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PREDICTION OF AIRCRAFT LOST OF CONTROL IN THE FLIGHT BY CONTINUATION, BIFURCATION, AND CATASTROPHE THEORY METHODS

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Abstract

Lost of Control in Flight (LOC-I) is ordinarily associated with flight outside of the normal flight envelope, with nonlinear behaviours, and with an inability of the pilot to control the aircraft. These results provide a means for analysing accident data to establish whether or not the accident should be classified as LOC-I. Moreover, they help identify when the initial upset occurred, and when control was lost. The analysis also suggests which variables were involved, thereby providing clues as to the underlying mechanism of upset. However, it does not provide direct links to the flight mechanics of the aircraft, so it cannot be used proactively to identify weaknesses or limitations in the aircraft or its control systems. Moreover, it does not explain how departures from controlled flight occur. The complexity of the disaster aetiology stems from both the scale and coupling of the systems (not only the physical aircraft systems but also the organizational systems that support the operation). This complexity creates a pattern of disaster that evolves or it is precipitated through a series of several small failures. The cusp catastrophe model facilitates the mapping of Reason's latent failure model, providing a descriptive and predictive illustration of the emergence of latent conditions under the trigger of situational factors. The risk of an accident increases as the situational and systematic factors combine to create an inherent instability resulting in the catastrophic event

Keywords: *nonlinear dynamics of flight, lost of control in the flight, catastrophe theory, bifurcation and continuation methods*

Nomenclature

C_i – coefficient of aerodynamic force or aerodynamic moment ($i=D, L, Y, L_a, M_a, N_a$)
D – drag force,
g – gravity acceleration,
H – altitude of flight,
I_X, I_Y, I_Z, I_{XZ} – aircraft moments of Inertia,
L – lift force,
L_a – rolling moment of aerodynamic forces,
M_a – pitching moment of aerodynamic forces,
m – mass of aircraft,
Na – yawing moment of aerodynamic forces,
P, p – roll angular rate,
Q, q – pitch angular rate,
R, r – yaw angular rate,
t – time,
T – trust,
T_x, T_y, T_z – components of trust force (in aircraft system of coordinates – Fig. 1),
V – aircraft velocity,
$\mathbf{u}$ – control vector – $\mathbf{u} = [\delta_w, \delta_a, \delta_v, T]^T \in \Re^4$,
$\mathbf{x}$ – state vector – $\mathbf{x} = [V, \alpha, \beta, P, Q, R, \Theta, \Phi]^T \in \Re^8$,
x_1, y_1, z_1 – aircraft centre of mass coordinates in inertial system of coordinates (Fig. 1),
α – angle of attack (AOA),

β	–	slip angle,
δ_a	–	aileron deflection,
δ_w	–	deflection of elevator,
δv	–	deflection of rudder,
Φ, ϕ	–	roll angle,
Θ, θ	–	pitch angle,
Ψ, ψ	–	yaw angle.

1. Introduction

Problems connected with recreation of aircraft crashes were the subjects of many works. One can mention here works by Calkins [1], Dietenberger, Haines and Luers [2], Luers and Ditenberger [3]. The works mentioned above contained analyses and simulations of specific occurrences. A very interesting work is treatise [1] containing reconstruction of a crash of Boeing 737-300 aircraft no. N513AU belonging to UA airlines (flight 427). The crash took place near Pittsburgh (the aeroplane fell near the town of Aliquippa, Pennsylvania) on September 8th 1994 [6]. Official statement of the commission of investigating aircraft crashes said that the direct cause of the crash was an uncontrolled descent, which leads to hitting the ground. Calkins showed that the loss of control over the aeroplane could have been caused by a vortex flowing off the wings of a Boeing 747 liner flying in front of N513AU aeroplane. The effect of this independent expert's report by Calkins was a range of theoretical works and in-flight investigations (undertaken, among others, by NASA) documenting the existence of threat to the safety of landing approach by such vortex (e.g. work by Nelson and Jumper [4]). The starting point, allowing reconstructing an aircraft crash, is the data from many sources. The main source of information is the data coming from an on-board flight parameter recorder. A part from that, also recordings of conversations made by the crew, photos of the occurrence, drawings, witness statements, way and direction of scatter of the wreckage, after-crash investigations of the wreckage and other investigatory actions are taken into account. Reconstruction of a crash of an aircraft of course cannot determine the guilt and level of responsibility of people for the crash. Nevertheless, it is a valuable source of information allowing better understanding the motion of the aeroplane right before the crash and during the crash. Results of computations can be used in the process of computer flight animation and in the process of correct assessment of the crash (by the Commission of Investigation of Aircraft Accidents, or in the courtroom). The ultimate goal of this effort is to contribute to the reduction of the fatal accident rate due to loss-of-control. Research activities have involved accident analyses, and piloted simulation. This paper provides a summary of research completed to date and includes discussion on key technical results, lessons learned, and future research needed.

