

Research Paper

Study of Effect of Linear Tip Relief Modification in Power Transmission Efficiency of Spur Gears

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This paper explores the influence of linear gear tip relief modification on power transmission efficiency. In real time applications gears experience transmission error (TE) during operation which increases noise and vibration and also results in increased tooth profile deformation during operation of the gear. By providing tip relief profile modification this TE can be decreased. Using MATLAB for computation and ANSYS for the simulation of deformation, stress, strain, life, and factor of safety results for the gear assemblies are obtained. Deformation results are used for the computation change in power transmission efficiency followed by the modal and harmonic analysis of the gears and gear assemblies to determine change in the first mode of natural frequency.

Keywords: linear gear tip relief profile modification; tooth profile deformation; transmission efficiency; transmission error (TE).

1. Introduction

Power transmission is one of the key components of any automobile or industrial setup. Among various modes of power transmission such as belts, pulleys, chains, etc., gears remain to be the simplest and viable option for efficient power transmission. However, during this process gears do experience several losses such as frictional, thermal, and transmission loss. Change of tooth profile, i.e. involute profile with characteristic improvement in performance of the gear, is known as profile modification (MEI *et al.*, 2016). The calculation of gear loading capacity is based on the assumption that gear teeth are absolutely rigid and that the

gear tooth profile is ideally accurate. However, during transmission, actual gear teeth experience deflection as a result of loading. These deflections result in deviations of the relative motion of tooth flanks from the theoretical one and consequently generate transmission errors. Therefore, the tip relief profile modification is implemented to compensate for the deflections of the tooth tip expected under load, which results in reduced transmission errors (MARKOVIĆ, VRCAN, 2016). The Transmission Error (TE) is defined as the difference between the positions of the output shaft in effective and the ideal case with reference to the input shaft in a gear assembly. The ideal position indicates a perfect gear set with errors and deformations. Generally, an-

gular or linear displacement along the line of action of base circle is used to express TE (BEGHINI *et al.*, 2006).

Transmission error arises due to tooth geometry errors, i.e. profile error, pitch error, and errors from the manufacturing process such as runout errors, elastic deformation, and imperfect mounting. Elastic deformation happens because of the load being transmitted transversely to the gear axis causing local contact deformation at each meshing point, which results in deflections of teeth. Imperfect mounting due to misalignment during mounting gives rise to geometric errors. Static and dynamic deflections in bearings and shafts can also induce geometric errors.

In order to overcome the TE tip relief, modification is done. Linear and parabolic modifications are currently studied the most. Linear profile modifications result in a small transmission error for a known load (NIEMANN, WINTER, 1983). On the other hand, a smooth tooth flank curve results from the parabolic tip relief modification which is suitable for variable loads but leads to an increase in TE (BEGHINI *et al.*, 2004). For the current work, linear tip relief modification is considered. This paper deals with the effect of this profile modification on power transmission efficiency during transmission.

BANODIYA and KARMA (2017) conclude TE to be one of the main causes of noise and vibration in gears. The paper aims to understand the cause and effect of TE. An experimental setup used to do the measurement of TE in the system is introduced. The results from the setup profile modification with the use of an automated profile generation tool became the basis for TE reduction. Adverse effects of TE and the factors which help in generating TE are discussed.

In (BEGHINI *et al.*, 2004), the authors discuss the issue of tip relief modification as well as its effect in surface loading of the tooth flank. They consider this modification to be dangerous as a result of contact pressure which is the main reason for the activation of micro-pitting. An effective profile on the basis of direct measurements is defined. As a result, they were able to determine more realistic contact pressure at start relief profile modification MARKOVIĆ and VRCAN (2016) analyse the effects of linear tip relief modification, especially with respect to the stress of tooth root and tooth flank in involute spur gears. To study the effects of this modification on gear stress, two matching FE models of the involute spur gear pairs were created, and tip relief profile modification was performed on one pair. Stresses at the tooth root and flank were compared to establish the effect of the tip relief modification.

BEGHINI *et al.* (2006) present a new topography for tip relief modification which is a linear profile modification with a parabolic fillet resulting in a Linear-Parabolic modification. Parabolic fillet taken as a parameter and its effects are investigated. Taking into ac-

count the contact boundary conditions and the anomalous contact conditions that were developed, maps were made to obtain optimum tip relief.

