bill to liberalise the retirement benefits sector. The Uganda Retirement Benefits Regulatory Authority (URBRA) was established under the URBRA act 2011, which is mandated to regulate the establishment, management and operations of retirement benefit schemes in Uganda.

The Parliamentary Pension Scheme (PPS) started its operations in 2001 and was formally established by an Act of Parliament in 2007, which was later amended in 2011. The scheme has two categories of members, the staff of the parliamentary commission (staff) and Members of Parliament (MPs). The PPS is a hybrid cash balance scheme which combines features of both DB and DC schemes, hence it operates like NDC systems. The benefits are indexed to the members' contributions, there is a guaranteed period of pension payment, and a guaranteed return on accumulated funds on members' notional accounts. The benefit rules are written into the law as mathematical formulae and this reduces political risk. The scheme sponsor undertakes to provide benefits expressed in form of annuity and lump sum calculation, based on work history and guaranteed return on contribution account, regardless of investment performance of the fund associated with the scheme, which therefore constitutes a contingent liability to the sponsor.

The aim of this study is a long term projection of the demographic and financial evolution of the PPS, and to analyse the long term financial sustainability of the scheme, with respect to each category of members and fairness of the scheme to the two categories of members. The projection is done for a horizon of 50 years from 2018 to 2068. We use data from the scheme's annual reports (available in [2]) and bio-data information provided by the scheme. The bio-data information includes; year of birth, year of joining the scheme, year of retirement or leaving the scheme for each member. Established abridged mortality tables are used for future expectation of life and survival probabilities.

The remainder of the paper is organised as follows. In Section 2 we use historical data to obtain the age distribution of new members and state transition matrices, and project the scheme members and their age distribution, together with future average ages and dependence ratios. In Section 3 we project the contributions, accumulated funds, benefits, cash flows, liabilities, assets, cash flow to asset value ratios and funding ratios. In Section 4 we evaluate the effect of changes in asset return, and Section 5 gives the conclusion and future research.

2. Demographic projections

The PPS is composed of staff members of the parliamentary commission (staff) and Members of Parliament (MPs). The remuneration and terms of service for the two groups are very different, hence we handle the two groups separately, and combine results from the two groups. The initial year of our projection is 2018. From the PPS annual report 2017/2018 the scheme had a total of 959 members of which 842 were active, 2 were deferred staff and 115 were pensioners. The initial active staff were 394 and 30 pensioners. The initial active MPs were 44 and 85 pensioners. In each year new members join, some members retire, some leave and others die. From the Parliamentary Pensions Act 2007 as amended and Parliamentary Pensions Regulations from 2013, a member can retire on attaining the retirement age of 45 years subject to being a member for a continuous period of 5 years. The staff always retire at the normal retirement age of 60 years for civil servants in Uganda, while the MPs have no restriction on continuing to work beyond the retirement age as long as they get elected.

A pension fund planning horizon can stretch for several decades due to the long term commitment to members. Fund population growth is therefore a key element of projection for pension funds. The evolution of the fund population over the horizon is shown in Figure 1.

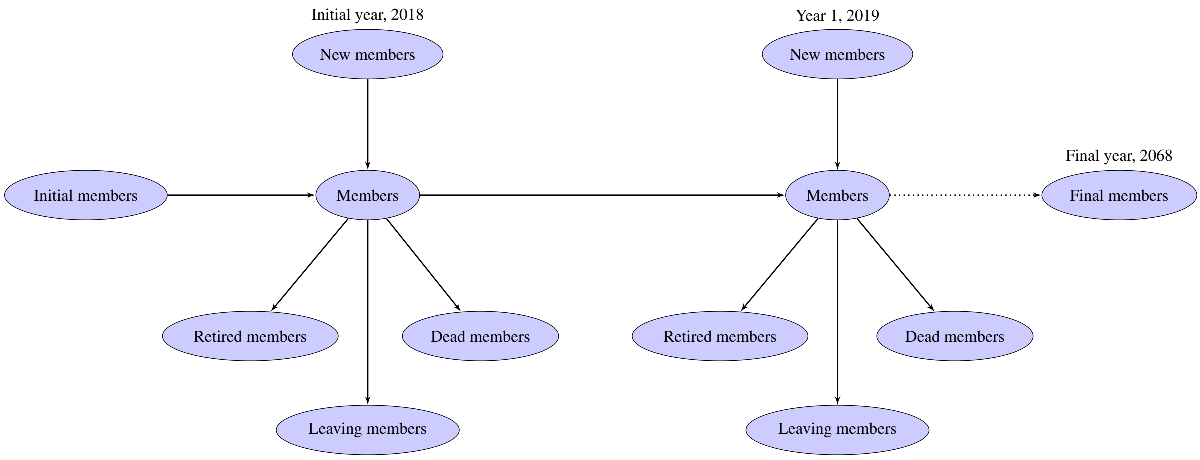


Figure 1. Evolution of the fund population

Based on [32], we consider a state transition matrix $\mathbf{\Pi}$ for n, different states which include ages of active members, leavers, retired, pensioners and dead, and defined as

$$\mathbf{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{12} & 0 & 0 & 0 & 0 & \dots & \pi_{1n-2} & \pi_{1n-1} & \pi_{1n} \\ 0 & \pi_{22} & \pi_{23} & 0 & 0 & 0 & \dots & \pi_{2n-2} & \pi_{2n-1} & \pi_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & \dots & \dots & 1 \end{bmatrix}$$

Here, the columns and rows represent states of the fund and π_{ijt} is the portion of the fund population that moves from state i to state j in year t, with

$$\begin{aligned} &\pi_{ijt} \geq 0 && i,j = 1,\dots,n \\ &\sum_{j=1}^{n} \pi_{ijt} = 1 && i,j = 1,\dots,n \\ &\pi_{ijt} = \frac{f_{ij}}{\sum_{j=1}^{n-1} f_{ij}} [1 - \pi_{int}] && i = 1,\dots,n-1 \end{aligned}$$

where f_{ij} is the frequency of transition from state i to state j and π_{int} is the death probability in state i in year t.

In each state, there is the risk of dying and each year the model moves a member to the next age or to death. If the age is near retirement, the model moves a survivor to the retirement state. Altogether, there are five transition possibilities: move to next age, leave, become inactive, die or retire. There are eight aggregate age states for staff and nine for MPs; the common ones are $20-24$, $25-29$, $30-34$, $35-39$, $40-44$, $45-49$, $50-54$ and $55-59$, with an additional state 60+ for MPs who are at least 60 years of age. These states are represented by the numbers $1, 2, 3, 4, 5, 6, 7, 8$ and 9 respectively in the figures. The Markov chains from the state transition matrices are given in Appendix A.3.

The population at time t is a vector $\boldsymbol{P}_t$ given by the relation

$$\boldsymbol{P}_t = \boldsymbol{\Pi}_t^T \tilde{\boldsymbol{P}}_{t-1} + N_t \boldsymbol{d}_N \tag{1}$$

where N_t is the number of new members in year t, $\boldsymbol{d}_N$ is a vector for the age distribution of new members in the different states, obtained using frequencies in Appendix A.2 and $\tilde{\boldsymbol{P}}_{t-1}$ is the population vector $\boldsymbol{P}_{t-1}$ with zeros for leaving and dead members in year $t-1$. This population status is updated throughout the planning horizon.

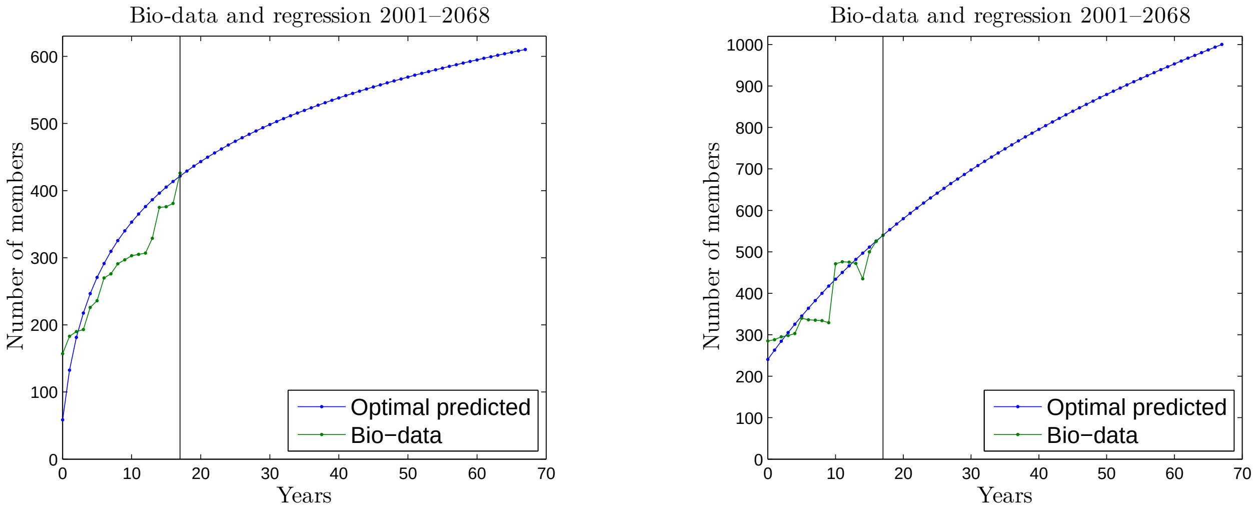
Historical data from the PPS was used to find the state transition frequencies, given in Appendix A.1 and age distribution of new members. The bio-data information used was from 2001 to 2018 and some reasonable assumptions about the future PPS members for the period 2018 to 2068. Using historical data for fund population, we use non-linear regression to estimate the future total population of staff and MPs. The best fitting models to the data are $y_t = \alpha + \beta \ln(t + \alpha)$ for the staff, and $y_t = \alpha + \beta \ln(t + \psi)$ for the MPs, where t is time, y_t is the number of members and α, β and ψ are parameters. These models are suitable for projecting the scheme populations, since in the beginning, the population increases rapidly. In later years, as the scheme matures, there is little increase in scheme population and it becomes relatively stable. The optimal values of parameters in the regression model are obtained iteratively by minimising the weighted sum of the squared relative residuals. The future total population of the staff is given in equation (2), while that of MPs is given in equation (3). The graphical results are shown in Figure 2, from which the PPS staff fund population in year t is given by

$$\mathrm{STF}_t = 1.5 + 144 \ln(t + 1.5) \tag{2}$$

while the MPs population in year t is given by

$$\mathrm{MPs}_t = 598 \ln(t + 26) - 1713 \tag{3}$$

where t is the year, with 2001 as year 0. The projections are done from 2018 to 2068, that is for $t = 17, 18, 19, \dots, 67$. The vertical line at $t = 17$ in Figure 2a) and 2b) indicates the starting year of our projections. The staff population increases from 425 members at the start of the horizon to 610 at the end, while the MPs population increases from 534 to 1000 at the end.



a). Projected member population, staff

b). Projected member population, MPs

Figure 2. Projected scheme populations

From the projections of [1], the probability of dying at a given age is expected to reduce in future and the expectation of life at a given age is projected to increase. Mortality and expectation of life are given in Table 2 and Table 3, respectively, where we compare for Uganda to high income countries by using the average values in the period 2020–2050. Mortality and expectation of life for members in the PPS are far different from those for Uganda in general, and are comparable to mortality for high income countries. This is because their remuneration and working condition is far ahead of the rest of the ordinary population in the country. In order to have a more realistic projection, we therefore use mortality and expectation of life for high income countries in our projections, which are provided in Table 4 and Table 5 respectively of Appendix A.4.

Table 2. Average mortalities 2020–2050

Age	Uganda	High income countries
20	0.00889	0.00232
25	0.01156	0.00312
30	0.01476	0.00371
35	0.01958	0.00458
40	0.02551	0.00630
45	0.03323	0.00952
50	0.04711	0.001473
55	0.06363	0.02244
60	0.09473	0.03324
65	0.14467	0.04661
70	0.22239	0.07411
75	0.33790	0.12825
80	0.50544	0.21517

Table 3. Expectation of life 2020–2050

Age	Uganda	High income countries
20	50.94	63.62
25	46.37	58.76
30	41.88	53.94
35	37.47	49.13
40	33.16	44.34
45	28.96	39.61
50	24.86	34.96
55	20.95	30.44
60	17.20	26.08
65	13.72	21.88
70	10.59	17.82
75	7.87	14.02
80	5.59	10.68

The projected average ages of the active members are shown in Figure 3. The average age of the staff increases from about 41 years at the start of the horizon to nearly 43 years after 10 years, and it then stabilises around 43 years up to the end of the horizon. The average age of the MPs increases from about 48 years at the start of the horizon to 51 years after 10 years, and then it oscillates around 52 years for the rest of the horizon. The increase in average ages for the active members is due to reduction in mortality, allowing more members to survive for more years. The stability in the average ages is caused by the policy for staff to recruit more of younger members. For MPs, those who have been in parliament have higher chances of winning and retain their seats, hence more older members remain in parliament, and the reduction in mortality also enables them to live longer.