2. The Loss of Control in Flight problem

Although the majority of fatal aircraft crashes over the past decade or so have been attributed to Lost of Control in Flight (LOC-I), its meaning is ambiguous. Generally, a pilot will report LOC-I if the aircraft does not respond as expected. Consequently, pilot experience can be a major variable in assessing LOC-I. What LOC-I is to one pilot may not be to another. Recently, Wilborn and Foster [10] have proposed quantitative measures of LOC-I. These Quantitative Loss-of-Control in Flight (QLC-I) metrics consist of envelopes defined in two dimensional parameter spaces. Based on the analysis of 24 data sets compiled by the Commercial Aircraft Safety Team (CAST) Joint Safety Analysis Team (JSAT) for LOC-I [7] five envelopes have been defined:

adverse Aerodynamics Envelope: (normalized) angle of attack vs. sideslip angle; unusual Attitude Envelope: bank angle vs. pitch angle; structural Integrity Envelope: normal load factor vs. normalized air speed; dynamic pitch control envelope: (dynamic pitch attitude ($\Theta+\delta\Theta/\delta\tau\Delta\tau$) vs. % pitch control command); dynamic roll control envelope, (dynamic roll attitude ($\Phi+\delta\Phi/\delta\tau\Delta\tau$) vs. % lateral control command);

The authors provide a compelling discussion of why these envelopes are appropriate and useful. Flight trajectories from the 24 CAST data sets are plotted and the authors conclude manoeuvres that exceed three or more envelopes can be classified as LOC-I, those that exceed two are borderline LOC-I and normal manoeuvres rarely exceed one. According to Ref. [7], the precipitating events of the CAST LOC incidents were stalls (45.8%), sideslip-induced rolls (25.0%), rolls from other causes (12.5%), pilot-induced oscillation (12.5%), and yaw (4.2%). These results are important, because can provide a means for analysing accident data to establish whether or not the accident should be classified as LOC-I. Moreover, they help identify when the initial upset occurred, when control was lost and suggests which variables were involved. However, because the approach does not directly connect to the flight mechanics of the aircraft, it does not identify weaknesses or limitations in the aircraft or its control systems. Moreover, it does not explain how departures from controlled flight occur. In particular, we would like to know how environmental conditions or actuator failures or structural damage impact the vulnerability of the aircraft to LOC-I. To do this we need a formal analytical definition of Loss of Control in Flight. Another important study [8] reviews 74 transport LOC-I accidents in the fifteen year period 1993-2007. Of these the major underlying causes of LOC-I are identified as stalls, ice contaminated airfoils, spatial disorientation, and faulty recovery technique.

An aeroplane must typically operate in multiple modes that have significantly different dynamics and control characteristics. For example, cruise and landing configurations. Within each mode, there may be some parametric variation, such as weight or centre of mass location that also affects aircraft behaviour. Each mode has associated with it a flight envelope restricting speed, attitude and other flight variables. Under normal conditions keeping within the flight envelope provides sufficient manoeuvrability to perform the mode mission while insuring structural integrity of the vehicle for all admissible parameter variations and anticipated disturbances. Abnormal conditions, e.g., icing, faults or damage, will alter aircraft dynamics and may require the definition of a new mode with its own flight envelope.

Ordinarily a flight envelope can be considered a convex polyhedral set, not necessarily bounded, in the state space. Thus, the aircraft speeds to operate within the state constraints imposed by the envelope. Insuring that an aircraft remains within its flight envelope is called envelope protection. Envelope protection is generally the responsibility of the pilot although there is an increasing interest in and use of automatic protection systems [9-11]. Because the controls themselves as well as the states are constrained, the question of whether it is even possible to keep aircraft within the envelope is not trivial. Questions like this have been considered in the control literature [11]. Besides the control bounds, other restrictions may be placed on the admissible controls that could further restrict the safe set. For instance, we could require that only smooth feedback controls be employed.

3. Bifurcation analysis of aircraft dynamics of flight

The dynamic nature of aeroplane motion and associated risks is a function of the situational factors and latent conditions present at the time. Both share a temporal and spatial element. The basic idea of the investigation of flight dynamics is the concept of steady states. The steady state of a flying vehicle is considered in a nonformal mathematical sense for a complete set of equations. Steady state or equilibrium is considered in a more general dynamical sense as the steadiness of all the external forces and moments, i.e. as the aerodynamic steady state.

