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roughness, contact,
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Pawel MACIOLKA[1]
Jerzy JEDRZEJEWSKI[1]

MODELLING THE BEHAVIOUR OF A CONTACT LAYER BETWEEN THE WORKPIECE AND A LOCATOR

The assurance of high precision alignment of workpieces with low stiffness requires thorough knowledge about the phenomena taking place in the contact area under the influence of forces exerted by actuators. It is indispensable to utilise the model of a contact and deformations taking place in it. In order to solve the problem of contact, the contact geometry was modelled by means of a flat and rough surface model based on stereometric measurements, as well as by means of the finite element method (FEM). In a FEM method, a spherical structure of the surface layer was assumed and only one zone was taken into consideration, i.e. $0.4{\cdot}R_t$ (maximum surface roughness). Zone structure allows easy changing of mechanical properties of material during the introduction of data defined by a strengthening curve in the plastic region. The possibility of expanding the model by consecutive zones allows taking into consideration the physical properties in complex analyses of the surface layer. This paper presents the problem of contact related to the alignment and fixing process, a model of contact deformations between a flat and a rough surface, as well as the algorithm for determining the model of a surface profile according to the bearing ratio curve.

1. INTRODUCTION

The precision of workpiece alignment in machining and assembly devices has a big impact on the precision of their mutual position in the assemblies and on the operational parameters of these assemblies and final products. Post-assembly and operational clearances, stiffness, susceptibility to vibrations, operational noise and lifetime, all depend on this precision. The deviations of tolerated dimensions relative to machining datums are caused, among other things, by the displacements taking place in the contact of a workpiece with the surface of machining device locator. Such deformations should be small and repeatable, which will assure the invariability of workpiece position relative to machining datums. The behaviour of the contact of a surface during its loading by the force exerted by an actuator, as well as during the approach of a workpiece to a locator in the moment of causing contact. The force exerted on a workpiece must not cause, in the alignment process, plastic deformations, which considerably change its geometry and dimensional

[1] Wroclaw University of Technology, Institute of Production Engineering and Automation, Poland,
50-371 Wrocław, ul. Łukasiewicza 5, email: Pawel.Maciolka@pwr.wroc.pl , Jerzy.Jedrzejewski@pwr.wroc.pl

precision.

Hereby work is devoted to the description of phenomena taking place in contact, in order to create a model, which can make it possible to analyse the workpiece alignment process. It is also aiming at the definition of the parameters in the alignment process, ensuring its required precision, while maintaining geometrical precision of the workpiece itself.

2. THE PROBLEM OF CONTACT IN ALIGNMENT

The alignment process in a high-performance and precise manufacturing of products is at the present time a highly organised process. It is based on the precise identification of requirements, the definition of rational actions, controlling these actions, their diagnostics and monitoring. High precision and performance of alignment is achieved by assisting individual actions with modelling and simulations. Such modelling also includes the phenomena taking place in contacts with locators.

Especially important, in order to reach high precision in the alignment process is the knowledge about possible disturbances, which can appear. In order to achieve this, the authors tried to identify the disturbances, which could appear in a conventional alignment process in a machining device (Fig. 1). There are successive actions shown on it, which lead to precise alignment of a workpiece, with the description of phenomena taking place in the actions, which are important for the precision of fixing and modelling of the process. In a complex analysis, the disturbances mentioned on Fig. 1 should be taken into considerations both in the experimental, as well as in the simulation-based alignment precision assessment.