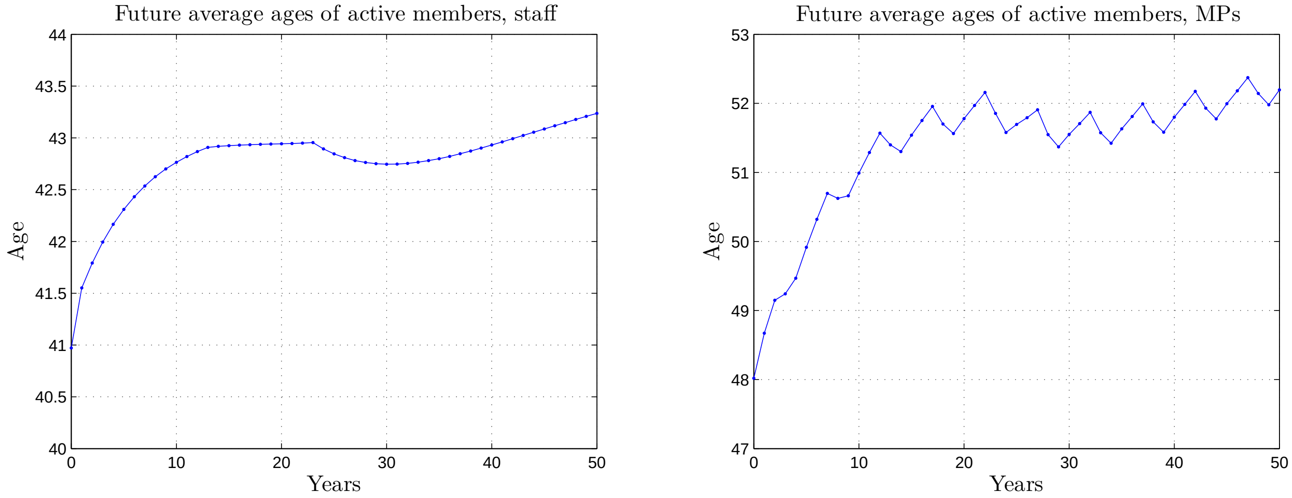


Figure 3. Projected average ages

The projected distribution of scheme members by age states after every decade is given in Figure 4 for the staff, while those for MPs are given in Figure 5. From Figure 4, initially the first aggregate age states for staff have no members in state 1 and few members in state 8, while the age distribution of members in the remaining states is higher. After 10 years, the average age has increased and the age distribution of members then remains stable over the horizon.

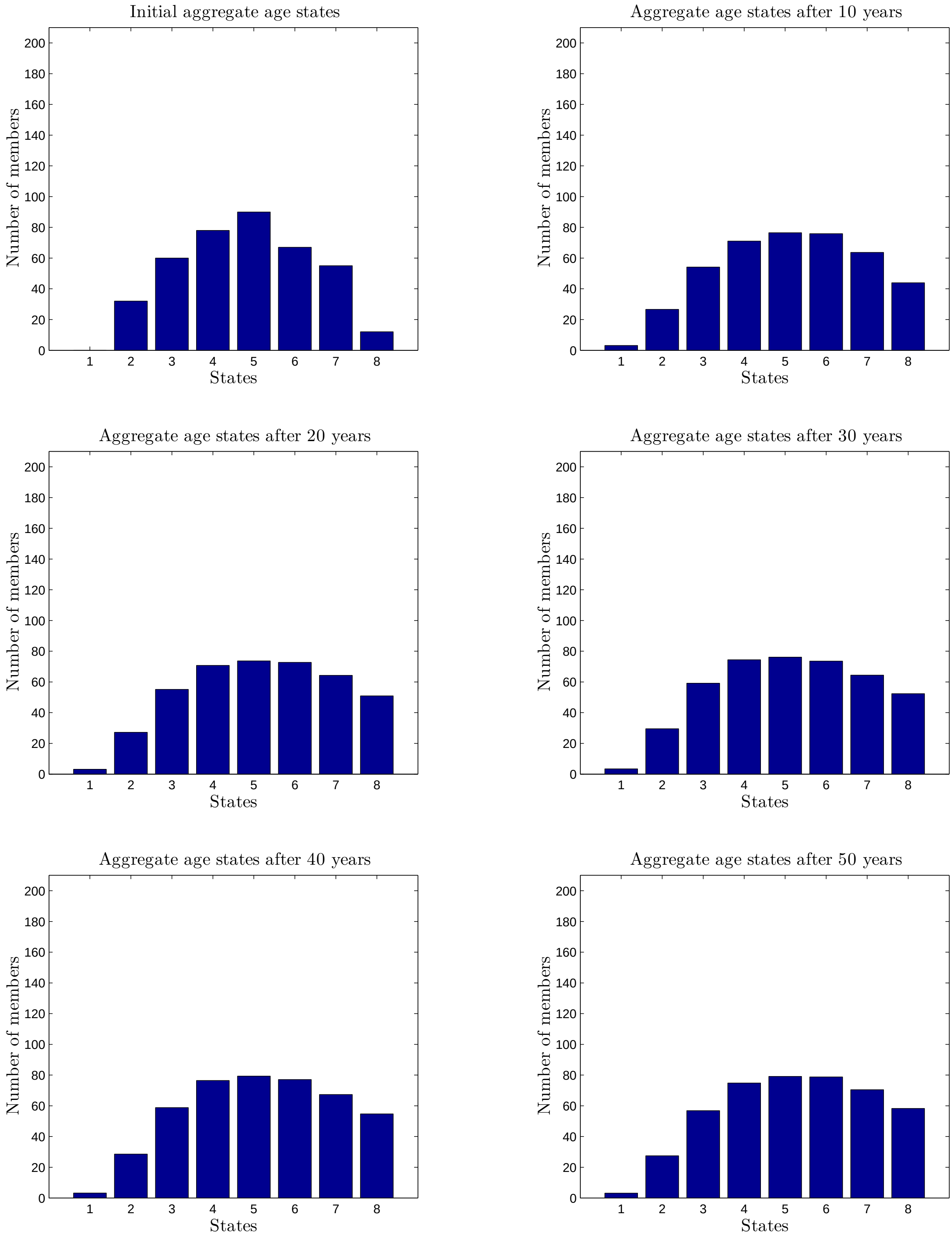


Figure 4. Active staff age distribution in 10 year intervals

The initial age states in Figure 5 for MPs have very few members in states $1, 2$ and 3 due to the difficulty faced by the youth to get elected into parliament. After 10 years, the average age of parliament has increased and there are more members in the older age states. The age distribution stabilises on the horizon with state 9 having more members, because more of the older members stay in parliament and the reduction in mortality enables them to live longer.

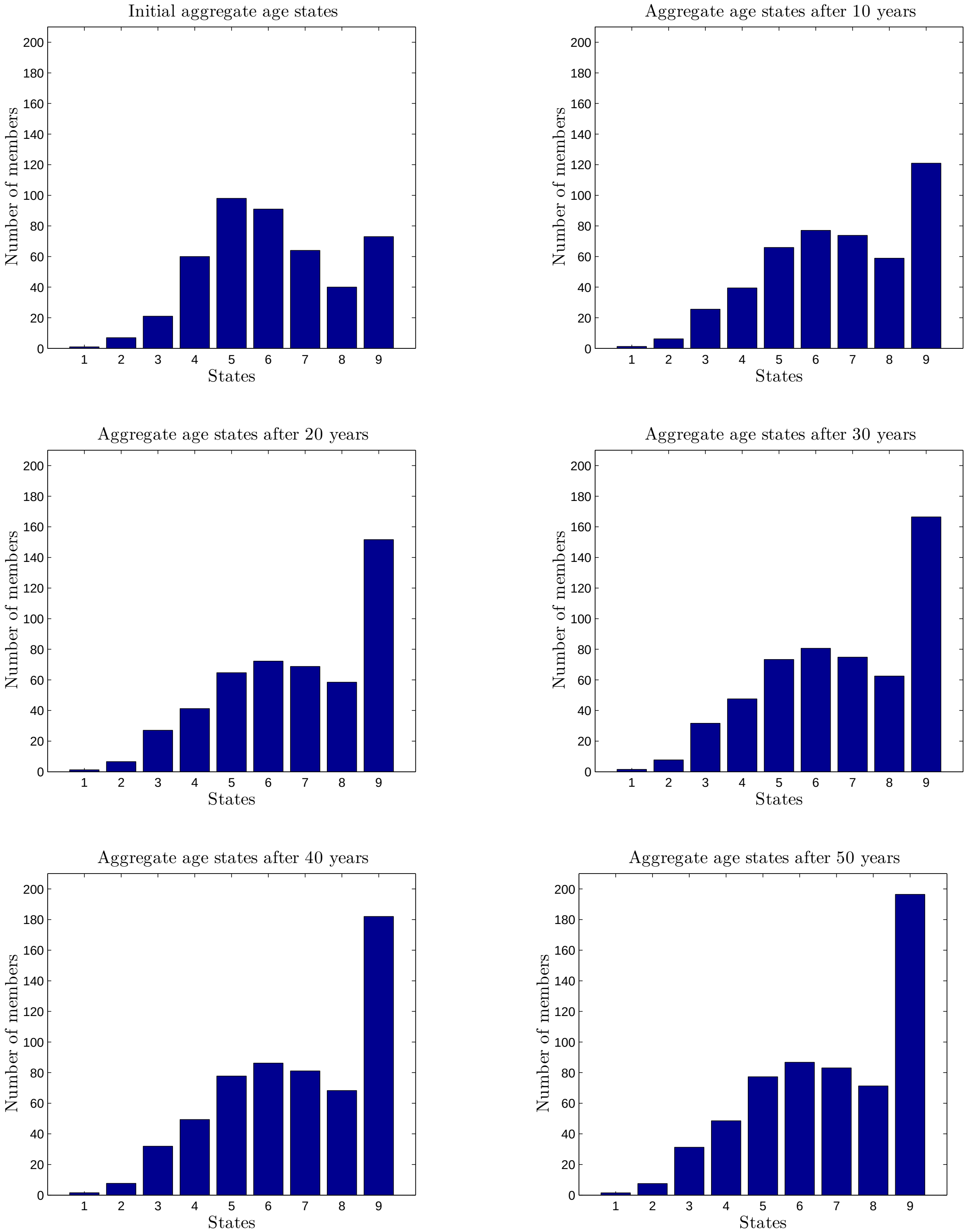


Figure 5. Active MPs age distribution in 10 year intervals

The dependence ratio in a pension fund is the ratio of the number of pensioners to the number of contributing members. The relative composition of active and retired members and the resulting dependence ratios increase over the horizon, as shown in Figure 6 and Figure 7, respectively. This is due to an increase in number of retirees as more members retire and as mortality reduces over the horizon. This increase in retirees poses a big challenge on the sustainability of the scheme, since contributions from active members can not cover benefits for the increasing number of retirees.