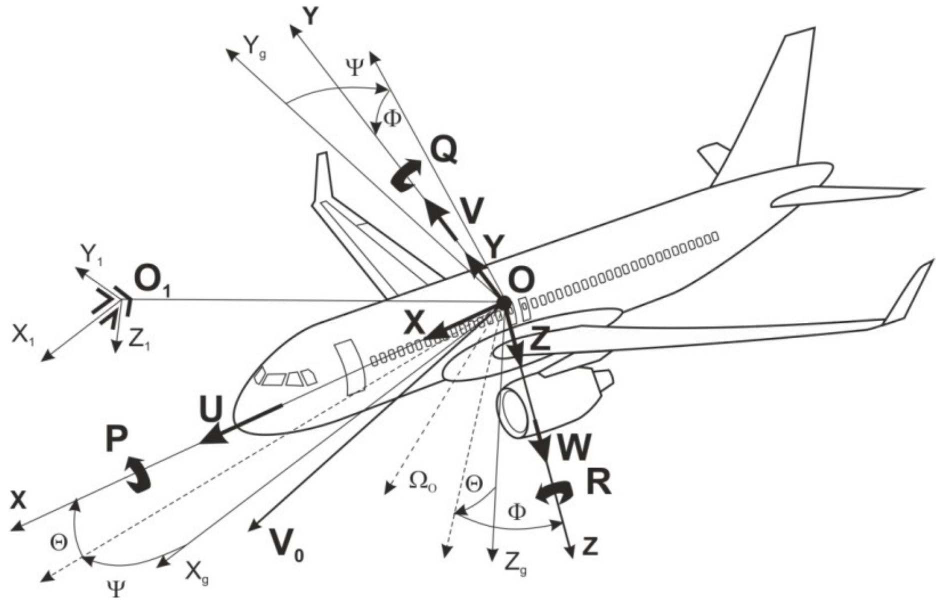

Fig. 1 Systems of coordinates, forces and aerodynamic moments, which influence the aircraft in flight

The full set of motion equations can be divided into the following four groups:

– dynamics of translational motion written in wind-body axes:

$$\begin{aligned}
\frac{dV}{dt} &= \left(Tx_a - D - mg\sin\Theta\right)/m \\
\frac{d\alpha}{dt} &= Q - \left(P\cos\alpha - R\sin\alpha\right)\tan\beta + \frac{Tz_a - L}{mV\cos\beta} + \frac{g}{V\cos\beta}\cos\Theta\cos\Phi, \\
\frac{d\beta}{dt} &= P\sin\alpha - R\cos\alpha + \frac{Ty - C}{mV} + \frac{g}{V}\cos\Theta\sin\Phi
\end{aligned} \tag{1}$$

– dynamics of angular motion written in principal body axes:

$$\begin{aligned}
\frac{dP}{dt} &= \left[\left(I_Y - I_Z\right)QR + \mathsf{L}\right]/I_X \\
\frac{dQ}{dt} &= \left[\left(I_Z - I_X\right)RP + \mathsf{M}\right]/I_Y, \\
\frac{dR}{dt} &= \left[\left(I_X - I_Y\right)PQ + \mathsf{N}\right]/I_Z
\end{aligned} \tag{2}$$

– kinematics of angular motion written for wind-body axes (the same equation valid for body axes without subscript (a):are:

$$\begin{aligned}
\frac{d\Theta}{dt} &= Q\cos\Phi - R\sin\Phi \\
\frac{d\Phi}{dt} &= P + \left(Q\sin\Phi + R\cos\Phi\right)\tan\Theta, \\
\frac{d\Psi}{dt} &= \left(Q\sin\Phi + R\cos\Phi\right)\sec\Theta
\end{aligned} \tag{3}$$

– and kinematics of translational motion written in earth axes:

$$\frac{dH}{dt} = V\sin\Theta, \tag{4}$$

where altitude of flight H = ~ z_1.

The basic idea of the investigation of flight dynamics is the concept of steady states. The steady state of a flying vehicle is considered in a nonformal mathematical sense for a complete set of equations. Steady state or equilibrium is considered in a more general dynamical sense as the steadiness of all the external forces and moments, i.e. as the aerodynamic steady state. This condition requires that the main motion parameters (V, α, β, P, Q, R) and gravity projections (i.e. the Euler angles Θ, Φ) are all constant in the body axes frame with time. The position in space x_1, y_1 and head angle Ψ do not affect on the aerodynamic steady state therefore can be considered separately, together with appropriated equations. The altitude of flight H (or z_1) defines the air density and can be added to the system of equations only in cases when the influence of density variation on dynamics behaviour is significant. If the variation of P with altitude can be neglected, the most general case of equilibrium flight is a vertical helicoidal trajectory. These may be climbing and gliding turns with large radius of curvature, or steady equilibrium spin modes with relatively small radius of trajectory curvature. The spatial case of such trajectories is the rectilinear motion. Therefore, to study flight dynamics the following autonomous system of equations can be extracted from the full system (Eqs (1)-(4)):

$$\frac{d\mathbf{x}}{dt} = \mathrm{f}\left(\mathbf{x},\mathbf{u}\right), \tag{5}$$

where $\mathbf{x} = \left[V,\alpha,\beta,P,Q,R,\Theta,\Phi\right]^T \in \Re^8$, $\mathbf{u} = \left[\delta_e,\delta_a,\delta_r,T\right]^T \in \Re^4$ are the state vector and the control vector. The steady state regimes, which are the vertical helicoidal trajectories, are defined by equilibrium solutions of this system of equations.