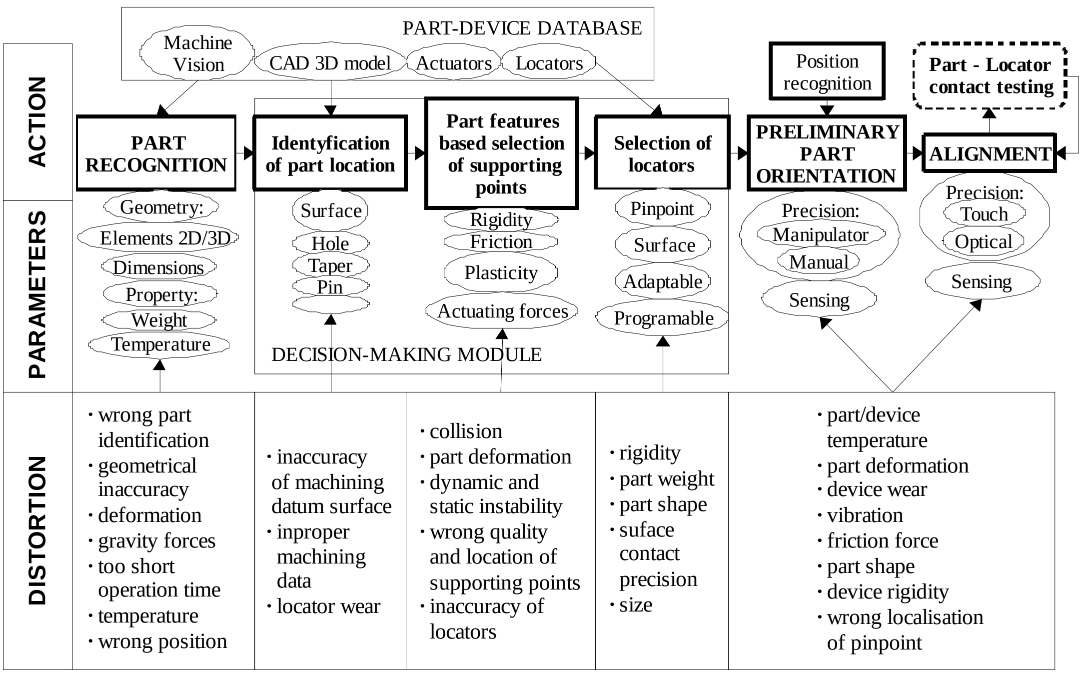

Fig. 1. Elements of a conventional alignment process

In a highly organised (automated), precise alignment of workpieces, process efficiency and error monitoring are very important, ensuring the prevention of the exceeding of their permissible values (Fig. 2). Errors must be then very quickly identified and modelled, which is the basis of their precise monitoring. In such actions, behaviour identification of contacts and their proper modelling during alignment are of key importance. It is the basis for realising precise alignment based on simulations, and generally based on knowledge.