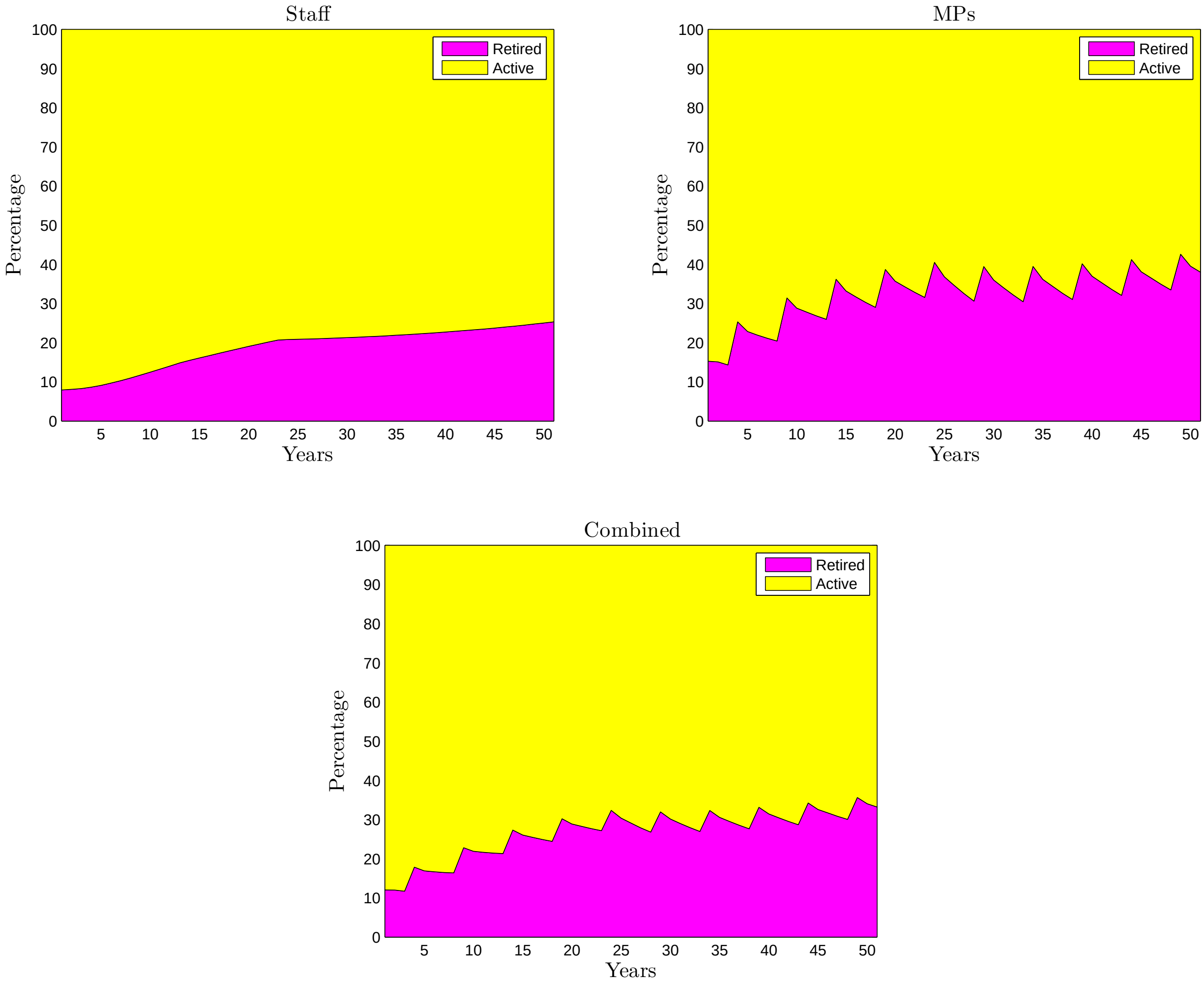


Figure 6. Percentage composition of active and retired members

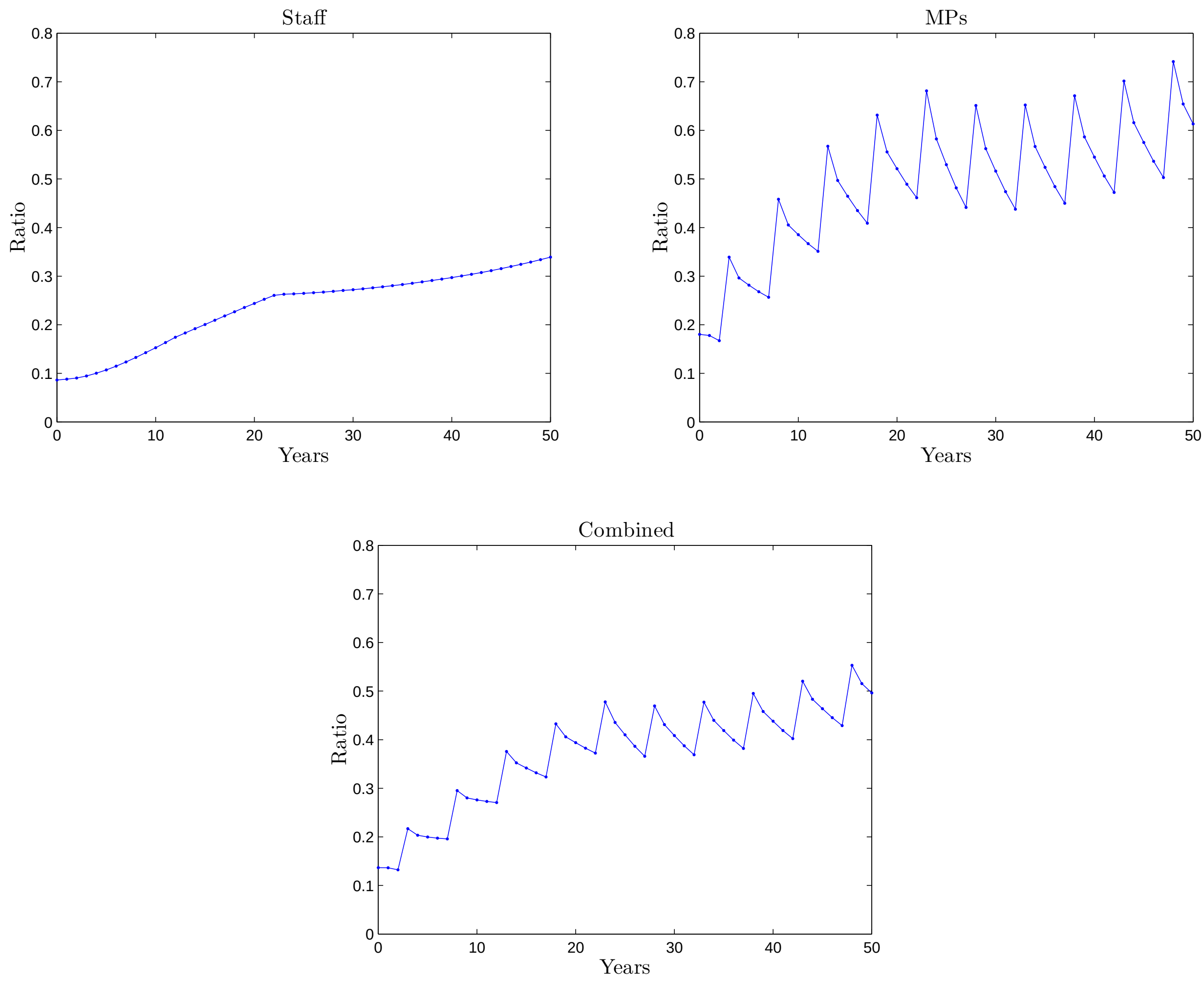


Figure 7. PPS dependence ratios

There is a gradual growth in staff total population, as shown in Figure 8. The population of active members experiences a very slow growth, caused by a small difference between number of new members and that of retiring members. The population of retired and old age members increases gradually for the first 22 years and then slows down for the remainder of the horizon. After 22 years, the initial surviving retirees and those from succeeding years are in advanced ages. Even though mortality reduces, at advanced ages there is a higher risk of death. The population of new members remains stable on the horizon, as the scheme matures there is little expansion in the work force. Leaving members stabilise to between 4 and 5 per year on the horizon, since only a handful of members leave the high paying jobs as civil servants. The number of retiring members each year grows fast for the first ten years and then more gradually for the rest of the horizon, because of the stabilisation in aggregate age states after ten years. In state transition for pensioners, mortality rates at age 60 is used from year 2018 to 2030, at age 65 from year 2031 to 2040, at age 70 from year 2041 to 2068.

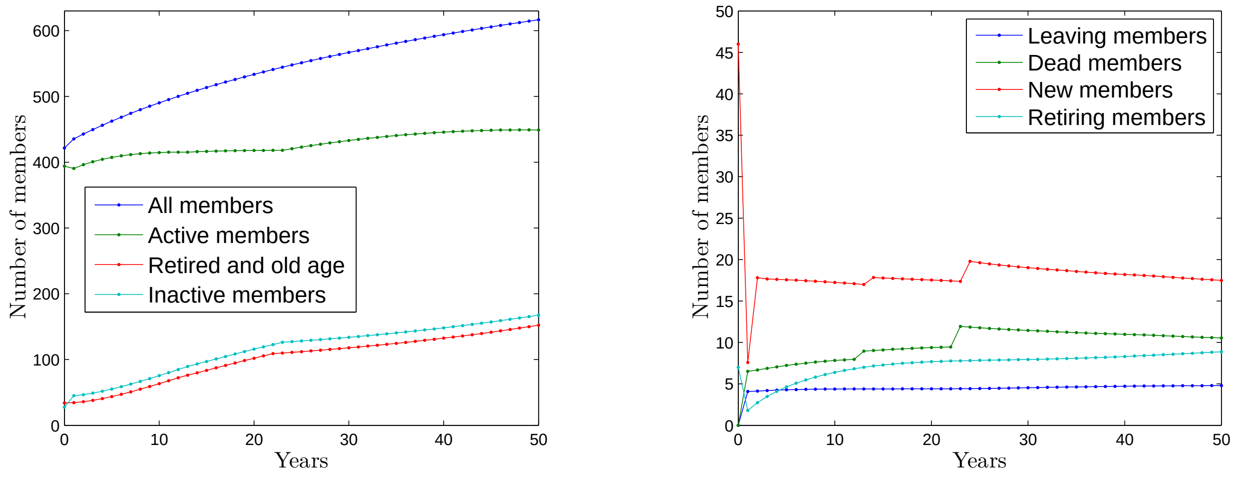


Figure 8. Evolution of fund population dynamics for staff (note the different scales)

There is a moderate growth in population of MPs as shown in Figure 9. The active members' population experiences a gradual growth, caused by creation of new elective positions. The MPs populations are affected by dynamic political cycles, caused by regular elections after every 5 years. There is a moderate growth in pensioners' population during election years, since those who loose their seats either retire or leave the scheme. Those retiring or leaving in non-election years are insignificant, hence pensioners' population gradually reduces due to deaths of some pensioners. The death pattern is similar to that for the staff, with the only difference being in active MPs above 60 years. For these, mortality rates at age 60 years is used from year 2018 to 2040, and at age 65 years from year 2041 to 2068. New members that join parliament complete their first year in the year that follows the election year.

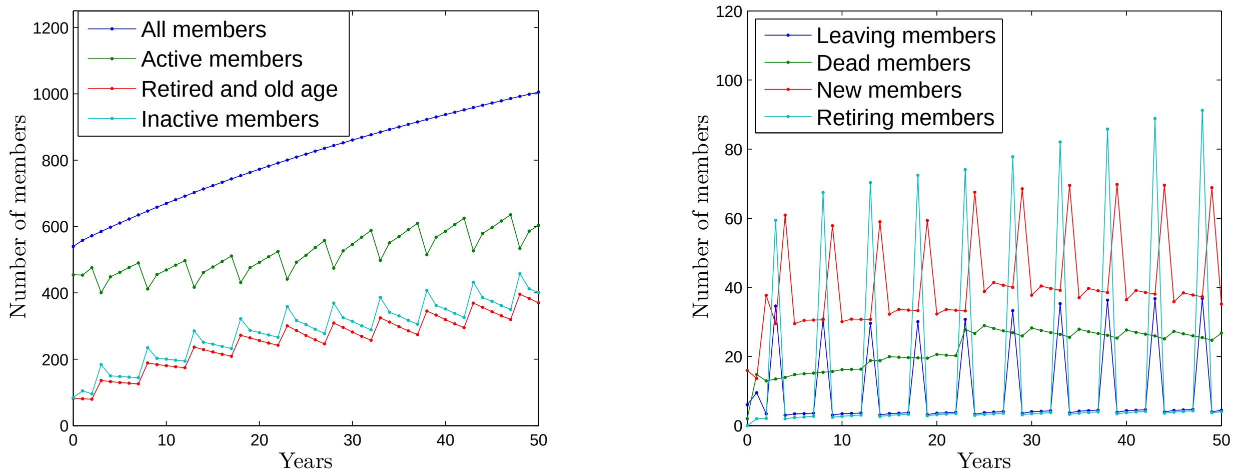


Figure 9. Evolution of fund population dynamics for MPs (note the different scales)

3. Financial projections

The most important thing to retirees is the amount of pension received regardless of how long they live, and this necessitates consideration of both the accumulation phase and decumulation phase. The members and government make contributions to the fund, part of which are invested while others are used to pay benefits to inactive and leaving members. Figure 10 shows the fund's financial process (here, and in the following, all monetary values are given in Uganda Shillings, UGX.)

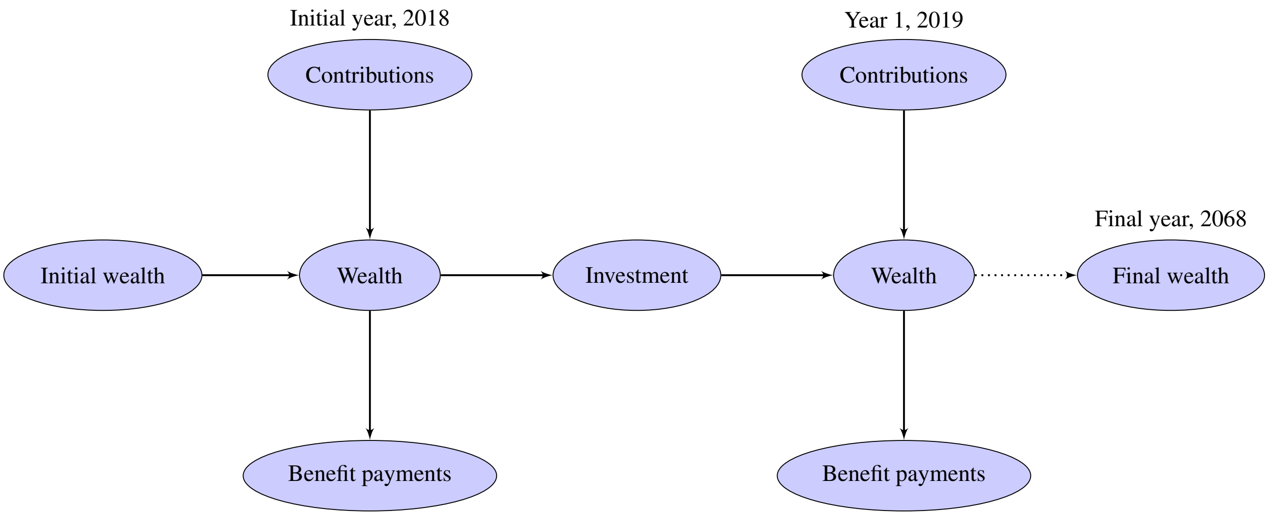


Figure 10. Financial process

3.1. Salary growth and contributions

If we denote by S_t the average annual salary of the members in year t, then

$$S_t = (1 + g)\, S_{t-1}$$

where g is the expected long term annual salary growth rate, we use $g = 6\%$. This value of g has been calculated from historical data as the average rate of salary increase each year, it therefore serves as the expected long term annual salary increase. This salary calculation is not unique, it is a special case of [30][Equation 2] with constant salary growth rate. Contributions are determined by the members' pensionable emoluments. The contribution rate is the percentage of the pensionable emolument that is remitted to the fund. According to the Parliamentary Pension Regulations from 2013, the contribution rate is 45%, of which the employee contributes 15% and the employer contributes 30%. The total annual contributions C_t in year t is given by

$$C_t = \iota \times S_t \times A_t$$

where A_t is the number of active members in year t and ι is the contribution rate.