Some additional physical assumptions, concerning the type of motion, such as the existence of the plane of symmetry, steadiness of some state variables, etc., can be taken into account for obtaining the approximate autonomous subsystems of equation of lower dimension for investigation the flight dynamics. Such subsystems can be derived for symmetrical flight in vertical plane, for studying the roll-coupling problem, etc. (see the following sections).

The aerodynamic model intended for spin conditions, i.e. high angles of attack and fast rotation, is based on the experimental data obtained from the different kinds of wind tunnel tests – static, forced-oscillation and rotary balance. There exist a number of methods for designing the “combined” mathematical model of aerodynamic coefficients, which implement the experimental data in a very similar manner.

The rotation at high angle of attack can significantly influence the flow pattern. As a result, the aerodynamic coefficients become nonlinear functions of the reduced rate of rotation. That is why the aerodynamic coefficients measured in rotary balance tests are considered as the basic or “nondisturbed” part of the aerodynamic model for high angle-of-attack conditions.

The disturbed motion is accompanied by the misalignment between the velocity and the rotation vectors. The projections of the rotation vector onto the wind-body axes can be used as parameters for describing the disturbed conical motion.

Assuming that the disturbances of the pure conical motion are small, the following representation of the aerodynamic coefficients can be used:

$$\begin{aligned} C_i &= C_{i_{RB}}\left(\alpha,\beta,p_a,\mathbf{u}\right) + C_{i_{q_a}}\frac{q_a\bar{c}}{2V} + C_{i_{\dot\alpha}}\frac{d\alpha}{dt}\frac{\bar{c}}{2V} + C_{i_{r_a}}\frac{r_a b}{2V} + C_{i_{\dot\beta}}\frac{d\beta}{dt}\frac{b}{2V} \\ &= C_{i_{RB}}\left(\alpha,\beta,p_a,\mathbf{u}\right) + \left(C_{i_{q_a}} + C_{i_{\dot\alpha}}/\cos\beta\right)\frac{q_a\bar{c}}{2V} + \left(C_{i_{r_a}} + C_{i_{\dot\beta}}\right)\frac{r_a b}{2V} - C_{i_{\dot\alpha}}\frac{q_{a_{sp}}\bar{c}}{2C\cos\beta} + C_{i_{\dot\beta}}\frac{r_{a_{sp}} b}{2V}\cdot \end{aligned} \tag{6}$$

The derivatives of the aerodynamic coefficients, standing together with the reduced rates of rotation and corresponding to the rotary flow, can be measured, by means of the oscillatory coning technique.

When the nonlinear term in (6) can be approximated by a linear function on the angular rate, the representation (6) becomes equivalent to the aerodynamic model commonly used for low angles of attack. The results of the static wind tunnel tests in this case can also be incorporated into the mathematical model.

Some additional physical assumptions, concerning the type of motion, such as the existence of the plane of symmetry, steadiness of some state variables, etc., can be taken into account for obtaining the approximate autonomous subsystems of equation of lower dimension for investigation the flight dynamics.

The transient and steady state of a system represented by a set of differential equations (1) can be solved by conventional numerical integration methods, by computing the trajectories and orbits using digital simulation [23, 24]. However, it is possible with bifurcations theory to predict the behaviour of trajectories and orbits without resorting to the solution of the differential equations. In this case, bifurcations analysis is applied to study the emergence of sudden changes in a system response arising from smooth, continuous variations on the system parameters (see for example refs. [15-21]. The results obtained with this analysis can be showed in a bifurcations diagram. The bifurcations diagram provides qualitative information about the behaviour of the system steady state (equilibrium) solutions, as physical parameters are varied. At a certain points (bifurcations points), infinitesimal changes in system parameters can cause significant qualitative changes in equilibrium solutions. General theory of bifurcation analysis can be found in book [25] for example. The region of attraction of critical deep stall regime in the general case will be more complicated with respect to the region, considered above in simplified manner. In the Fig. 2 is shown in the steady states in middle and higher angles of attack. In this figure for the elevator deflection between –13.7 deg. and –10.1 deg., the steady state trim conditions of the aircraft are unstable as a result of two Hopf bifurcations. Hopf bifurcations can lead to periodic motions, so it is possible that for elevator deflections between –13.7° and –10.1° the aircraft will undergo periodic motion.

The main feature of the problem considered is that even in the case of linear representation of aerodynamic coefficients the existence of multiple stable steady-state solutions, e.g. equilibrium and periodic, is possible. The bifurcation analysis of all the possible steady-state solutions and their local and global stability analysis can show the genesis of stability loss and explain in many cases the very strange aircraft behaviour.