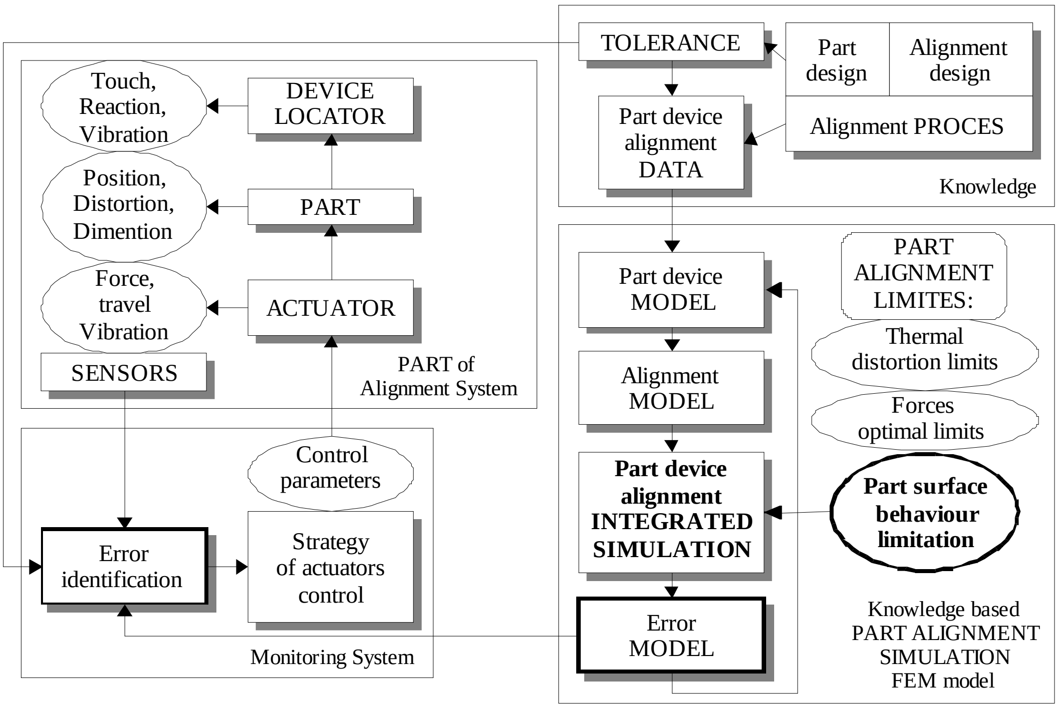

Fig. 2. Part alignment system with monitoring errors

In the analysed case of alignment, displacements take place in two planes: horizontal and vertical (Fig. 3).

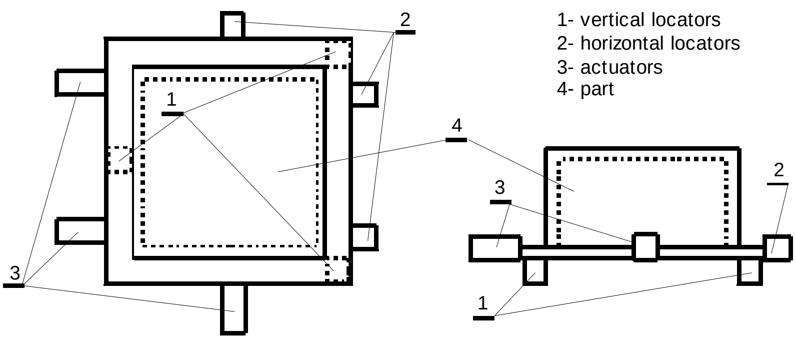

Fig. 3. Alignment device: 1 – alignment locator in the horizontal plane, 2 – alignment locator in the vertical plane, 3 – actuator for positioning in the vertical plane, 4 - workpiece

The source of displacements in the horizontal plane are actuators 3, while in the vertical plane – force of gravity. The constraint of movement in both planes are base elements – locators 1 and 2. During alignment in the horizontal plane, apart from the load from forces exerted by actuators, workpiece 4 is subject to the constant action of the force of gravity. Additionally, friction force on locators 1 also influences the aligning process.

3. INTERFERENCE MODEL OF A FLAT AND A ROUGH SURFACE UNDER THE INFLUENCE OF EXTERNAL LOAD

The condition of locator surfaces should be possibly constant during the alignment process. This imposes specific requirements on the locator surface properties, which should be characterised by a low susceptibility to deformations and by a high resistance to fatigue. In order to fulfill such requirements, it must have low roughness, high hardness and resistance to abrasive wear.

The surface of a workpiece coming in contact with the locator has roughness dependent on the kind of processing. The higher workpiece surface roughness, the smaller contact surface, therefore larger deformations are created under the action of the force exerted by an actuator, that is larger workpiece position changes relative to locators.

A geometry model of the contact between two surfaces was built based on the contact between a rough workpiece surface with a flat locator or actuator tip surface and was used for the analyses of the contact zone. Larger roughness contributed to the decrease of friction in the contact between a rough and a flat surface. Therefore in order to overcome the static friction, compared to the contact of two flat surfaces, lower external forces are needed in the contact of a rough and a flat surface.

3.1. THE APPROXIMATION OF THE SHAPE OF A SINGLE MICRO-ROUGHNESS

The shape of a micro-roughness has the important influence on the participation of a surface bearing ratio of a contact and influences the area of contact [6, 8]. Based on the ellipsoidal models of micro-roughnesses it was proven in paper [6], that with the increase of the curvature R of a single micro-roughness, the contact area increases (Fig. 4). The interference ω signifies the approach of two profiles.