Figure 11 shows the contributions growth for the projection period 2018–2068, and it behaves the same way as the growth in number of active members. The contributions start with a slow growth in the first 20 years and then grow faster in the remaining years. The contributions for staff increase gradually on the horizon due to gradual increase in their pensionable emoluments and in number of staff, which stabilises in the last 20 years from 2048 to 2068. The increase in contributions of MPs is as a result of growth in their pensionable emoluments over the horizon and also due to an increase in number of MPs on the horizon. The political cycles are evident and affect MPs contributions, while the staff contributions are not affected since they are civil servants.

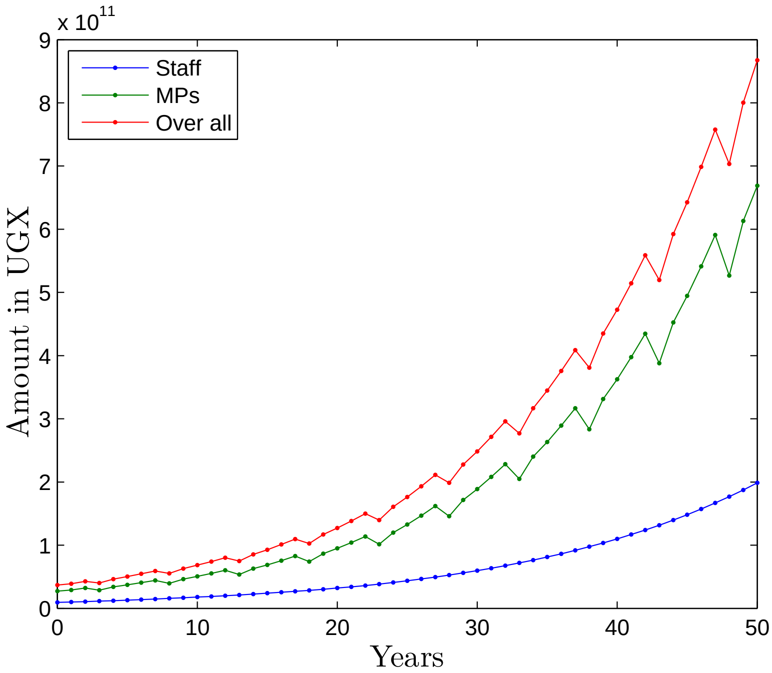


Figure 11. The PPS contributions growth

3.2. Accumulated fund for a member

Consider the pension fund of a member in year j, who has contributed a portion ι of its pensionable emolument S_j in year j to the scheme for the last $j-\nu$ years, where ν is the year this member joined the scheme. Each year the entire value of the fund, including the previous returns, are re-invested and earn a rate of return ϑ. If the pensionable emolument of this member in the year he starts contributing to the fund is S_ν and there is a constant rate of growth of the pensionable emoluments of g per year, then the pensionable emoluments S_j after $j-\nu$ years is given by

$$S_j = S_\nu \left(1+g\right)^{j-\nu} \tag{4}$$

Since the members contributions are remitted monthly, the interest on funds collected from members in a given year are considered to earn half-year interest while the funds from the previous years earn full yearly interest. After putting all this into consideration, we obtain the fund value $\mathrm{AF}_{t,\nu}$ in year t for this member as follows.

- Year 1:

$$\begin{aligned}
\text{Contributions} &= \iota S_\nu \\
\text{Half year interest} &= \frac{1}{2}\vartheta \iota S_\nu \\
\mathrm{AF}_{\nu+1,\nu} &= \iota S_\nu \left(1+\frac{1}{2}\vartheta\right)
\end{aligned}$$

- Year 2:

$$\begin{aligned}
&\text{Contributions} = \iota S_\nu \left(1+g\right)\\
&\text{Half year interest} = \frac{1}{2}\vartheta\iota S_\nu \left(1+g\right)\\
&\text{Growth in }\mathrm{AF}_{\nu+1,\nu} = \iota S_\nu \left(1+\frac{1}{2}\vartheta\right)\left(1+\vartheta\right)\\
&\mathrm{AF}_{\nu+2,\nu} = \iota S_\nu \left(1+\frac{1}{2}\vartheta\right)\left(\left(1+g\right)+\left(1+\vartheta\right)\right)
\end{aligned}$$

Continuing in this way, we obtain the fund value in year t for this member as

$$\mathrm{AF}_{t,\nu} = \iota S_\nu \left(1+\frac{1}{2}\vartheta\right)\sum_{j=\nu}^{t-1}\left(1+g\right)^{t-j-1}\left(1+\vartheta\right)^{j-\nu} \tag{5}$$

where we assume no return earned in the last year of contribution.

3.3. Benefit payments

According to the Parliamentary Pensions Regulations from 2013, the different kinds of benefits are: retirement (commuted benefit and monthly pension), retirement on health grounds, death of a member, leavers benefit (resignation and refund of contributions), and deferred benefits. These benefits depend on the number of years that the member has been contributing to the fund and the age at retirement. In our analysis we consider the leavers benefit, death benefit, commuted benefit and monthly pensions. These are the only benefits reflected in the payments in the PPS annual reports. Pension is paid to a member who retires and ceases to be active on or after attaining an age of 45 years, subject to service as a member for a continuous period of five years or more. In our projections, we take the retirement age for staff as 60 years, since they work up to the normal retirement age of 60 years for civil servants in Uganda, while we use 57 years as the retirement age for MPs. The latter retirement age has been obtained using historical data for the normal average age for retiring MPs. The MPs can continue to be active members above their retirement age, while staff cannot continue to work beyond 60 years of age.

The monthly pension for the PPS is given for life to a retiree and is guaranteed for a period of $\tau = 15$ years. It is given by

$$MB = \frac{\mathrm{AF}_{t,\nu}}{\mathrm{CAF}} \times 75\% \times \frac{1}{12} \tag{6}$$

CAF is the expected present value of a conversion of life annuity of 1 per annum, payable monthly at the time of retirement of a member, based on appropriate terms of interest and expense factors. The values of CAF for retirees at different ages used by the PPS are given in Table 6 of Appendix A.5. Since the model developed will not treat men and women separately, we use historical data on composition of scheme to obtain the values of CAF to use as a weighted average of the values given in Table 6. The women are given a weight of 35% to obtain weighted CAF values in Table 7 of Appendix A.5. We convert the monthly pension to annual benefit in equation (7), which we use in calculations that follow.

$$AB = MB \times 12 \tag{7}$$

The commuted benefit at the time of retirement is given by

$$CB = \mathrm{AF}_{t,\nu} \times 25\% \tag{8}$$

The leavers benefit is paid as a lump sum to a member who leaves the scheme, and this appears as a cash outflow from the fund in the year in which the payment is done. This then reduces the scheme's future liabilities, and hence leavers benefits are risk neutral with respect to the fund's liabilities. Only part of the sponsor's contributions are vested under the leaver's benefit for those leaving before five years of service, while sponsor's contributions vest for those who leave after at least five years of service. From historical data, the average years of service for leavers is 5.17 years for MPs and 8.7 years for staff, hence we assume that all sponsor's contributions vest. The leavers benefit in year t is given by

$$LB_t = NL_t \times \overline{AF}_{L,t} \tag{9}$$

where NL_t is the number of leavers in year t and $\overline{AF}_{L,t}$ is the average value of the leavers accumulated funds in year t.

The death benefit in year t is given by

$$DB_t = ND_t \times \overline{AF}_{D,t}, \tag{10}$$

where ND_t is the number of members dying in year t and $\overline{AF}_{D,t}$ is the average value of accumulated funds for members dying in the year.

Using Equations (7)–(10), the total benefit payouts BP_t in year t is given by

$$BP_t = NR_t \times CB + NO_t \times AB + LB_t + DB_t, \tag{11}$$

where NR_t is the total number of members retiring in year t and NO_t is the total number of old age pensioners in the same year.

Figure 12 shows that all the different kinds of benefits and hence the total benefit payments increase on the horizon. Commuted and leaver's benefits increase due to salary growth and high guaranteed return, causing increases in average accumulated funds on member's notional accounts by the time they retire. The spikes in election years result from many MPs who loose their seats. Annual benefits increase significantly due to increase in number of pensioners, and growth in their pension. The death benefits increase gradually due to growth in average accumulated funds of dying members and number of pensioners dying during the guaranteed period of payment of benefits.

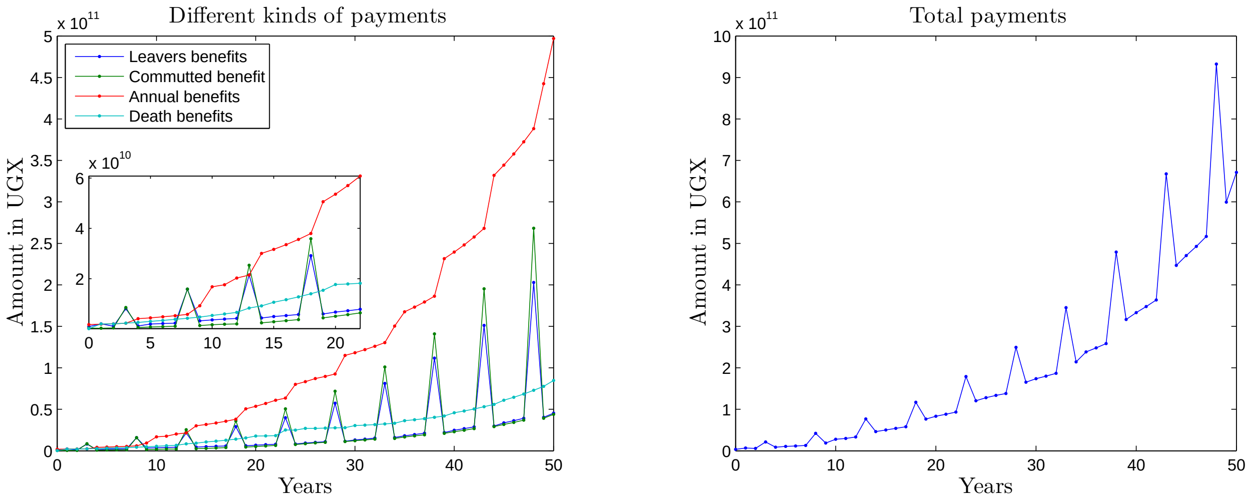


Figure 12. PPS benefit payments (note the different scales)

3.4. Cash flow

Cash flow is the net amount of money moving into the fund. A positive cash flow implies that cash inflow is greater than cash outflow, and excess amount is invested to boost the fund's assets. If the cash flow is negative, part of the fund's assets must be used to pay benefits. The cash flow in year t is given by

$$CF_t = C_t - BP_t \tag{12}$$

As seen in Figure 13, the staff cash flows have a gradual growth for the first 12 years followed by a gradual decline for the rest of the horizon. For the second half of the horizon, the staff cash flows are negative so that contribution can not fully cover the benefits. For the first 12 years, the number of retirees is small and when this number increases the contributions ultimately become less than benefits paid. The MPs cash flows decline significantly in election years due to huge lump sum benefits paid to retiring and leaving MPs after losing their parliamentary seats. However, in all non-election years there is gradual growth in MPs cash flows in the first half and significant growth in the second half of the horizon. This is caused by the increase in active MPs and their average pensionable emoluments. For the PPS as a whole, there is a big boost from MPs cash flows so that contributions fully cover benefits in all non-election years, although this is not the case in election years. The scheme is not fair to the two categories of members, since contributions from MPs are used to subsidise payment of benefits for staff.