In (FAGGIONI *et al.*, 2011), the authors show a global optimisation method for reducing gear vibration with the help of profile modifications. A nonlinear dynamic model was developed by them for the vibrational behaviour study and the model from previous literature was validated. The objective function of their studies was the minimisation of static TE or amplitude minimisation of vibration. They developed a Random-Simplex optimisation algorithm to find the optimal profile of spur gear pair to reduce the vibration for maximum range of operating condition. The Monte Carlo simulation was used to calculate the optimum reliability. The result of their studies shows good efficiency of the transmission system as well as reliable performance. The authors presented the application of a high contact ratio (HCR) gear and applied the above algorithm to get optimal parameters, which resulted in an extremely good performance.

The (SANKAR *et al.*, 2010) paper analyses the increase in tooth strength of a spur gear as a result of profile modification. The root fillet was used as the design variable in increasing tooth strength. The authors suggested the use of a circular instead of a trochoidal root fillet for spur gears having less than 17 teeth. The effects of the different types of fillets used were analysed using ANSYS version 11.0 and it was concluded that the circular root fillet is better for spur gears having less than 17 teeth. The results from the modified and standard gears were compared. The circular root fillet proved to be better for lesser number of teeth while the trochoidal root fillet was suitable for more than 17 number of teeth.

The paper by THARMAKULASINGAM (2009) discusses in detail the effect of tooth profile modification on TE of spur gears. Analysis and simulation of a finite element modelled spur gear was done under static conditions. Differences in static and dynamic TE were observed during the meshing and analysis of a spur gear pair model and the differences were investigated in detail.

GUPTA *et al.* (2012) performed FE analysis using ANSYS 13 on spur gear model. The authors were able to obtain results of the contact stress between the two spur gears. These results were then compared with results from the Hertzian theoretical equation. They proved that resistance to pitting failure can be improved by increasing the hardness of the gear tooth. Increase in module was found to indicate a decrease in maximum contact stress. A higher module spur gear was found to be preferred if contact stress minimisation was a concern, along with transmission of large amount of power.

In (TIWARI, JOSHI, 2012), the authors discussed and compared the results of contact as well as bend-

ing stresses for involute spur gears in mesh obtained using AGMA/ANSI equations, Hertz equation, and Lewis formula with that obtained using finite element method. It was concluded that Lewis formula is the basic formula for calculating bending stress and Hertz theory for determination of contact stress.

MANURWAR *et al.* (2017) is an overview of various literature regarding profile modification and its effects. The authors also discuss issues of optimisation of the gear tooth profile to get optimal gear parameters in order to maximise the power transmitting capacity of the system. They come to a conclusion about the necessity of profile optimisation and the need for optimal gear tooth profile to decrease transmission error and to decrease the effects of vibration in the system.

The following research gap has been identified: power transmission efficiency changes for tip relief modified spur gear in comparison to nonmodified gear has not yet been done. The current research problem can be articulated as follows: “Does applying linear tip relief modification to spur gears indicate changes in power transmission efficiency?”.

2. Methodology

The involute gear along with the gear with linear tip relief modification are both to be designed in SolidWorks. Figure 1 by (MARKOVIĆ, VRCAN, 2016) describes the method of how the tip relief modification is performed.

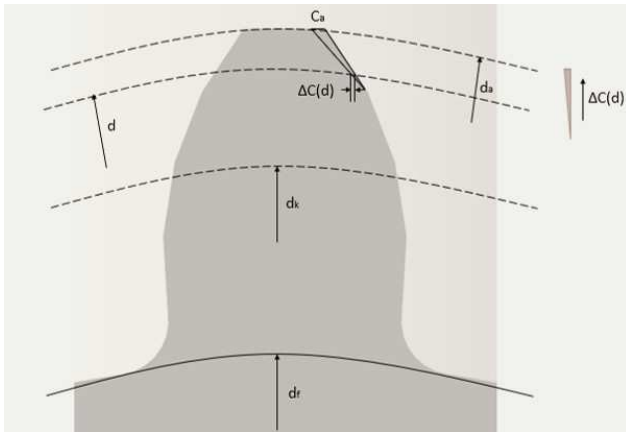


Fig. 1. Linear tip relief profile modification.

The subsequent formulas and variables for tip relief profile modification, deformation due to Hertzian stress, elastic deformation of spur gear teeth, and tooth tip relief have been taken from (MARKOVIĆ, VRCAN, 2016).