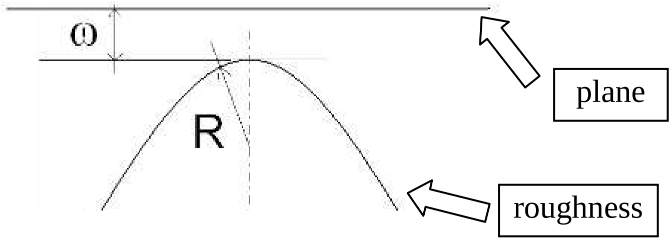

Fig. 4. Model of a single micro-roughness: *R*- radius, ω- interference

Contact models, which were proposed so far, usually reduce the outline of a micro-roughness peak to a semi-sphere. A different approach, proposed and applied in paper [10] and continued by the authors of this paper, is the assumption of a hyperboloidal shape of a micro-roughness, which is described by a more complex mathematical relationship. A similar level of complexity can be met in a model, which substitutes a micro-roughness with a paraboidal shape [8]. The hyperboloidal micro-roughness model has two important characteristics: its equation, most accurately among the ellipsoidal curves, approximates the shape of the real surface after machining [9], and additionally the equation of a theoretical model almost in the entire height of a micro-roughness ensures the least deviations from the actual bearing surface of a micro-roughness [7, 10]. In paper [10] it was proven that besides peaks, the large role in micro-roughness deformations is also played by the sides of a micro-roughness.

3.2. CONTACT MODEL FOR ELASTIC DEFORMATIONS

In order to make it possible to compare the results of modelling obtained by the use of various contact models, e.g.: semi-spherical and from the ellipsoidal family, contact surfaces and mean contact pressure must be made independent from the material properties and the radius of a micro-roughness peak. To this purpose, the models with a dimensionless variable are used, in form of $\omega^*=\omega/\omega_c$ [3], i.e. the ratio of interference value – ω (Fig. 4) to the value of the interference in the beginning of plastic deformations – ω_c.

In the range of elastic deformations, the Hertz theory is useful. It is commonly agreed that elastic deformations under the contact surface develop as early as from $\omega^*=1$. In paper [5], the range of elastic deformations was defined for the value of $\omega^* < 1.9$. In this work it was also proven that in the range of $1< \omega^* < 1.9$ the Hertz model is still little sensitive to the load, while plastic deformations are still relatively low. Therefore for $\omega^* < 1.9$ the contact is treated as elastic, and it is only after exceeding the value of $\omega_t^* = 1.9$ when it becomes elastic-plastic.

In the modelling of the range of elastic contact deformations, the force P_e [N] causing them is described by the following relationship:

$$P_e=\frac{4}{3}ER^{1/2}\omega^{3/2}=P_c\left(\frac{\omega}{\omega_c}\right)^{3/2} \tag{1}$$

where P_c – the contact load respectively at $\omega/\omega_c=1$,

ω – interference value [m],

ω_c – value of ω at the beginning of plastic deformations,

E – Young Modulus [GPa],

R – micro-roughness peak radius [m].

The equation for a dimensionless/normalised contact force, marked by the "*" index, is in the following form: $P_e^*=P_e/P_c$.

3.3. CONTACT MODEL FOR PLASTIC DEFORMATIONS

Mean contact pressure increases nonlinearly together with the deformation of contact until it reaches the value equal to the hardness of the deformed material. Based on the investigations of the surface wear process for the contact in the range of elastic deformations, it was proven in paper [1] that the force in a contact is equal to the hardness multiplied by the contact area, according to the formula $F=H\cdot A_c$. While the contact area A_c is equal to the area of the truncation of the undeformed profile in place of the contact of two touching profiles.

Assuming the isotropic micro-roughness model, the contact surface of a single single-hull hyperboloid of revolution, loaded in the symmetry axis, will be in approximation the area of a circle: $A_c=\pi\cdot(l_{ni}/2)^2$, where l_{ni} is equal to the bearing ratio at the depth of h_n dependent on the interference ω. The index by h_n represents the number of consecutive depth, on which the linear bearing ratio l_n is achieved (e.g.: at h_3, $l_3=l_{31}+l_{32}$). The interference ω smaller than zero means that the contact has not occurred yet. When $\omega=0$, then the contact starts and develops for the increasing values of ω, until the bearing ratio is achieved, which is capable of handling the external load P.