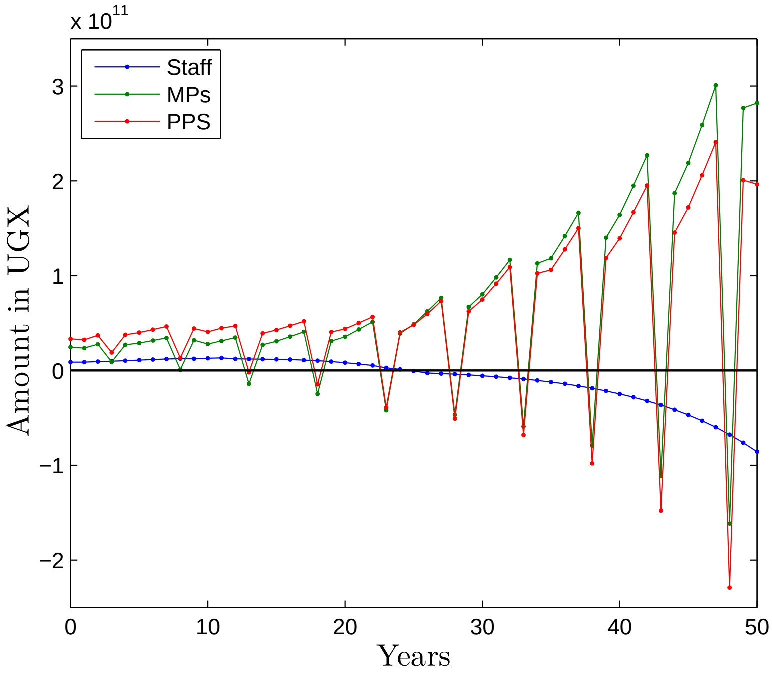


Figure 13. Cash flows

3.5. Total liabilities

Liabilities are the future benefits to be paid to members when they retire and the value of the liabilities is the present value of the expected benefit payments. From the Parliamentary Pensions Regulations from 2013, commuted benefit depends on the accumulated amount on the member's notional account, while the monthly pensions depend on both the accumulated amount on the member's notional account and the age at retirement.

From equation (5), the accumulated fund on the member's notional account depends on the initial salary and number of years of service. The total expected commuted benefit in year t for members of age j is given by

$$CB_{tj} = P_t^{r_{\text{age}}-j} \times n_j \times \overline{AF}_{tj} \times 25\% \tag{13}$$

where $P_t^{r_{\text{age}}-j}$ is the probability that a member aged j years in year t lives for $r_{\text{age}} - j$ more years until the retirement age r_{age}, n_j is the number of members aged j years in year t, and $\overline{AF}_{tj}$ is the average value of fund for members of age j in year t.

The total expected yearly benefits in year t for members aged j years with a guaranteed monthly pension of τ years after retirement is

$$AB_{tj} = P_t^{r_{\text{age}}-j} \times n_j \times \frac{\overline{AF}_{tj}}{\text{CAF}} \times 75\% \times \left(\tau + P_t^{r_{\text{age}}+\tau-j} \times \mathbb{E}P_{(r_{\text{age}}+\tau)_{tj}}\right) \tag{14}$$

where $P_t^{r_{\text{age}}+\tau-j}$ is the probability that a member aged j years in year t lives for $r_{\text{age}} + \tau - j$ more years after retirement, and $\mathbb{E}P_{(r_{\text{age}}+\tau)_{tj}}$ is the expected remaining life in year t for a member aged j years, when he reaches the age of $r_{\text{age}} + \tau$ years. Extending some ideas developed in [38] we incorporated the guaranteed period of payment for the annual benefits.

The total expected benefit in year t for members aged j years is

$$B_{tj} = CB_{tj} + AB_{tj}$$

The liability at time t is the discounted present value of expected total benefit. The total liability in year t is hence given by

$$L_t = \sum_{j=j_0}^{r_{\text{age}}-1} \frac{B_{tj}}{(1+r)^{r_{\text{age}}-j}} \tag{15}$$

where j_0 is the minimum entry age and $r\%$ is the discounting factor. Based on [9], because the fund is accumulated to meet the liabilities, the expected long term asset return also represents the discounting factor at which the level of liabilities is determined. Hence, it is used to represent both the gross yield earned on the fund as well as the rate of return used to discount the liabilities. Usually it will be a long term rate of return, allowing for re-investment.

The initial total liability is UGX 192.98 billion, assuming a 106% initial funding level. We assume that 30% of the initial liability was created by staff and 70% by MPs. The liabilities created and their distribution is given in Figure 14 and follow the pattern explained for growth in the number of active members. The final liability created by staff is UGX 15.14 trillion and that created by MPs is UGX 35.90 trillion leading to a total of UGX 51.04 trillion.

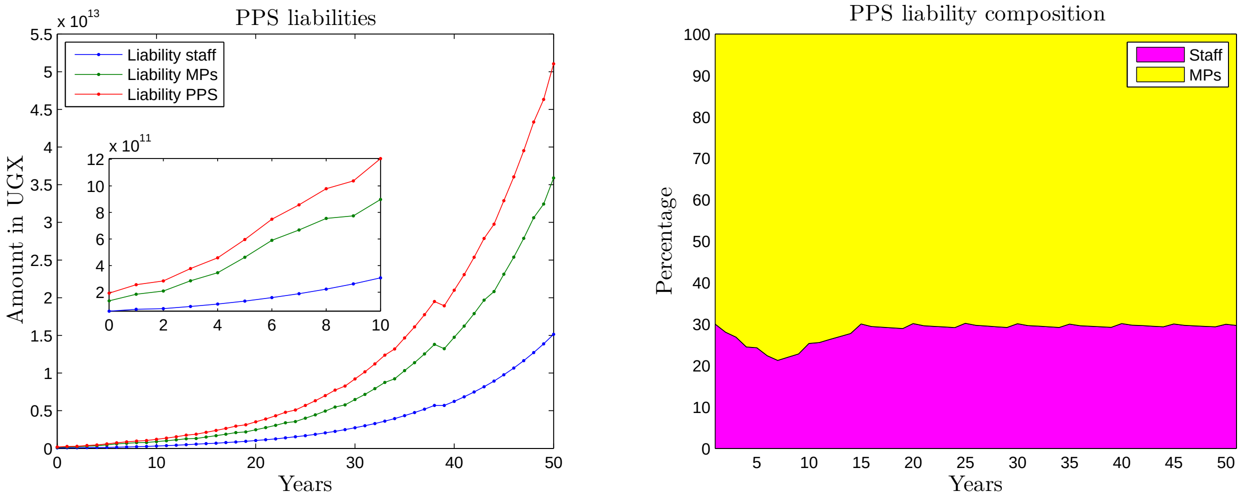


Figure 14. PPS liability distribution

3.6. Asset value

Growth in asset value depends on the cash flow and return rate from investments. When the fund receives contributions from active members, current benefits are paid and any positive balance is invested to boost the asset value. If contributions are less than current benefits, part of the asset value is used in paying benefits, instead of being re-invested. The PPS starts with an initial asset value, or initial wealth, in 2018. From the PPS Annual Report 2017/2018, this value is given as UGX 204.55 billion, and we assume that 30% of initial asset is for the staff and 70% for the MPs. The average return rate on assets after tax from 2011 to 2018 is 11% per year. This is however expected to reduce in future, and we therefore use 8% as the long term return rate on asset.

Each year the net cash flow CF_t together with return on asset AS_{t-1} gives the new asset value. The asset value in year t can be expressed recursively by the equation

$$AS_t = CF_t + (1+\gamma) \times AS_{t-1} \tag{16}$$

where γ is the return rate on investment after tax.

As seen in Figure 15, there is gradual growth in asset values for MPs in the first 25 years and moderate growth in the last 25 years, while staff asset growth is gradual over the horizon. In the first 25 years, the cash flows for both categories are small and asset growth is mainly due to returns earned on investments. In the last 25 years, however, there is moderate growth in MPs cash flows while those for staff are negative and decreasing gradually. Since asset return remains the same, this accounts for moderate growth in MPs assets and gradual growth in staff assets. A combination of the two leads to a gradual PPS asset growth in the first 25 years, followed by moderate growth in the last 25 years. The staff share of the asset value is about 30% in the first 25 years, and is than reduced gradually in the last 25 years to about 24% at the end of the horizon.

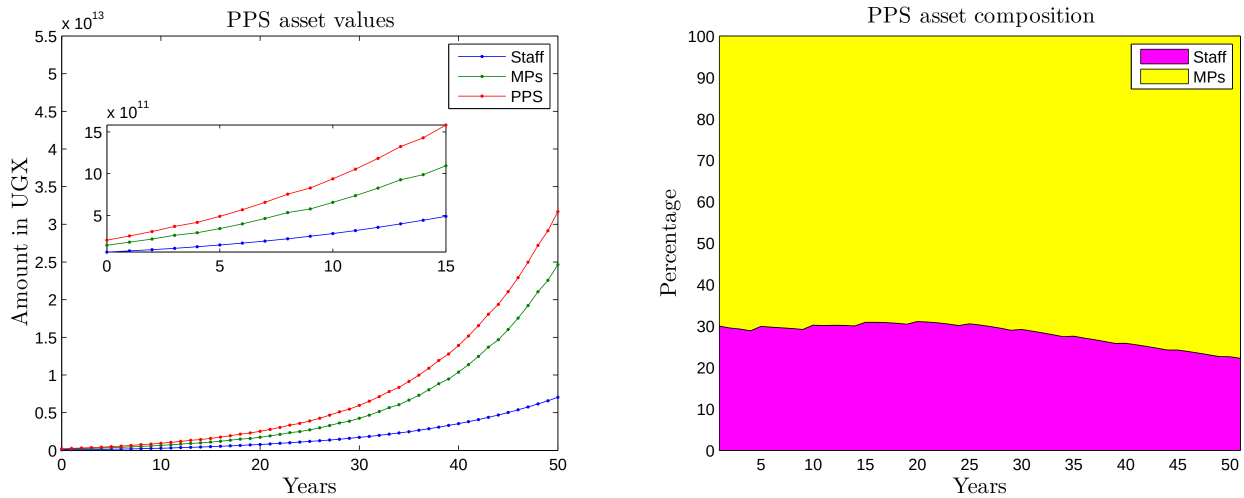


Figure 15. PPS asset distribution

Figure 16 compares the scheme's assets and liabilities. There is gradual growth in liabilities for the first half of the horizon, followed by a fast growth for the remaining horizon. In the first half of the horizon, smaller liabilities are created as the average age of MPs is low, and those losing their seats can leave the scheme because they are below the retirement age. Since the staff have working careers of 26 years on average, staff that retire in earlier years do not create a lot of liabilities. For the last part of the horizon, the staff create a lot of liabilities after contributing for many years with a guaranteed return on their accumulated funds. Further, during the horizon more MPs are re-elected, and thereby accumulate larger liabilities. The long guaranteed period of pension of 15 years also plays a significant role in accumulation of huge liabilities.

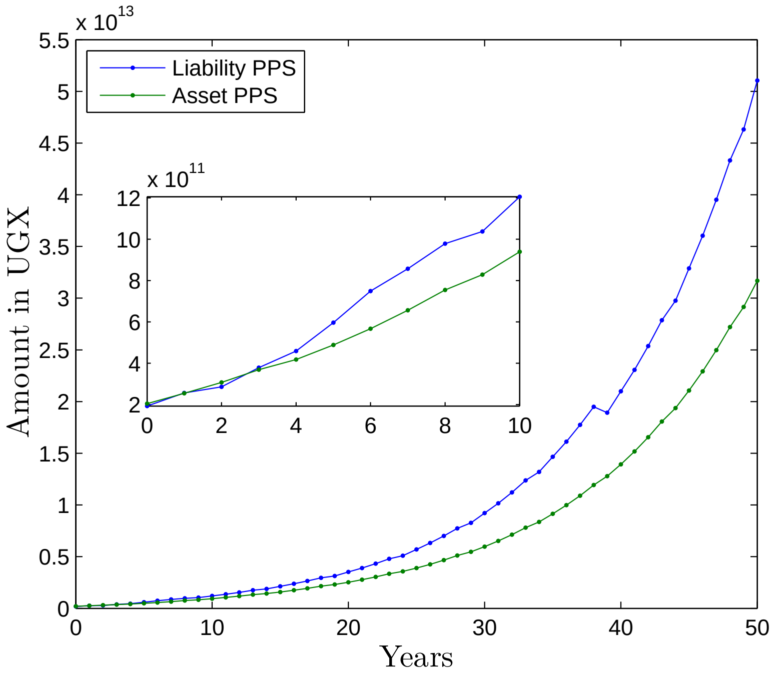


Figure 16. Projected asset and liability values

The final liability created by staff is UGX 15.14 trillion and that created by MPs is UGX 35.90 trillion, leading to a total of UGX 51.04 trillion. This can not be offset by the accumulated assets, which are UGX 7.04 trillion by staff, UGX 24.65 trillion by MPs, and a total of UGX 31.69 trillion for PPS at the end of the horizon.

3.7. Cash flow to asset value ratios

The ratio of cash flow to asset value is given by

$$CAR_t = \frac{CF_t}{AS_t} \times 100\% \tag{17}$$

where AS_t is the asset value from equation (16) and CF_t is the cash flow from equation (12).

In Figure 17 we examine which of the two categories of members contributes more to the negative cash flows. The negative cash flows on the part of the MPs are due to huge benefit payments to those who have lost their seats in parliament during election years. Although MPs cash flow to asset value ratios are positive in non-election years, they are gradually reducing up to the end of the horizon.