2.1. Tip relief profile modification

Tip relief modification can be defined as the removal of material of thickness $\Delta C(d)$ along the tooth

flank of a particular diameter d with reference to the nominal involute profile.

$$\Delta C(d) = C_a \frac{d - d_k}{d_a - d_k}. \quad (1)$$

Deformation due to Hertzian stress:

$$\delta_{H1,2} = \frac{2F_{bti} (1 - \nu^2)}{\pi b E} \left(1.27 + 0.781 \ln \frac{m_n}{b_H} \right). \quad (2)$$

Elastic deformation of spur gear teeth:

$$\delta_{b1,2} = \frac{2F_{bti} (1 - \nu^2)}{b E} (A + B e^{C \bar{y}_p} + D), \quad (3)$$

where

$$A = -1.05 + 153 e^{-8.1x} z^{-(1.75-1.6x)}, \quad (4)$$

$$B = 0.63 + (7.35 - 0.924x) z^{-1}, \quad (5)$$

$$C = 1.28 - (2.88 + 3.68x) z^{-1}, \quad (6)$$

$$D = -1.06 + 0.638 \ln(m_n z), \quad (7)$$

$$\frac{\bar{y}_p}{m_n} = \frac{y_p}{m_n} = \frac{r_p \cos(\alpha_b - \omega_b) - r_f}{m_n}, \quad (8)$$

$$\alpha_b = \arctan \frac{\rho}{r_p}, \quad (9)$$

$$r_p = \arctan \frac{r_b}{\cos \alpha_b}, \quad (10)$$

$$r_b = \frac{m_n}{2} (z + x - 2.5), \quad (11)$$

$$\omega_b = \frac{\rho}{r_b} - \varphi, \quad (12)$$

$$\varphi = \frac{(4x \tan \alpha_n)}{2z} + \text{inv} \alpha_n, \quad (13)$$

$$r_f = \frac{m_n z}{2} \cos \alpha_n, \quad (14)$$

$$\text{inv} \alpha_n = \tan \alpha_n - \alpha_n. \quad (15)$$

Tooth tip relief:

$$C_{a1,2} = \delta_{1,2} = \delta_{b1,2} + \frac{1}{2} \delta_{H1,2}. \quad (16)$$

2.2. Design of gear

The initial design of gears was done using manual calculations. The design torque was chosen from the data of a W16 engine used in the Bugatti Chiron which gives an output torque of 1600 Nm in 2000–6700 rpm range. The data used for the calculation are presented in Table 1.

The material used for the gear was chosen as AISI 4340 steel as opposed to structural steel, which is the default material used in ANSYS. AISI 4340 has better structural strength compared to structural steel.

Table 1. Input data for gear design.

Design torque, M	1600 Nm
Speed, N	6700 rpm
Gear ratio, i	1
Module, m	5

Moreover, AISI 4340 is sought for its toughness and its ability to sustain high stress. These properties are valuable especially in gears since they are subjected to high stress during their functioning. The values for the physical properties of AISI 4340 were taken from SolidWorks and input as a custom material in ANSYS.

Table 2. Physical properties for AISI 4340.

Poisson’s ratio	0.285
Elastic modulus	$2.05 \cdot 10^{11}$ Pa
Thermal conductivity	44.5 W/(m·K)
Mass density	7850 kg/m ³
Tensile strength	$7.45 \cdot 10^8$ Pa
Shear modulus	$8 \cdot 10^{10}$ Pa
Specific heat	475 J/(kg·K)
Thermal expansion coefficient	$1.23 \cdot 10^{-5}$ K ⁻¹
Yield strength	$4.7 \cdot 10^8$ Pa

The gear was designed using the design torque (mentioned above) using (ACHCHAGAM, 2017). The dimensions of the gear obtained after calculations are as shown in Table 3.

2.3. Calculation of tooth tip relief modification

The equations were input into MATLAB to generate the relief modification at various diameters along the tooth flank. MATLAB was chosen over manual cal-

Table 3. Dimensions of gear.