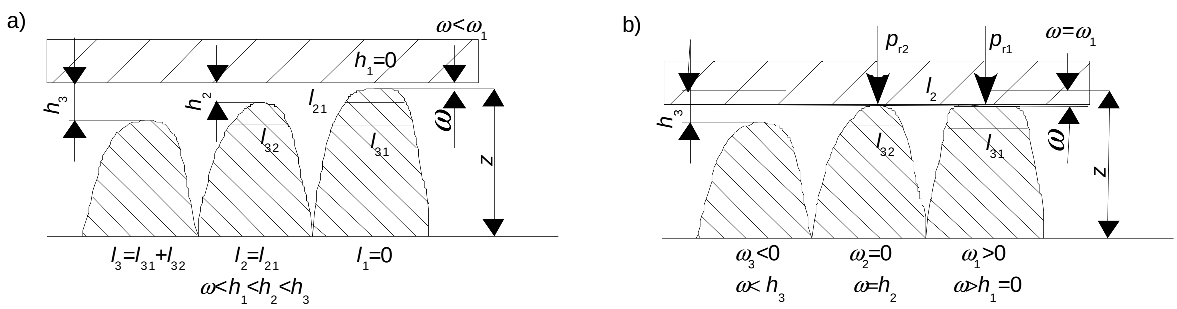

Fig. 5. A rough surface model: a) unloaded, b) under load

3.4. APPROXIMATION OF THE CONTACT PRESSURE FOR THE ELASTO-PLASTIC MODEL

In paper [5] the influence of material properties on the mean contact pressure between the semi-sphere and the surface is shown. A relationship has been achieved, describing the boundary value of the mean pressure – yield strength, H_g/Y, as a function of material properties E, Y and ν:

$$\frac{H_g}{Y}=2.84\left[1-\exp\left(-0.82\left(\frac{\pi\cdot C\cdot e_y}{2}\sqrt{\omega^*}\left(\frac{\omega^*}{\omega_t^*}\right)^{B/2}\right)^{-0.7}\right)\right] \tag{2}$$

where H_g – hardness geometric limit, Y – yield strength [MPa], C – critical yield stress coefficient, dependent on ν, ν – Poisson's ratio, $e_y=Y/E'$ – uniaxial yield strain,

E' – equivalent Young modulus, ω^* – dimensionless interference, B – constant dependent on e_y, ω_t^* – value representing the point of transition from elastic to elastic-plastic deformations.

Thanks to it, mean contact pressure for the elastic-plastic range can be approximated by means of the following equation:

$$P_F^* = [\exp(-\frac{1}{4}(\omega^*)^{5/12})](\omega^*)^{3/2} + \frac{4 \cdot H_G}{Y}[1-\exp(-\frac{1}{25}(\omega^*)^{5/9})]\omega^* \quad (3)$$

where symbol “*” indicates the normalised value, related to the value in the beginning of the plastic deformations [3] and is a dimensionless value.

4. HYPERBOLOIDAL SURFACE PROFILE MODEL ACCORDING TO THE BEARING RATIO CURVE

The actual contact area is lower than the nominal surface area of contacting parts [8]. This reflects the structure of the surface layer, which is also described by surface roughness [11]. Data about the participation in bearing, determined based on the Abbot-Firestone curve, as well as data about the amount of roughness peaks on different depths h is easily obtained from the surface profilograms measured with the use of a profilometer. In order to take into consideration the changes of surface bearing in a contact during alignment, a rough surface model was proposed, based on the bearing ratio curve.

4.1. ALGORITHM FOR DETERMINING THE SURFACE PROFILE MODEL

The algorithm for building a rough surface model (Fig. 6) takes into considerations the selected, actual parameters of a rough surface, i.e.: surface bearing capacity – L_N and the amount of micro-roughness peaks – *pic*, as well as the actual shape of micro-roughnesses, approximated to hyperboloidal.