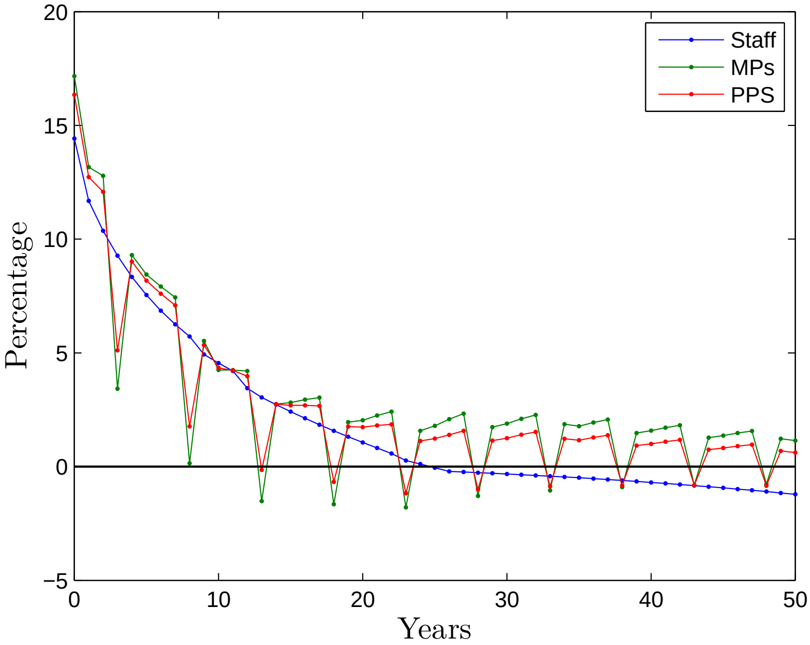


Figure 17. Cash flow to asset value ratios

The initial cash flow to asset value ratios are 14.4% for staff, 17.2% for MPs, and 16.3% for PPS. They reduce on the horizon so that the final values are -1.2% for staff, 1.1% for MPs, and 0.6% for PPS. The MPs cash flow to asset value ratios have spikes of sharp decline in election years due to payment of huge lump sum benefits. The lowest value of the ratio for MPs is -1.8% in 2046, meaning that 1.8% of the asset value is used in payment of benefits. The asset for MPs therefore only grows by 6.2% although an 8% return on member's notional accounts is guaranteed. The staff have negative cash flow to asset value ratios in the last 25 years, and it continues to decrease on the entire horizon.

3.8. Funding ratios

The funding ratio in a pension fund is the ratio of assets to liabilities. The regulating authority of Uganda, URBRA, sets the limits of the funding ratio for solvency of the pension funds. A funding ratio of 100% means that assets are equal to liabilities. If the funding ratio is greater than 100%, there is over-funding and the plan sponsor can reduce its contribution rate. If the funding ratio is less than 100%, there is underfunding and the plan sponsor must increase its contribution rate to keep the fund solvent. The funding ratio in year t is given by

$$FR_t = \frac{AS_t}{L_t} \times 100\% \tag{18}$$

where AS_t is the asset value in equation (16) and L_t is the value of liabilities in equation (15).

Funding ratios begin at 106%, but there is underfunding for the biggest part of the horizon. The final funding ratios are 46% for staff, 69% for MPs, and 62% for the PPS. To keep the fund solvent, the plan sponsor therefore has to make extra contributions, which is however not sustainable since the 30% contribution rate by the sponsor is already too big.

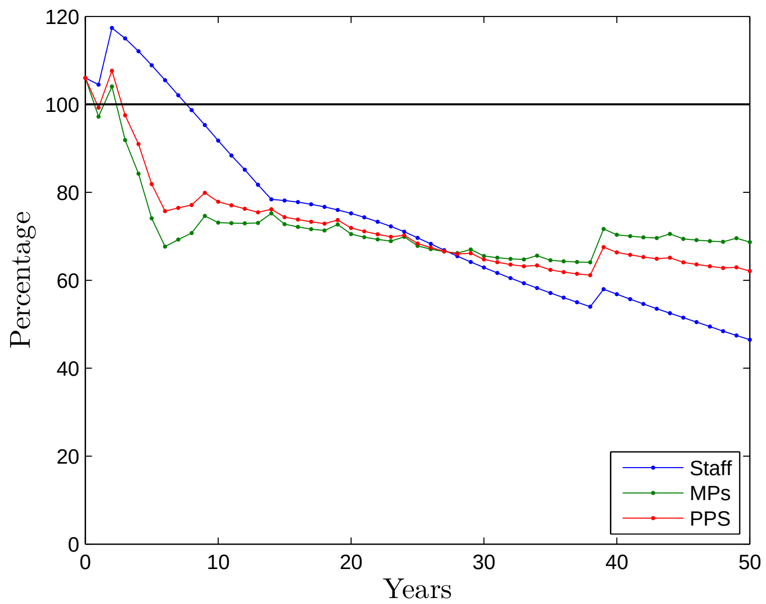


Figure 18. Funding ratios

4. Effect of varying return on assets

The funding ratios and cash flow to asset value ratios analysed in the previous sections show that the scheme is not sustainable. The guaranteed return on members' notional accounts and the guaranteed period of pension payment are fixed by law. We thus analyse the effect of changing the long term return rate on asset on the sustainability of the scheme.

4.1. Projected asset values at different return rates

Figure 19 shows that an increase in return rate on asset will increase the staff and MPs asset values substantially. Figure 20 shows the growth of the PPS asset values.

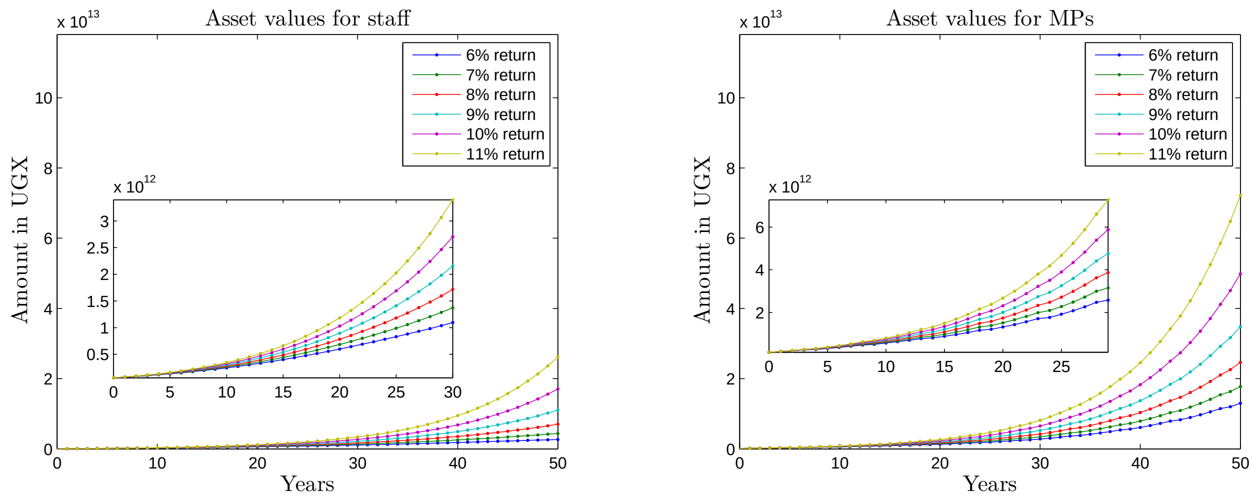


Figure 19. Asset values for staff and MPs

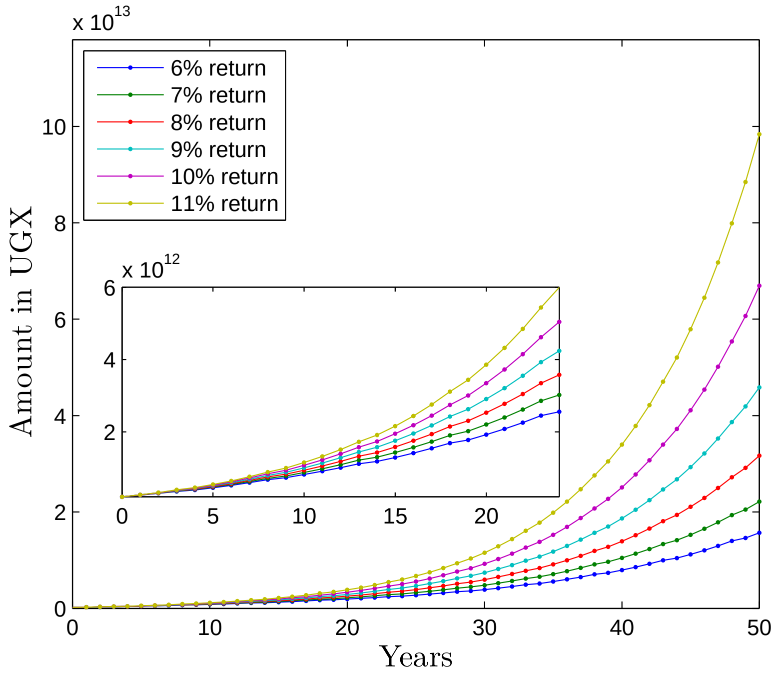


Figure 20. Asset values for PPS

4.2. Projected liability values at different discounting factors

The discounting factor is set to match the corresponding return rate on asset, as stated in Section 3.5. Figure 21 shows that an increase in discounting factor causes an increase in liability value. Figure 22 shows the increase in overall scheme liabilities with increasing discounting factor.

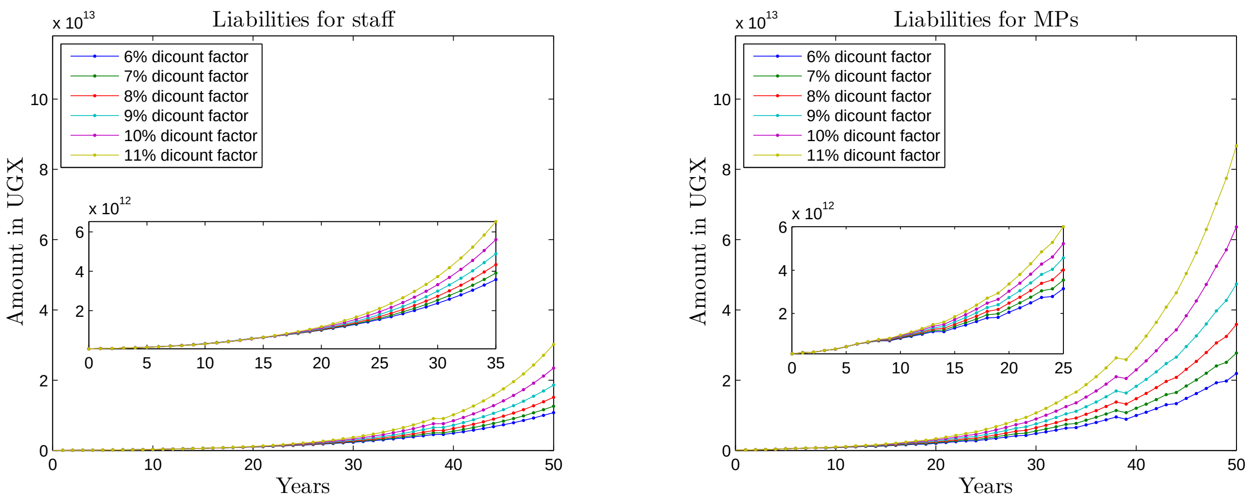


Figure 21. Liability values for staff and MPs at given discounting factors

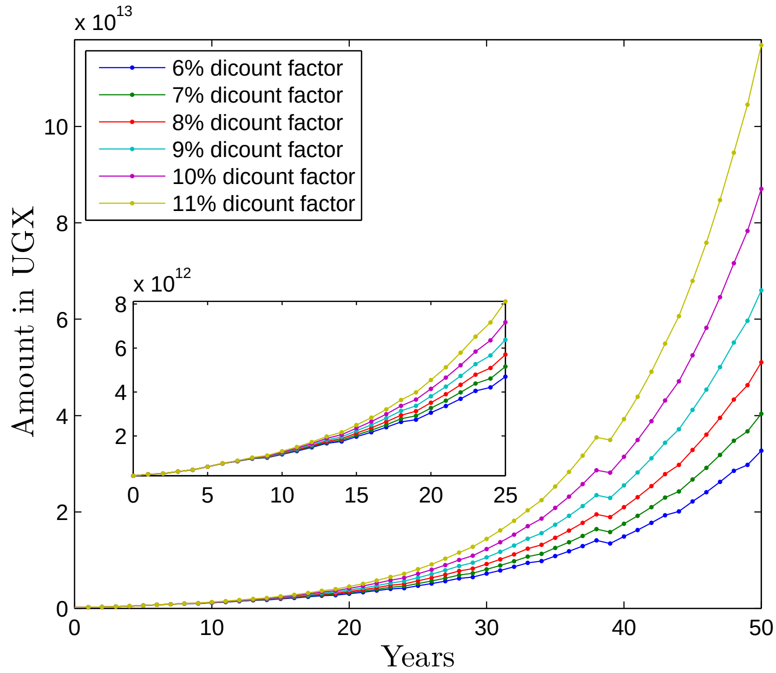


Figure 22. Liabilities for PPS

4.3. Projected cash flow to asset value ratios

The cash flow to asset value ratios follow the same pattern as described in Section 3.7. The cash flows do not change when the return rate on asset changes. Hence, as shown in Figure 23, when return rate on asset increases, the cash flow to asset value ratios decrease for positive cash flows and increase for the case of negative cash flows. The positive values mean that asset growth increases by that percentage, while negative values imply that asset value reduces by that percentage being used in benefit payment. For staff, the ratios are positive in the first half of the horizon and negative in the other. The ratios for MPs reduce gradually and negative values are observed only during election years.