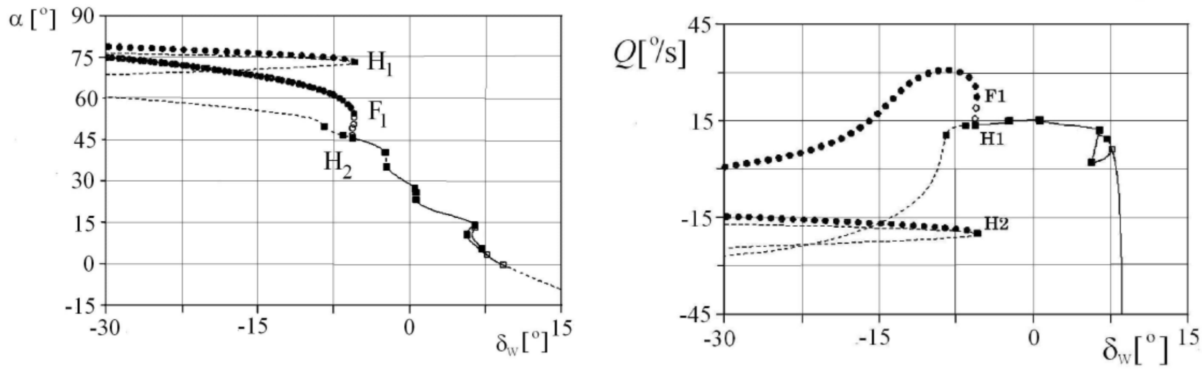

Fig. 2 Steady states at middle and high angles of attack — variation

To take into account the interaction between phugoid and angular modes the first four equation of (1), which are the autonomous nonlinear system with state vector $\mathbf{x}=[V,\Theta,\alpha,Q]^T$, have to be considered with entire ranges of α and Θ variations [$-\pi$, π]. To represent the stability region in the fourth-order state space it is possible only by means of drawing its two-dimensional cross-sections considering the disturbances only in two selected state variables.

For example, in Figure 3 two different cross-sections of the region of attraction of deep stall regime, i.e. stable point α_3, are shown. The disturbances in the plane of pitch angle Θ and velocity V are considered. Two other state variables at the initial moment are the same for all points of cross-section. In the first case (A) the angle of attack is trimmed in a lower stable point $\alpha = \alpha_1$ with zero pitch rate $Q = 0$, and in the second case (B) the angle of attack is trimmed in the critical position $\alpha = \alpha_3$, also with zero pitch rate $q = 0$. In every point of the considered cross sections the initial path angle can be calculated using the following formula $\gamma = \Theta - \alpha$.

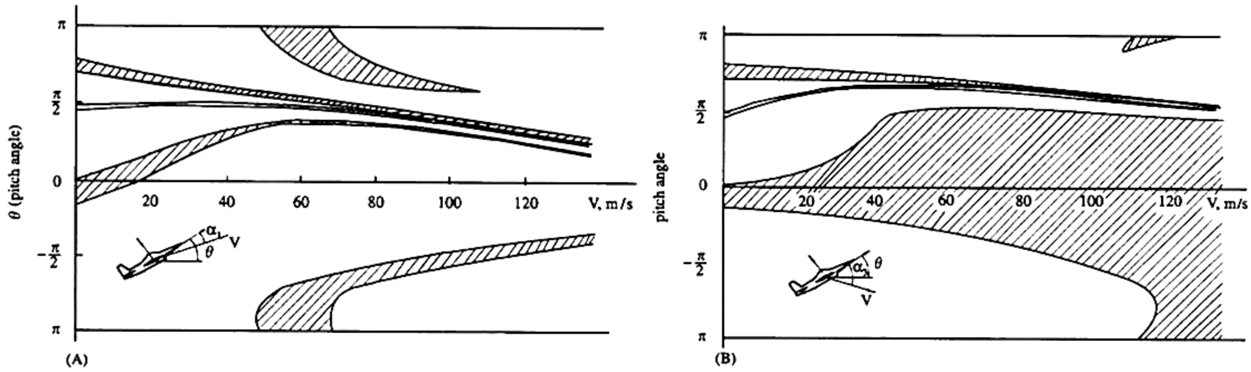

Fig. 3 Cross section of asymptotic stability region for deep stall regime (cf. [17])

The dashed areas on the cross sections define the initial points, starting from when the aircraft enters in the deep stall regime, α_3. In the second case (B), the probability of entering into the deep stall is much greater, especially in flights with large velocities, than in the first case (A). The number of cross sections of multidimensional stability region can offer global information about the aircraft dynamics.