Number of teeth, Z	60
Face width, b	90 mm
Root circle diameter, d_b	287.5 mm
Tip circle diameter, d_a	310 mm
Pitch circle diameter, d_k	300 mm
Pressure angle	20°
Contact ratio	1
Addendum	5 mm
Dedendum	6.25 mm

culatation due to ease of use, better accuracy of results, and the fast computation speed that it provides. Table 4 shows the relief modifications at various diameters along the tooth flank (in mm).

Table 4. Reliefs at various diameters along tooth flank.

Sr. No.	Diameters	Relief	Sr. No.	Diameters	Relief
1	300.0000	0.0000	6	305.4317	0.6061
2	300.8064	0.0900	7	306.6581	0.7430
3	301.9200	0.2143	8	307.9118	0.8829
4	303.0623	0.3417	9	309.1925	1.0258
5	304.2330	0.4724	10	310.0000	1.1159

2.4. Modelling of gears in SolidWorks

GearTrax is an add-on to SolidWorks used to produce gears of good accuracy that are used in transmission systems in team vehicles. Thus, it was used to generate the gears required for the paper.

Figure 2 shows the interface of GearTrax which is the software used for the design of the gear assem-

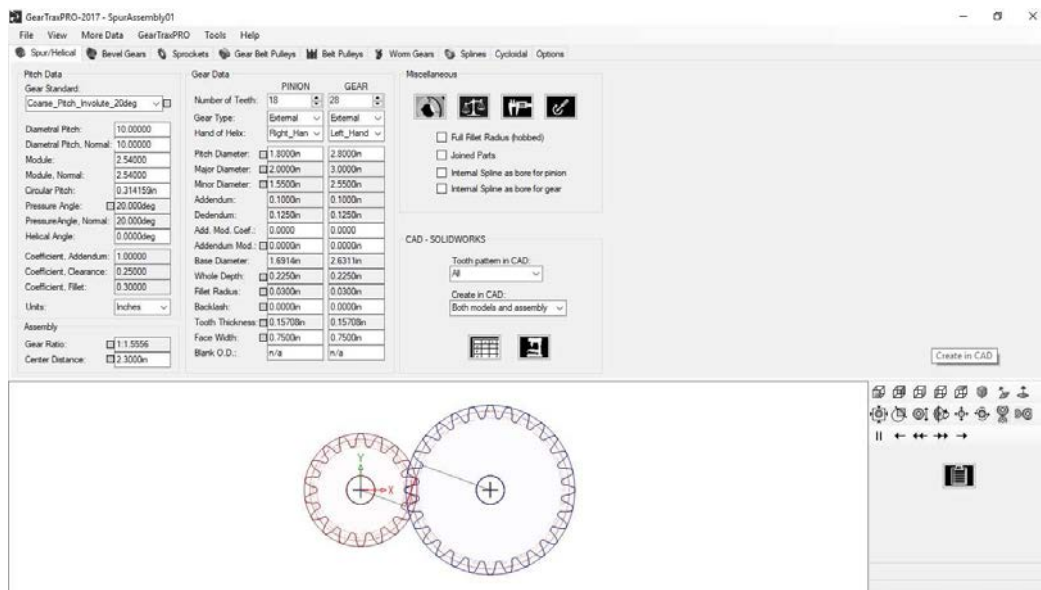


Fig. 2. Gear trax interface.

bly. In the SolidWorks model of the gear and pinion, changes can be made to the profile of the gear tooth. Thus using the output from MATLAB, modification was made to the involute tooth profile of both the spur and pinion. Figure 3 shows the general gear assembly generated in SolidWorks using GearTrax.

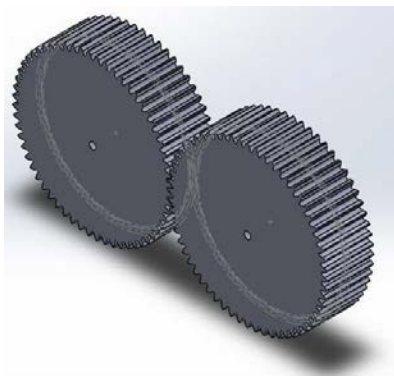


Fig. 3. GearTrax generated gear assembly.

Two separate gear assemblies were created; in the first one the gear tooth profile was involute and the other one had a modified tooth profile for both pinion and gear. Figure 4 shows both the involute and modified gear tooth profiles. Once the modification was done to the tooth profiles the changes automatically took place in the respective assemblies as well.