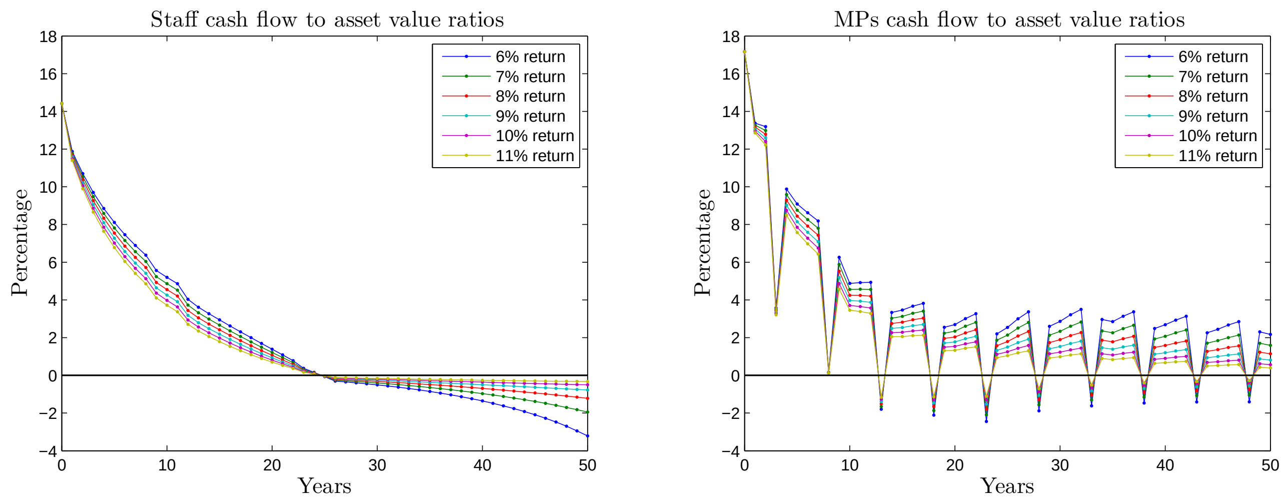


Figure 23. Staff and MPs cash flow to asset value ratios

As seen in Figure 24, the MPs component has a large impact on the PPS cash flow to asset value ratios. The ratios reduce gradually on the horizon and negative values are observed only in election years. These trends are caused by an increase in number of pensioners, more years beyond the horizon of our projection, these ratios will all become negative. This will have a negative impact on the fund's sustainability, since the assets will be consumed in payment of benefits instead of being invested.

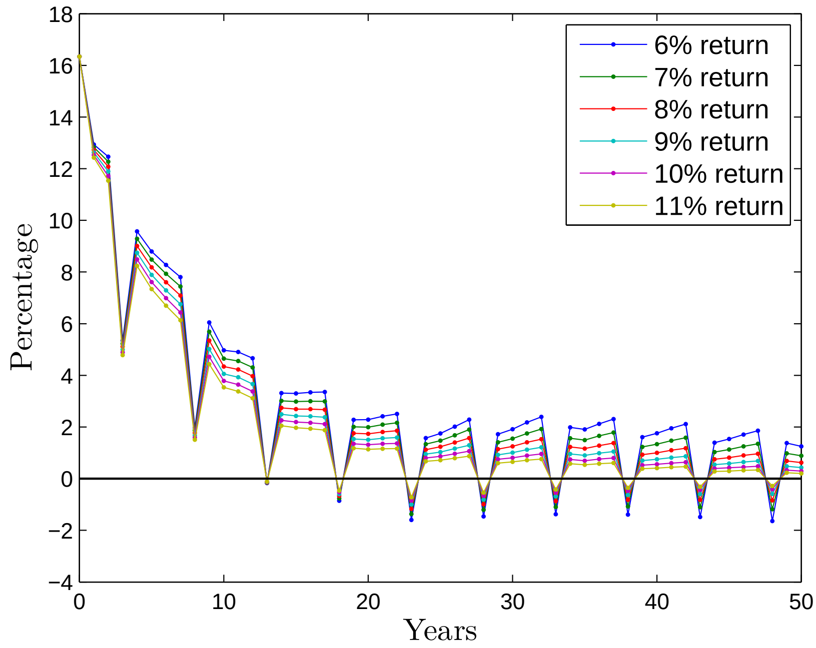


Figure 24. PPS Cash flow to asset value ratios

4.4. Funding ratios at different rates of return on investment

We analyse here the solvency of the scheme at different rates of return. Figure 25 shows that the funding ratios for staff mainly decrease on the horizon but when the rates increase, the level of funding improves. The funding ratios for MPs are more stable on the horizon, but again when the rates increase the level of funding improves.

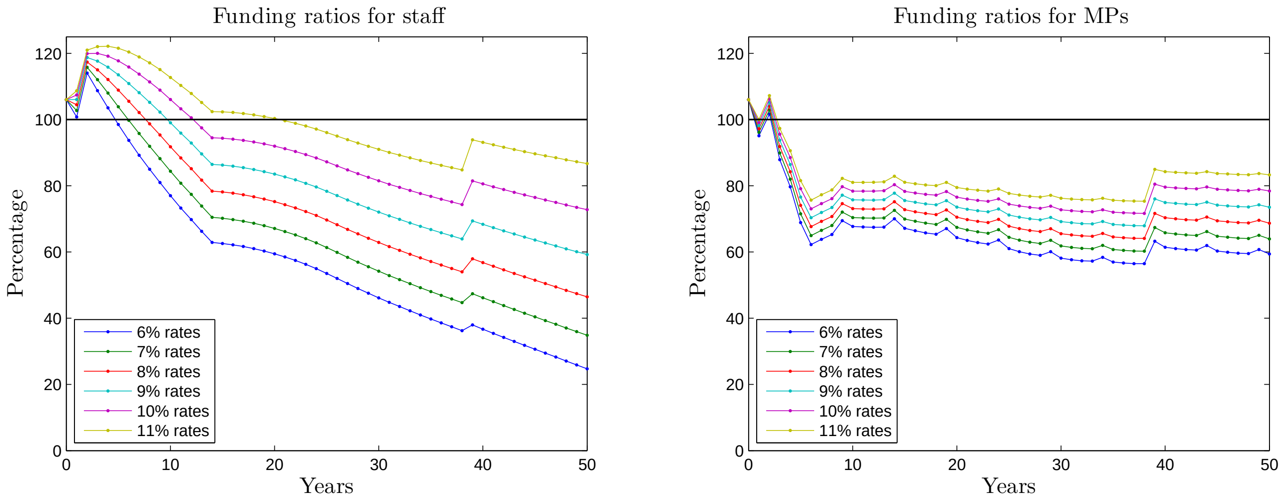


Figure 25. Funding ratio for staff and MPs at given rates

Figure 26 shows that when the rates increase, the overall funding level of the scheme improves. It is evident that at all the considered rates, the scheme remains insolvent on nearly the entire horizon of our projection. Hence, to keep the fund solvent, the sponsor will have to make extra contributions. Because the scheme is not sustainable, policy changes regarding guaranteed return on members' notional accounts and guaranteed period of pension payment should be considered.

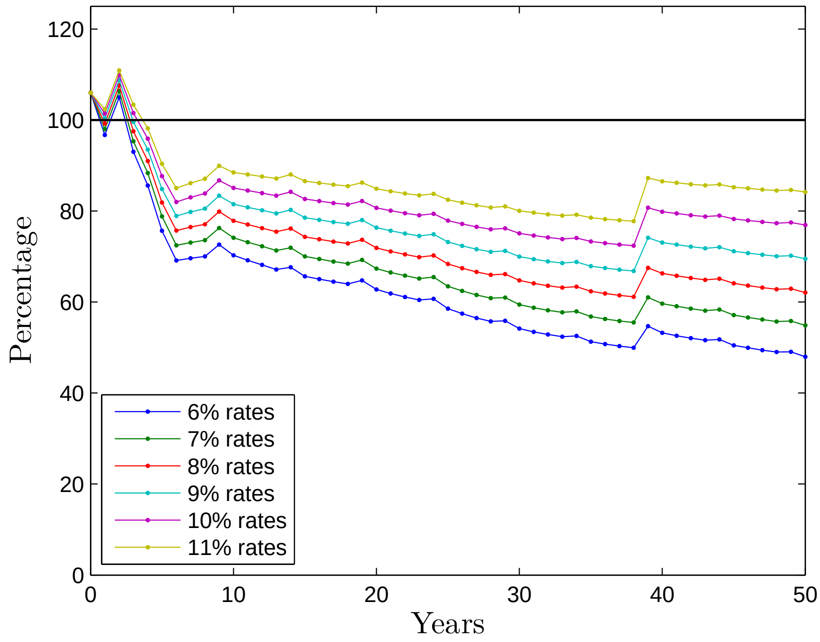


Figure 26. Funding ratios for PPS

5. Conclusion

We have studied the long term sustainability of the PPS of Uganda. The PPS is a contributory scheme with two categories of members, the staff and the MPs. The results show that the fund is not sustainable, since the accumulated asset values can not cover the liabilities on long term. We projected the future staff and MPs populations, respectively, including both active and inactive members, and the future average ages for active members for staff and MPs, respectively. The future distribution of members by aggregate age states was also projected.

We derived a formula for finding the accumulated fund on members' notional accounts on which calculation of benefits are based, and were able to incorporate the guaranteed period of payment of pension by using two survival probabilities. The cash flows obtained show that the scheme is not fair with respect to the two categories of members, since the cash flows for MPs are used to subsidise payment of benefits for staff. Further, in the long run the percentage composition of staff liabilities is much bigger than that of its assets. The funding ratios obtained in the projection show that the fund is not sustainable, higher asset returns could improve the funding ratio but still there is underfunding.

The model shows that the staff create more liabilities than MPs. This arises from staff having long working careers of up to 26 years on average, compared to 9 years on average for MPs. During this period, they earn a guaranteed return of 8% on their contribution accounts, regardless of the investment performance of the fund. There is therefore need to have different benefit indexation parameters for staff and MPs. The combined results also show that the benefits received by both categories of members are generous. This arose from the fact that legislators determine the scheme benefits, and since the staff implement proposed benefits indexation, they also accorded themselves the same form of benefits. Since in the event of underfunding, remedial contributions are obtained from the government treasury, the fund managers are hesitant about proposing reforms for sustainability of the fund.

In future research, policy reforms regarding adjustment of benefits indexation parameters on scheme's sustainability will be studied.

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A. Appendix

A.1. State transition frequencies from historical data

A.1.1. State transition frequencies for MPs in election years

	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60+	Leavers	Retired
20–24	3	4	0	0	0	0	0	0	0	1	0
25–29	0	40	20	0	0	0	0	0	0	2	0
30–34	0	0	114	45	0	0	0	0	0	24	0
35–39	0	0	0	191	73	0	0	0	0	43	0
40–44	0	0	0	0	274	68	0	0	0	80	0
45–49	0	0	0	0	0	240	76	0	0	17	70
50–54	0	0	0	0	0	0	203	62	0	6	71
55–59	0	0	0	0	0	0	0	176	36	4	76
60+	0	0	0	0	0	0	0	0	446	11	104

A.1.2. State transition frequencies for MPs in non-election years

	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60+	Leavers	Retired
20–24	6	2	0	0	0	0	0	0	0	1	0
25–29	0	62	38	0	0	0	0	0	0	1	0
30–34	0	0	267	92	0	0	0	0	0	2	0
35–39	0	0	0	498	154	0	0	0	0	4	0
40–44	0	0	0	0	756	184	0	0	0	9	0
45–49	0	0	0	0	0	692	158	0	0	8	4
50–54	0	0	0	0	0	0	630	132	0	4	2
55–59	0	0	0	0	0	0	0	514	101	3	6
60+	0	0	0	0	0	0	0	0	699	6	11

A.1.3. State transition frequencies for staff

	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	Leavers	Retired
20–24	33	36	0	0	0	0	0	0	0	0
25–29	0	401	151	0	0	0	0	0	1	0
30–34	0	0	810	229	0	0	0	0	7	0
35–39	0	0	0	928	211	0	0	0	14	0
40–44	0	0	0	0	789	177	0	0	13	0
45–49	0	0	0	0	0	560	112	0	10	0
50–54	0	0	0	0	0	0	346	67	3	0
55–59	0	0	0	0	0	0	0	186	3	35

A.2. Frequencies for age distribution of new members from historical data

A.2.1. Frequencies for the age distribution of new MPs in election years

State	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60+
Frequency	4	42	80	124	121	96	71	47	56

A.2.2. Frequencies for the age distribution of new MPs in non-election years

State	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60+
Frequency	1	4	10	8	11	12	7	5	15

A.2.3. Frequencies for the age distribution of new staff

State	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59
Frequency	31	112	100	42	37	9	1	0

A.3. Markov chains for state transition probabilities

The Markov chains for the state transition matrices used in Section 2 are given in Figures 27–29.