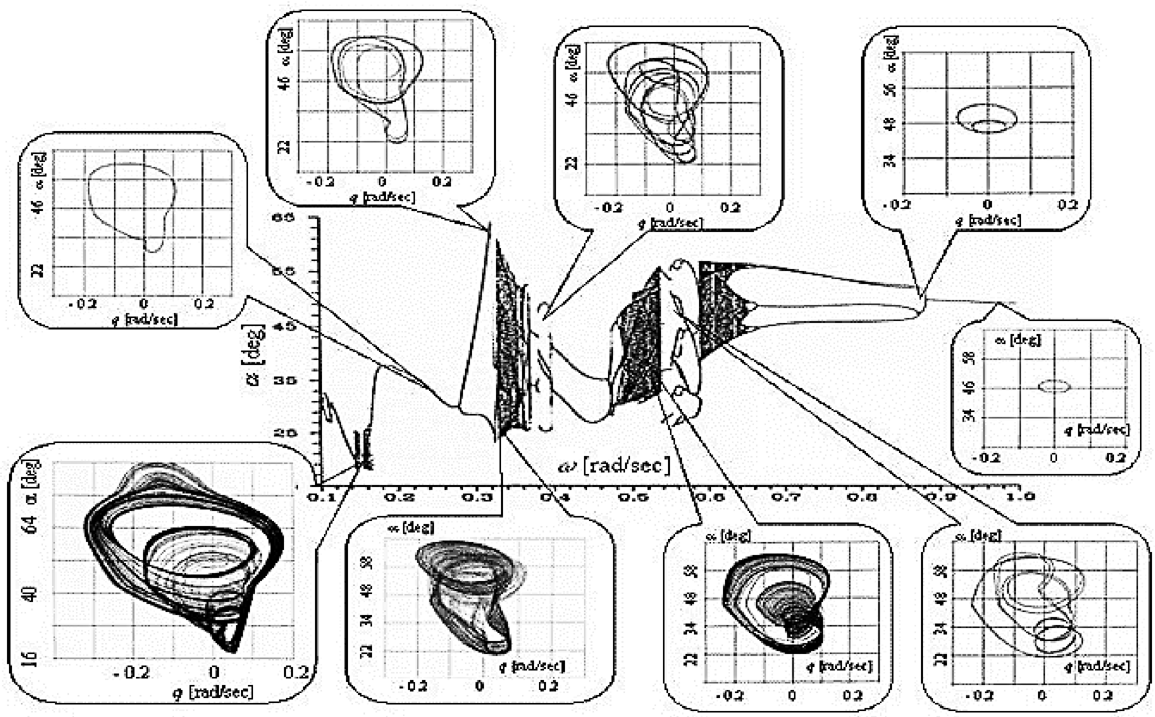

Fig. 4 Bifurcation diagram for aircraft dynamics (ω – rudder excitation frequency).

4. Lost of Control in Flight – Catastrophe theory approach

Lost of Control in Flight phenomenon can be mapped onto the 3-D space of the cusp model (Fig. 5), in order to illustrate the instability created when active errors v and latent conditions u combine to place the point at the pleat, thereby leading to a catastrophe event (see ref. [22]). In this model, the active errors are contained as futures within the situational factors. Mistakes or errors within the catastrophe model will result in a move toward a more negative situational factor. These active errors may act as a trigger enabling the latent factors, thereby activating a series of effects or consequences with possible disastrous outcomes. The latent conditions are contained within systemic factors and range in value from low to high based on their emergence during a flight safety event.

In Fig. 5, point A represents a scenario characterized by positive situational factors and low systematic factors where the potential for flight safety incidents is very low. As the situation deteriorates the scenario develops a dynamic nature characterized as a movement along the axis *x*. This situational factor triggers the latent (systematic) conditions characterized as a movement from point A to point B in Fig. 1. If uncorrected, a catastrophe event may occur beginning with the movement from point B to D where the instability of the situation results in the catastrophic event characterized by the movement from *D* to *C*. Similarly, situational factors may continue for some time before triggering the latent conditions, as characterized as the movement from *A* to *D*. The complexity of the disaster aetiology stems from both the scale and coupling of the systems (not only the physical aircraft systems but also the organizational systems that support the operation). This complexity creates a pattern of disaster that evolves; or is precipitated through a series of several small failures. The cusp catastrophe model facilitates the mapping of Reason's latent failure model, providing a descriptive and predictive illustration of the emergence of latent conditions under the trigger of situational factors. The risk of an accident increases as the situational and systematic factors combine to create an inherent instability resulting in the catastrophic event.