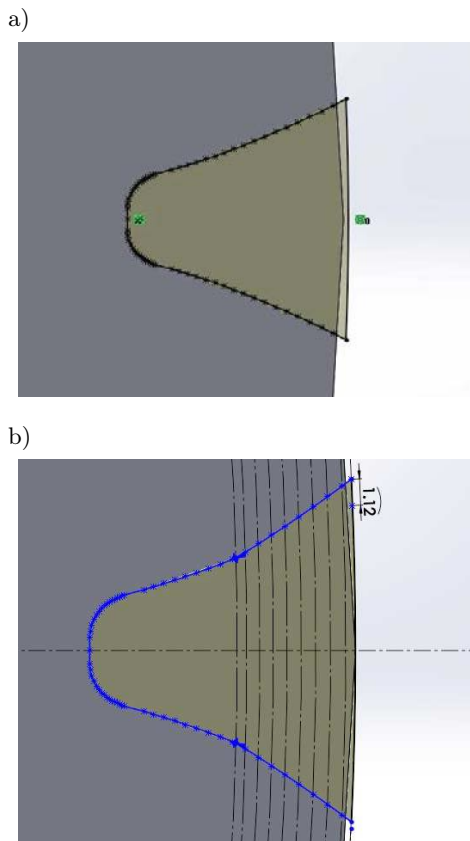


Fig. 4. Involute tooth profile (a) and modified tooth profile (b).

The assembly, as well as the gear model, were saved in the parasolid (x.t) format. Parasolid was chosen over IGES format because ANSYS is able to work on parasolid format faster than it does on IGES format.

2.5. Analysis in ANSYS

The following analyses were performed on each of the gear assemblies:

- 1) static structural,
- 2) modal analysis,
- 3) harmonic analysis.

After this, the pinion of each of the assembly was taken individually and a modal analysis was performed.

2.5.1. Static structural analysis

The assembly was imported in x.t (parasolid format). During meshing the entire system was given a quad element mesh of size 8 mm using the sweep method. Also to increase the fineness of the mesh at the contact region, a 1 mm contact sizing was given. The pinion was given fixed support in the centre, while the gear was given frictionless support. Also, the faces of the gear were given a remote displacement condition of zero displacements in x , y , z directions so that the gear remained in one place during the analysis. A moment of $1600 \text{ N} \cdot \text{m}$ was given to the gear in a clockwise direction so that it did not cause significant inertia and dampening effects.

The following solution parameters were found: total deformation, Von-Mises stress, equivalent strain, life, and factor of safety.

2.5.2. Modal analysis

Modal analysis is performed to determine the natural frequencies of the assembly. The analysis shows us different modes of the system under vibration. In a freely vibrating system, a mode represents the oscillations that are restricted to certain characteristic frequencies. These are known as normal modes. Here modal analysis is performed on both the two assemblies and the gears individually.

Modal analysis was done to determine the first 3 modes or natural frequency of the system and the pinion. During analysis for the system, only supports were applied, i.e., fixed and frictionless support, while the pinion analysis was done using the fixed support condition for each of the centres of the pinion.

2.5.3. Harmonic analysis

Harmonic analysis predicts the dynamic response of a structure in a steady state when it is subjected to sinusoidal varying loads of different frequencies and modes.

The frequency response of different parameters such as deformation, stress, and strain during the operation of the assembly in the presence of vibrational forces can be determined from this analysis. The frequency responses of these parameters were taken at the tip of the tooth in each case, i.e., for the involute and modified spur gear assemblies. For this analysis the first natural frequency for each of the assemblies was considered and the behaviour of the system parameters was observed.

3. Results and discussions

3.1. Static structural analysis

3.1.1. Deformation

Figure 5 shows the total deformation results for the involute spur gear tooth at the tip, pitch, and root. Figure 6 shows the total deformation results for the modified spur gear tooth at the tip, pitch, and root.

Deformations are found to be $5.0114 \cdot 10^{-5}$ m, $5.1144 \cdot 10^{-5}$ m, and $4.9565 \cdot 10^{-5}$ m at the tip, pitch,

and root, respectively, for the involute spur gear, and $6.5301 \cdot 10^{-5}$ m, $6.5265 \cdot 10^{-5}$ m, and $6.6184 \cdot 10^{-5}$ m at the tip, pitch, and root, respectively, for the modified profile spur gear. The profile modification has shown that the deformation has increased, especially at the root of the teeth.