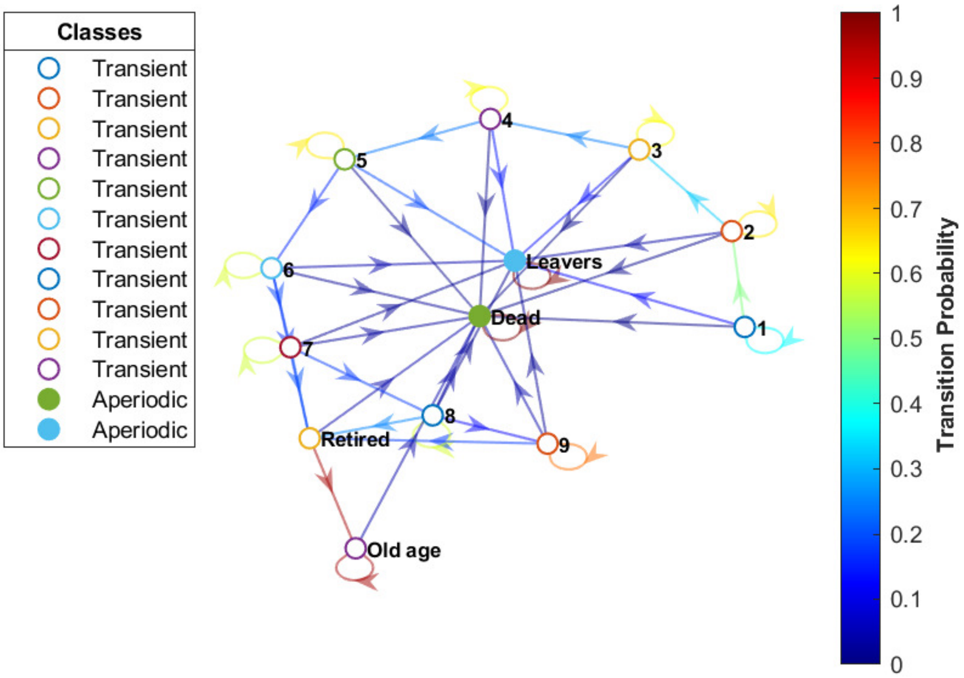

Figure 27. Markov chain for MPs in election years

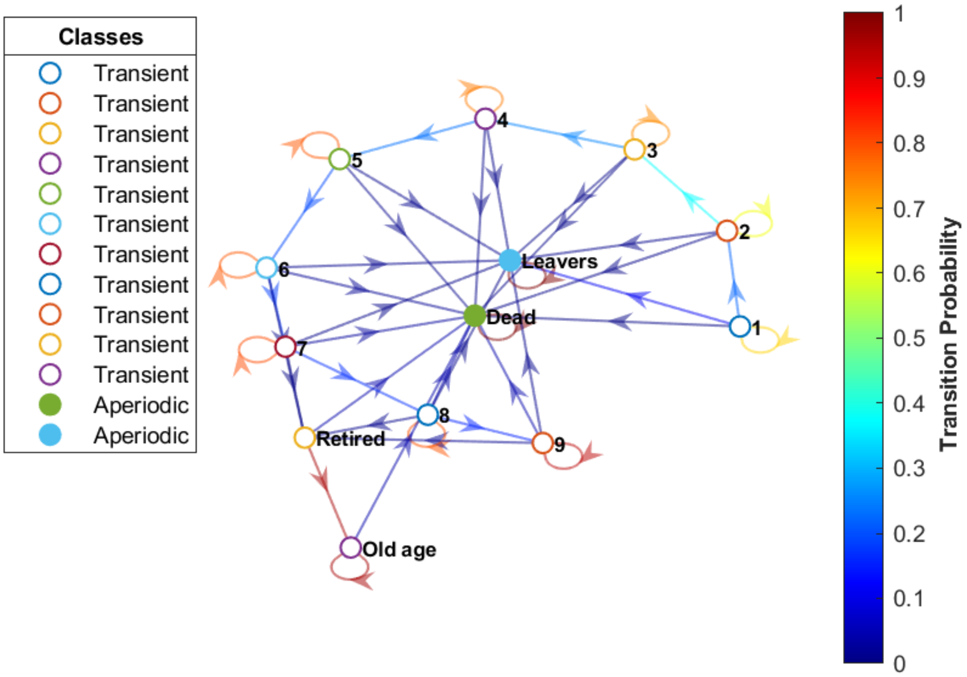

Figure 28. Markov chain for MPs in non-election years

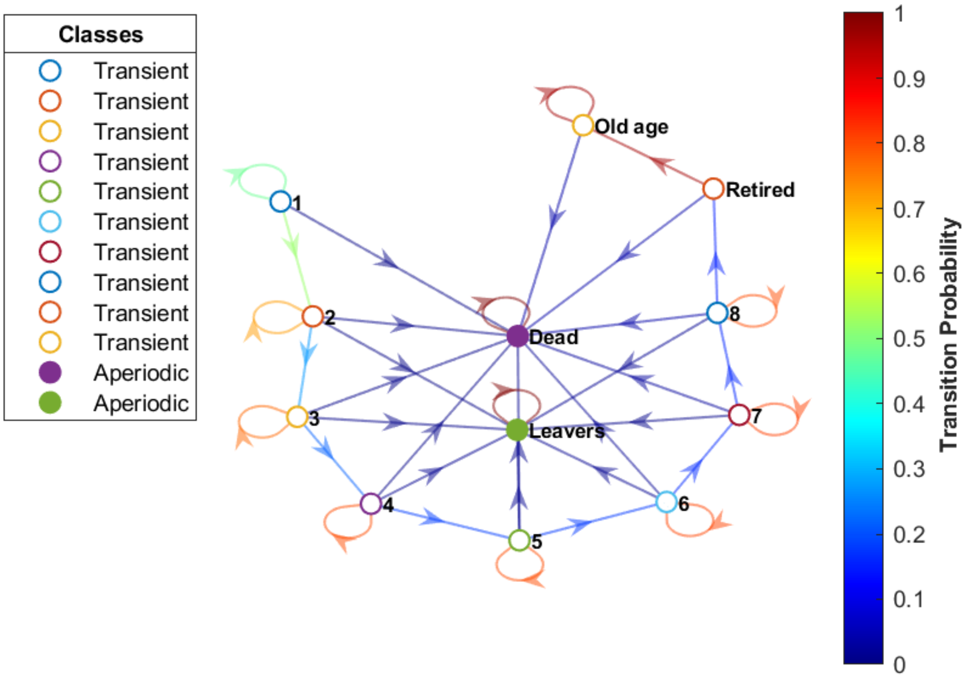

Figure 29. Markov chain for staff

A.4. Mortality high income countries 2015–2070

The data used to calculate the average values in Tables 4 and 5 was obtained from [1]. A high income country is defined by the World Bank as a nation with gross national income per capita of United States dollars $12,736$ or more in 2014, calculated using the World Bank Atlas method [21].

Table 4. Probabilities of dying

Age	15/20	20/25	25/30	30/35	35/40	40/45	45/50	50/55	55/60	60/65	65/70
20	0.00306	0.00273	0.00258	0.00237	0.00221	0.00208	0.00196	0.00184	0.00172	0.00161	0.00151
25	0.00359	0.00350	0.00337	0.00321	0.00303	0.00286	0.00275	0.00266	0.00253	0.00240	0.00226
30	0.00412	0.00412	0.00399	0.00381	0.00365	0.00344	0.00326	0.00314	0.00303	0.00289	0.00273
35	0.00527	0.00515	0.00500	0.00471	0.00445	0.00421	0.00394	0.00371	0.00354	0.00338	0.00320
40	0.00747	0.00725	0.00692	0.00650	0.00610	0.00570	0.00535	0.00499	0.00467	0.00442	0.00418
45	0.01160	0.01103	0.01048	0.00976	0.00922	0.00860	0.00801	0.00751	0.00700	0.00653	0.00616
50	0.01845	0.01715	0.01613	0.01508	0.01418	0.01337	0.01249	0.01165	0.01092	0.01018	0.00950
55	0.02864	0.02644	0.02454	0.02281	0.02157	0.02022	0.01908	0.01781	0.01659	0.01556	0.01452
60	0.04244	0.03965	0.03663	0.03362	0.03168	0.02987	0.02798	0.02637	0.02456	0.02285	0.02145
65	0.05905	0.05599	0.05194	0.04741	0.04410	0.04138	0.03882	0.03630	0.03406	0.03180	0.02941
70	0.09327	0.08797	0.08227	0.07580	0.07069	0.06610	0.06182	0.05813	0.05456	0.05143	0.04762
75	0.15648	0.14885	0.14046	0.13087	0.12326	0.11626	0.10979	0.10399	0.09835	0.09319	0.08745
80	0.25361	0.24350	0.23211	0.21894	0.20846	0.19853	0.18947	0.18097	0.17259	0.16456	0.15663

Table 5. Expectation of life

Age	15/20	20/25	25/30	30/35	35/40	40/45	45/50	50/55	55/60	60/65	65/70
20	61.52	62.07	62.67	63.39	63.97	64.54	65.08	65.61	66.14	66.67	67.17
25	56.70	57.23	57.83	58.53	59.11	59.67	60.20	60.72	61.25	61.77	62.27
30	51.89	52.42	53.02	53.72	54.28	54.83	55.36	55.88	56.40	56.91	57.41
35	47.10	47.63	48.22	48.91	49.47	50.01	50.53	51.04	51.56	52.07	52.56
40	42.33	42.86	43.45	44.13	44.68	45.21	45.72	46.23	46.73	47.24	47.72
45	37.63	38.15	38.73	39.40	39.93	40.46	40.96	41.44	41.94	42.43	42.91
50	33.04	33.55	34.11	34.76	35.28	35.78	36.26	36.74	37.22	37.69	38.16
55	28.61	29.09	29.63	30.25	30.75	31.23	31.69	32.14	32.60	33.05	33.50
60	24.38	24.80	25.31	25.90	26.37	26.82	27.25	27.67	28.10	28.54	28.95
65	20.34	20.72	21.17	21.71	22.14	22.57	22.96	23.35	23.74	24.14	24.53
70	16.45	16.79	17.19	17.66	18.05	18.43	18.78	19.13	19.49	19.85	20.19
75	12.87	13.15	13.48	13.88	14.21	14.54	14.84	15.14	15.46	15.78	16.06
80	9.77	9.99	10.25	10.57	10.83	11.10	11.34	11.59	11.85	12.12	12.34

A.5. The CAF rates

The annuity rates for converting members' accumulated funds into pension of UGX 1000 per annum at retirement are given in Tables 6 and 7.

Table 6. CAF rates for males and females

Age	Male	Female	Age	Male	Female
45	11,202	11,553	73	6,277	7,197
46	11,101	11,473	74	6,046	6,970
47	10,994	11,388	75	5,815	6,740
48	10,882	11,298	76	5,585	6,509
49	10,764	11,202	77	5,357	6,277
50	10,640	11,101	78	5,132	6,046
51	10,511	10,994	79	4,911	5,815
52	10,375	10,882	80	4,693	5,585
53	10,234	10,764	81	4,480	5,357
54	10,087	10,640	82	4,272	5,132
55	9,934	10,511	83	4,069	4,911
56	9,774	10,375	84	3,873	4,693
57	9,609	10,234	85	3,683	4,480
58	9,438	10,087	86	3,500	4,272
59	9,260	9,934	87	3,324	4,069
60	9,077	9,774	88	3,155	3,873
61	8,888	9,609	89	2,993	3,683
62	8,693	9,438	90	2,838	3,500
63	8,493	9,260	91	2,690	3,324
64	8,287	9,077	92	2,548	3,155
65	8,077	8,888	93	2,411	2,993
66	7,863	8,693	94	2,275	2,838
67	7,644	8,493	95	2,138	2,690
68	7,422	8,287	96	1,989	2,548
69	7,197	8,077	97	1,815	2,411
70	6,970	7,863	98	1,581	2,275
71	6,740	7,644	99	1,224	2,138
72	6,509	7,422	100	600	1,989

Table 7. Weighted CAF rates

Age	Rate	Age	Rate
45	11,325	73	6,599
46	11,231	74	6,369
47	11,132	75	6,139
48	11,028	76	5,908
49	10,917	77	5,679
50	10,801	78	5,452
51	10,680	79	5,227
52	10,552	80	5,005
53	10,420	81	4,787
54	10,281	82	4,573
55	10,136	83	4,364
56	9,984	84	4,160
57	9,828	85	3,962
58	9,665	86	3,770
59	9,496	87	3,585
60	9,321	88	3,406
61	9,140	89	3,235
62	8,954	90	3,070
63	8,761	91	2,912
64	8,564	92	2,760
65	8,361	93	2,615
66	8,154	94	2,472
67	7,941	95	2,331
68	7,725	96	2,185
69	7,505	97	2,024
70	7,283	98	1,824
71	7,056	99	1,544
72	6,829	100	1,086