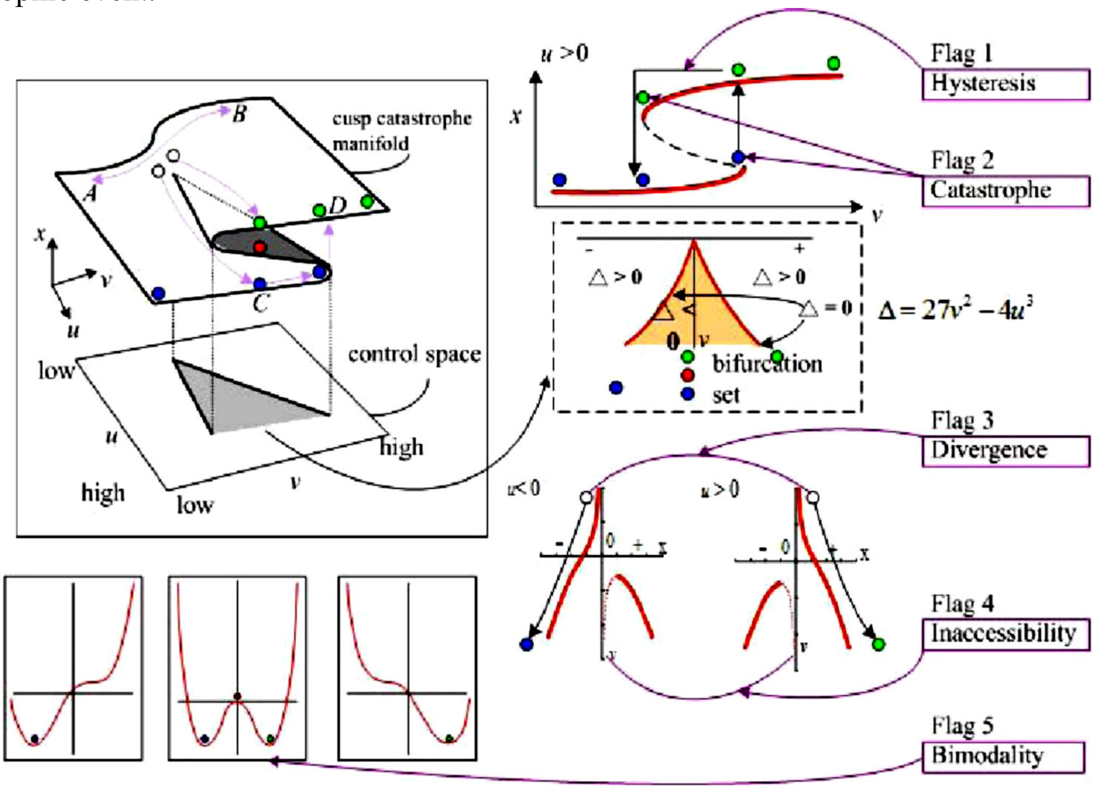

Fig. 5. A cusp catastrophe model

5. Conclusions

Lost of Control in Flight phenomenon is directly connected to near stall and post stall motion of aircraft. The behaviour of a commercial aircraft at high angles of attack (AOA) flight is so complex that it is very difficult to predict it exhaustively. Usually, this flight domain is investigated by means of systematic or Monte Carlo numerical simulations before the first flight and by means of extensive and expensive flight tests.

Thanks to bifurcation theory and computer capabilities, a methodology and software have been set up to investigate asymptotic behaviour of nonlinear differential equations depending on parameters. It can be seen that bifurcation theory has been used to identify an aerodynamic model suitable for the analysis of high-AOA flight regimes. Considering results of calculations, it can be said that this technique has great potential and is appropriate for the investigation of aircraft behaviour, using only wind-tunnel data. However, one cannot forget that the quality of prediction is directly related to the quality of the aerodynamic database of the aircraft model. Results presented above show the efficiency of the methodology, based on the qualitative methods of dynamical systems theory, for investigation of nonlinear flight dynamics problem. Nonlinear aircraft dynamics are too complex to be thoroughly studied using analytical methods. That is why the advances in the development of new methodology are closely coupled with the development of special numerical methods and software.

References

[1] Calkins, D. E., *Aircraft accident flight patch simulation and animation,* Journal of Aircraft, Vol. 31, No. 2, pp. 376-36, 1994.

[2] Dietenberger, M. A., Haines, P. A., Luers, J. K., *Reconstruction of Pan Am New Orleans accident*, Journal of Aircraft, Vol. 22, No. 8, pp. 719-728, 1985.

[3] Luers, J., Dietenberger, M. A., *Analyses of Arrow Air DC-8-63 Accident of December 12, 1985: Gander, Newfoundland*, Journal of Aircraft, Vol. 27, No. 6, pp. 543-550, 1990.

[4] Nelson, R. C., Jumper, E. J., *Aircraft wake vortices and their effect on following aircraft, AIAA Meeting Papers*, Atmospheric Flight mechanics Conference and Exhibit, Montreal, Canada, August 2001, American Institute of Aeronautics and Astronautics, AIAA 2001-4073 TP, http://arc.aiaa.org/doi/abs/10.2514/6.2001-4073

[5] Anon, Rapport NTSB, Aircraft Accident Report; United Airlines Flight 232, 1990.