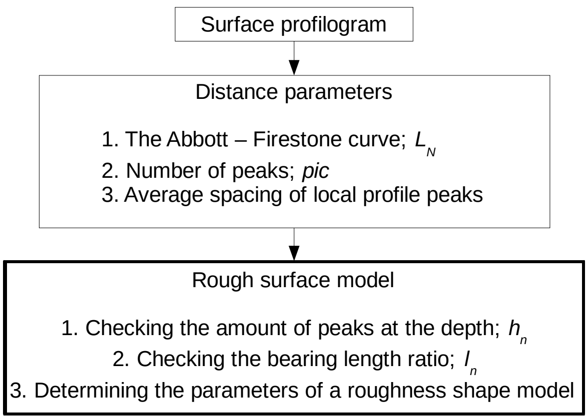

Fig. 6. Algorithm for building a rough surface model

In the algorithm shown on Fig. 6, two elements were distinguished: a surface profilogram and distance parameters, which lead to obtaining a rough surface model. The basis for such algorithm is an eccentric of a hyperbola, i.e. such selection of hyperbola semi-axes a and b, so the bearing capacity l_n of a model and a real surface L_N are not much different on depths h_n in the entire sub-zone layer (Table 1). The facilitation in case of such solution is a non-linearly changing bearing capacity of a hyperbola l_{ni} in function of the distance to its peak. For different parameters a_i, b_i it is possible to obtain the wide range of load bearing values l_{ni} and as a result to minimise the deviation (see Table 1) between the bearing capacity of a model - l_n and the actual bearing capacity from the Abbott-Firestone curve, L_N.

Table 1. Deviation of the bearing capacity l_n from the bearing capacity L_N, obtained from the Abbott-Firestone curve on various depths h

	Depth h_n		a_i	b_i	pic	l_{ni}	l_n	Deviation $\lvert L_N - l_n \rvert$	fault
	nr n	distance ω							
SUB ZONE	1	0	3	67	1	0	0	0	0
	2	3,55	1	92	4	1,98	1,98	0,01	0,56%
	3	7,1	2	71	4	0,8	5,18	0,03	0,54%
	4	10,65	2	32	2	2,27	14,27	0,94	6,60%
	5	14,2	2	26	2	4,17	22,6	2,37	10,49%
	6	17,75	-	-	-	5,41	33,43	5,16	15,45%

Thanks to a hyperboloidal shape of microroughnesses, the rough surface model contains information about the actual surface bearing capacity change, used in the FEM.

4.2. MAIN ASSUMPTIONS FOR A GEOMETRICAL SURFACE PROFILE MODEL

In a geometrical surface model it was assumed that there are no interactions between the microroughnesses. From paper [6] it is known that the magnitude of deformations is in the range of 40% of a maximum roughness R_t. It is assumed that the peaks of micro-roughnesses from the range of $0.4{\cdot}R_t$ are separated from each other and their deformations do not affect each other. It was preliminarily assumed that in the remaining part of the R_t range, micro-roughnesses do not participate in the deformations with a flat surface, because of the increase of the influence of the amount of pits. Although there is a possibility that the micro-roughnesses from lower depths h_n (from out of the range of $0.4{\cdot}R_t$) will be directly participating in bearing of stresses. Because of their small amount it is assumed that it has the unimportant influence on the positioning error and therefore it was assumed that the rest of the R_t range does not participate in the contact.

In a FEM model, the bottoms of micro-roughnesses have the possibility of translation in the plane perpendicular to the load. This results from the assumption about no mutual interactions between micro-roughnesses. Micro-roughnesses, which peaks, in the amount dependent on depth, create a sub-zone (40% R_t) in the FEM model. The sub-zone layer makes up only a part of the entire surface layer. For it, hardening characteristics same as for a soft steel were assumed (Table 2.)

Table 2. Yielding parameters

Yield Stress [MPa]	Plastic Strain [%]
300	0
350	0.25
375	10
394	20
400	35

The division of a material in layers, because of the complex structure of a surface layer [2], consists in a significant facilitation in the FEM modelling. Layers are characterised by different material properties than the core, appropriate for a specific kind of material. The core itself is usually treated as a bulk material. With the knowledge of hardness changes in a function of the distance from the outer surface, the rough surface profile model can be extended to successive layers. The analysis of phenomena taking place there and their mutual interaction will be possible then.