3.1.2. Strain

Figure 7 shows the strain results for the involute spur gear tooth at the tip, pitch, and root. Figure 8 shows the strain results for the modified spur gear tooth at the tip, pitch, and root.

Strain is found to be $4.5981 \cdot 10^{-5}$, $1.6721 \cdot 10^{-4}$, and $3.6276 \cdot 10^{-5}$ along the tip, pitch, and root, respectively, for the involute spur gear, and $2.3999 \cdot 10^{-6}$, $3.385 \cdot 10^{-4}$, and $3.1583 \cdot 10^{-4}$ along the tip, pitch, and root, respectively, for the modified profile spur gear. The strain at the pitch of the modified teeth has increased due to the modification. However, the results at the tip of the modified teeth indicate that the strain has been significantly reduced. There is a higher chance of failure at the tip of the teeth and hence a lesser strain indicates a better tooth life.



Fig. 5. Deformation results of the involute tooth profile.



Fig. 6. Deformation results of the modified tooth profile.

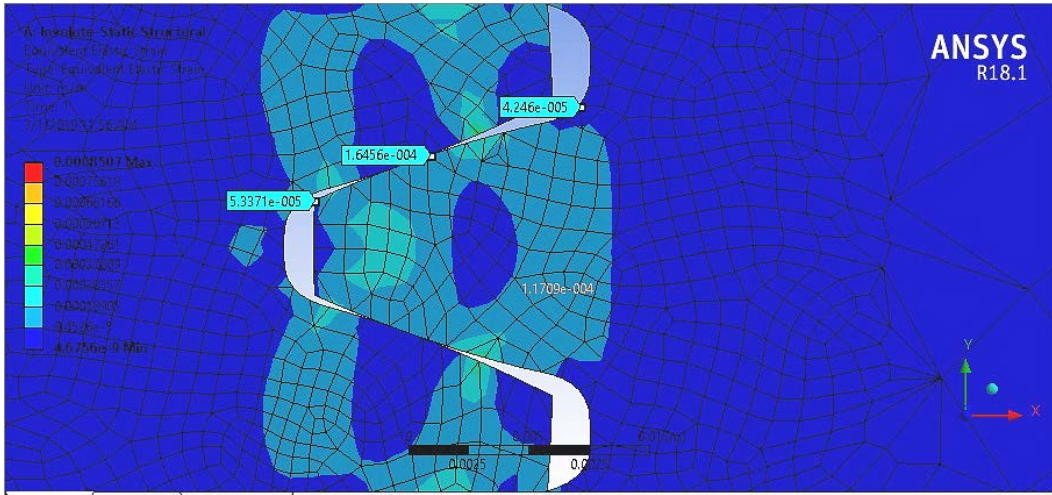


Fig. 7. Strain results of involute tooth profile.

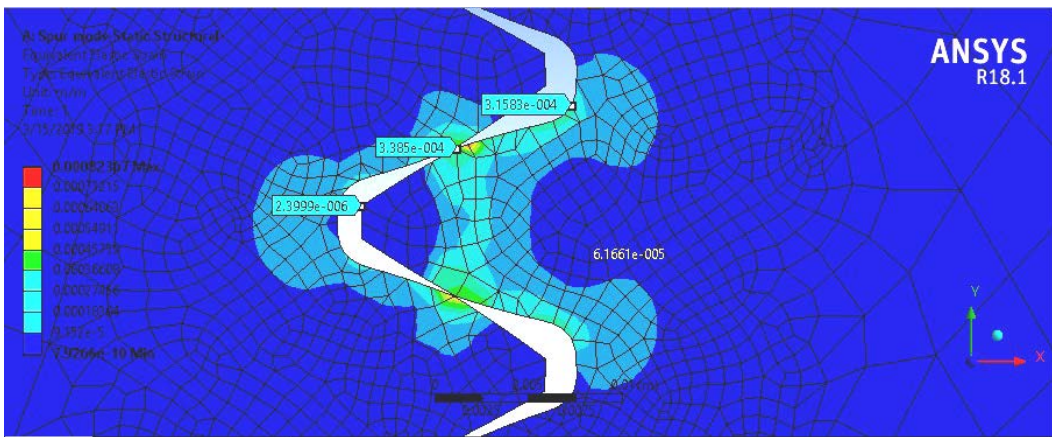


Fig. 8. Strain results of modified tooth profile.