[6] Anon, Rapport NTSB, Aircraft Accident Report; Uncontrolled descent and collision with terrain USAir flight 427 Boeing 737-300, N513AU near Aliquippa, Pennsylvania, September 8, 1994,

[7] Russell, P., Pardee, J., *Final Report: JSAT Loss of Control: Results and Analysis*, Tech. rep., Federal Aviation Administration: Commercial Airline Safety Team, 2000.

[8] Ranter, H., *Airliner Accident Statistics 2006*,Tech. rep., Aviation Safety Network, 2007.

[9] Lambregts, A. A., Nesemeier, G., Wilborn, J. E., Newman, R. E., *Airplane Upsets: Old Problem, New Issues,* AIAA Meeting Papers, AIAA Modeling and Simulation Technologies Conference and Exhibit, 18-21 August, Honolulu, Hawaii, 2008, American Institute of Aeronautics and Astronautics, http://arc.aiaa.org/doi/abs/10.2514/6.2008-6867, AIAA 2008-6867 TP,.

[10] Wilborn, J. E., Foster, J. V., *Defining Commercial Aircraft Loss-of-Control: a Quantitative Approach, AIAA Meetong Papers,* Atmospheric Flight mechanics Conference and Exhibit, Providence, Rhode Island, American Institute of Aeronautics and Astronautics, AIAA 2004-4811TP, http://arc.aiaa.org/doi/abs/10.2514/6.2004-4811, 16-19 August 2004.

[11] Kwatny, H. G., Dongmo, J-E. T., Bor-Chin Chang, *Aircraft Accident Prevention: Loss-of-Control Analysis*, AIAA Meeting Papers, AIAA Guidance, Navigation, and Control Conference, AIAA 2009-6256 TP, http://arc.aiaa.org/doi/pdfplus/10.2514/6.2009-6256, Chicago, Illinois 2009.

[12] Pamadi, B., *Performance, Stability, Dynamics and Control of Aeroplanes*, 2nd Edition, AIAA Educational Series, Richmint, VA, 2007.

[13] Etkin, B, Reid, L. D., *Dynamics of Flight: Stability and Control*, 3rd Edition, J. Willey

& Sons, New York 1996
[14] Carroll, J. V., Mehra, R. K., *Bifurcation Analysis of Non-Linear Aircraft Dynamics, Journal of Guidance Control and Dynamics*, Vol. 5, No. 5, pp. 529-536, 1982.
[15] Jahnke, C. C., Culick, F. E. C., *Application of Bifurcation Theory to the High-Angle-of-Attack Dynamics of the F-14, Journal of Aircraft*, Vol. 31, No. 1, pp. 26-34, 1994.
[16] Goman, M. G., Khramtsovsky, A. V., *Application of continuation and bifurcation methods to the design of control systems,* Phil. Trans. R. Soc., No. A 356, pp. 2277-2295, London 1998.
[17] Goman, M. G., Zagainov, G. I., Khramtsovsky, A. V., *Application of Bifurcation Methods to Nonlinear Flight Dynamics Problems,* Progress in Aerospace Science , Vol. 33, pp. 539-586, 1997.
[18] Guicheteau, P., *Bifurcation Theory: a Tool for Nonlinear Flight Dynamics,* Phil. Trans. R. Soc., No. A 356, pp. 2181-2201, London 1998.
[19] Lowenberg, M. H., *Bifurcation Analysis of Multiple-Attractor Flight Dynamics*, Phil. Trans. R. Soc., No. A 356, pp. 2297-2319, London 1998.
[20] Marusak, A. J., Pietrucha, J. A., Sibilski, K. S., *Prediction of Aircraft Critical Flight Regimes Using Continuation and Bifurcation Methods,* AIAA Meeting Papers , 38th Aerospace Sciences Meeting and Exhibits, Reno, NV, USA, American Institute of Aeronautics and Astronautics, AIAA-2000-0976-TP, http://arc.aiaa.org/doi/abs/10.2514/6.2000-976, 2000.
[21] Foster, J. V., et. all., *Dynamics Modeling and Simulation of Large Transport Aircraft in Upset Conditions,* AIAA Meeting Papers, AIAA Guidance, Navigation, and Control Conference, San Francisco, CA, American Institute of Aeronautics and Astronautics, AIAA 2005-5933TP, http://arc.aiaa.org/doi/abs/10.2514/6.2005-5933, 2005.
[22] Masys, A. J., *Aviation accident aetiology: catastrophe theory perspective, Diasert Prevention and Management*, Vol. 13, No. 1, pp. 33-38, 2004.
[23] Sibilski, K., *Numerical reconstruction of aircraft accidents flight dynamics, Archives of Transport,* Vol. IX, No. 1-2, 1997.
[24] Sibilski, K., *Some thoughts on mathematical models for aircraft accidents simulation*, Aviation Safety (H. Soekha (eds.), VSP Publishing Company, Utrecht, Nederland 1997.
[25] Wiggins, S., *Introduction to Applied Non-linear Dynamical Systems and Chaos*, Springer-Verlag, New York 1990.