5. SEMI-SPHERICAL SURFACE PROFILE MODEL ACCORDING TO THE BEARING RATIO CURVE

The Hertz solution assumes that interferences ω are small and the geometry of a single micro-roughness does not change significantly. During the alignment process it is easy to damage the surface of a workpiece as a result of the action of the external forces. The material of locators is harder and has a smaller roughness than the workpiece. In the analysis, main focus was made to the elastic-plastic and pure plastic deformations of a workpiece. It was assumed that, in alignment, the participation of plastic deformations in a contact is significant and larger from the pure elastic deformations. Such assumption was based on the analysis of determined interferences ω under influence of the external load. The interference ω_c, at which plastic deformations start to occur, is described by the following formula:

$$\omega_c = \left(\frac{\pi \cdot C \cdot Y}{2 \cdot E'}\right)^2 \cdot R \tag{4}$$

where C – constant dependent from ν, ν– Poisson's ratio, Y – yield strength, E' – equivalent Young modulus ($1/E'=(1-\nu_1^2)/E_1+(1-\nu_2^2)/E_2$), R – micro-roughness peak radius ($1/R=1/R_1+1/R_2$) , indexes 1 and 2 apply to two contacting surfaces.

At the same time, the value of mean pressure in a contact, at which plastic deformations start to occur, is equal to:

$$P_c = \frac{4}{3} \cdot E' \cdot R^{1/2} \cdot \omega_c^{3/2} \tag{5}$$

The amount of micro-roughnesses elastically and plastically deformed can be connected to the interference of both surfaces ω. The model according to the bearing ratio curve carries information about the amount of micro-roughnesses in a roughness sub-zone. By checking the interference value ω^* it is possible to determine the condition of deformed micro-roughnesses. Similarly as in paper [5] it was assumed that for the elastic-plastic state ω^*=1.9 and for the pure plastic state - ω^* =110.

When analysing the test piece, finished by machining in the 5th class of precision with R_z=17μm it was found that the most micro-roughnesses are on the level of $0.2 \cdot R_t$, i.e. ca. 4μm. In the range to $0.4 \cdot R_t$, ca. 97% of micro-roughness peaks is located. The remaining of the R_t range is taken by pits.

A contact model was analysed, between a flat and a rough surface, which profile is characterised by a certain known change of a bearing ratio curve. Bearing profile for the range of 0.4 R_t was approximated by a semi-sphere model. Maximum deviation from the bearing profile for such model is about 30%, while mean deviation does not exceed 5%. The contact between a plane and a semi-sphere with radius of $R=4 \cdot 10^{-5}$m was analysed, representing different roughnesses from the range of $0.4 \cdot R_t$. It was assumed that the roughness is isotropic, while material properties typical for steel: E=210GPa, ν=0.32, Y=0.30GPa. It results from the calculations that the interference ω_c, at which the elastic deformations start to appear is equal to $\omega_{(\omega^*)}=\omega_{(1)}=4.36 \cdot 10^{-10}$m, while the interference, at which the respective states appear are: elastic-plastic state at $\omega_{(1.9)}=8.28 \cdot 10^{-10}$m and pure plastic state at $\omega_{(110)}=4.80 \cdot 10^{-8}$m. With such interferences, the change of micro-roughness deformation states takes place, causing stresses in the bulk material.

The locator-workpiece and actuator-workpiece contact consists of many such contact zones, as the above presented hyperboloidal model. If the measurement section of a model is equal to $8 \cdot 10^{-5}$m, then on the actual surface, e.g.: equal to $441 \cdot 10^{-6}$m^2, the interference ω_c would equal to proportionally more, that is $\omega_c=\omega_{(1)}=1.14 \cdot 10^{-7}$m.

6. THE INFLUENCE OF MICRO-ROUGHNESS SHAPE OF A ROUGH CONTACT AREA ON THE ALIGNMENT PRECISION

Taking into consideration the workpiece mass, as well as formerly presented analyses, it can be assumed that the large influence on the precision in the alignment process will have mainly plastic deformations in a contact. The type of deformations in the contact is dependent on the elementary pressure. The elementary pressure decreases as the area of contact becomes larger. Therefore in the contact of locators or actuators with a workpiece, elastic-plastic deformations become significant in the moment of defining enough large nominal area of contact. Precisely modelled surface influences the results of deformation change courses, caused by load. Based on the model precision and simulation results, it is possible to define the precision of the alignment process.

With the use of a FEM model it was possible to define the influence of roughness shape type on the relationship between the force and the displacement (Fig. 7). This graph was obtained for earlier analysed models of the interference of a flat surface with a surface approximated to a semi-sphere (Section 5) and to a hyperboloid (Section 4) The modelled micro-roughnesses had the same mechanical properties. The approach of a flat surface took place as a result of roughness deformations under the influence of the actuator load. In both cases the external force from the actuator was normal to the approximated surface and acted in its axis of symmetry. On account of the ratio of the radius R to the contact area radius, which for a semi-sphere cannot be larger than 1, the investigations were restricted to the interferences ω from 0 to $10\cdot10^{-6}$m. In case of a semi-sphere, the force needed for the interference of a flat surface into a rough surface by 10 micrometers may differ even by 300% from the hyperboloidal model, built based on the bearing ratio curve.

Therefore the selection of a proper micro-roughness shape plays an important role in contact modelling, which then influences the alignment precision.

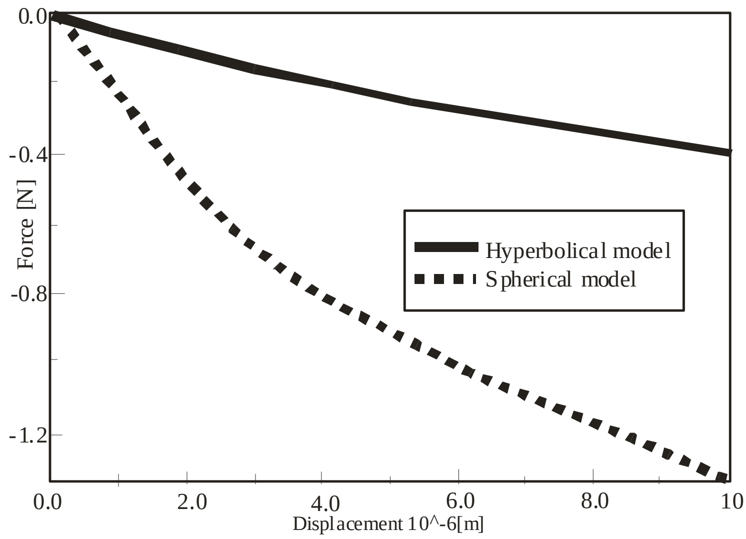

Fig. 7. Two shape distortion models

7. CONCLUSIONS

Modelling of phenomena in the contact between the workpiece and locator/actuator is necessary with the regard of defining the precise value of the actuator force and reaction values on these locators. Precise knowledge about loads and bearing capacity of surfaces, from which the actual contact area is dependent, can be used for the selection of surface bearing capacity and the nominal areas of locator/actuator surfaces. Such surface must be selected in order to be able to bear the load without any excessive contact deformations and without any workpiece deformations during alignment.

The contact model was reduced to modelling the contact between a rough surface made of many single hyperboloids and a flat surface with the nominal area of locators/actuators. Thanks to the Wrocław Centre for Networking and Supercomputing, calculations with the use of Abaqus/Standard system allowed comparing the models with different micro-roughness shapes. The selection of a shape from the ellipsoidal family is more reasonable in case of the analysed contact is misaligned or when the bearing capacity is to be taken into considerations. Deformations determined for a semi-spherical shape are limited by the ratio of the semi-sphere radius to the radius of a contact area. In case of the model consists of many individual semi-spheres, which peaks lie on different heights, the interferences larger than the radii of the highest located, smallest semi-spheres are also a limitation. Hyperboloidal shape does not pose such limitations and it is possible to apply it in more complex investigations, in which, besides peaks, also sides of micro-roughnesses play an important role.

The model of a contact between a rough surface with a flat one is still being developed and expanded. The next step will be planned construction of a test stand and the verification of the results obtained from the theoretical model with the results from experiments on a real object.

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