3.1.3. Stress

Figure 9 shows the stress results for the involute spur gear tooth at the tip, pitch, and root. Figure 10 shows the stress results for the modified spur gear tooth at the tip, pitch, and root.

Stresses are found to be $8.1822 \cdot 10^6$ Pa, $3.3959 \cdot 10^7$ Pa, and $7.6997 \cdot 10^6$ Pa at the tip, pitch, and root, respectively, for the involute spur gear, and $4.8747 \cdot 10^5$ Pa, $1.4999 \cdot 10^7$ Pa, and $6.1565 \cdot 10^7$ Pa at the tip, pitch, and root, respectively, for the modified profile spur gear. The linear modification has reduced

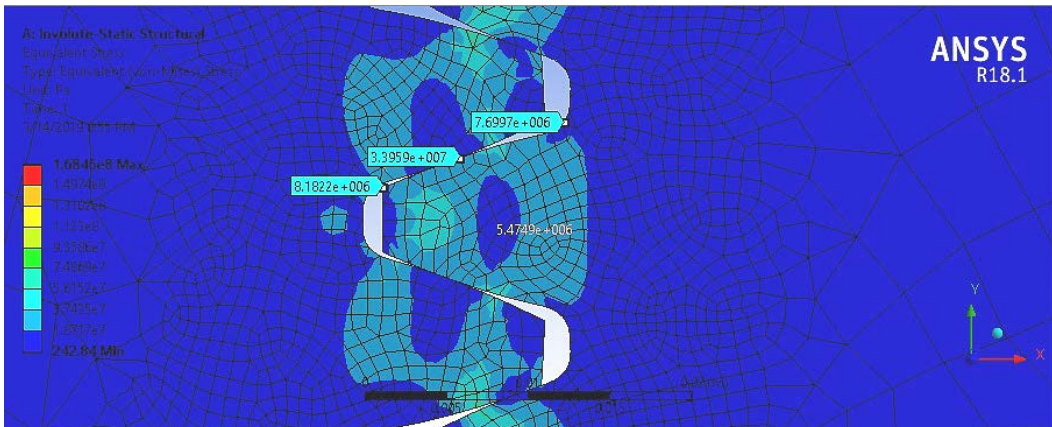


Fig. 9. Stress results of involute tooth profile.

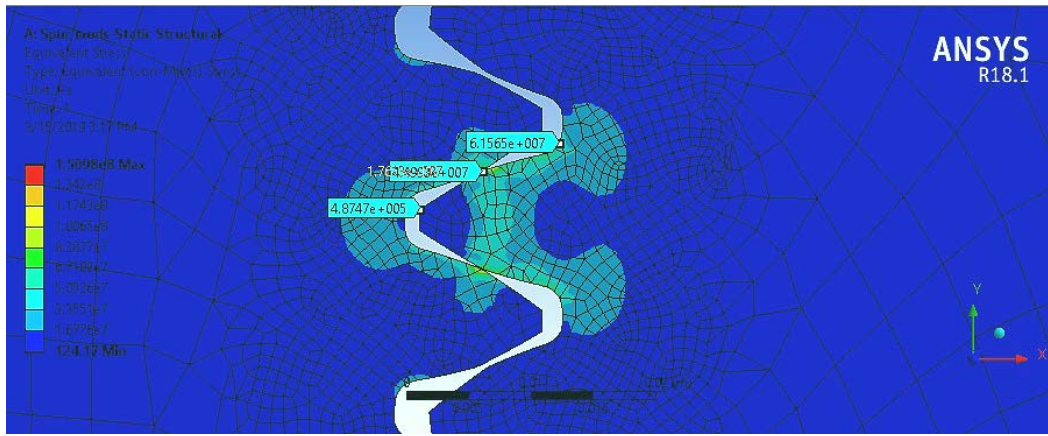


Fig. 10. Stress results of modified tooth profile.

the stresses at the tip, pitch, and root of the tooth, which increases the life of the teeth.

3.1.4. Life

Figure 11 shows the life results for the involute spur gear tooth and the modified spur gear tooth at the tip, pitch, and root.

Minimum life of the assemblies was found to be 48116 and 71911 cycles respectively for the involute and modified profile spur gear. The life of the modified

assembly has shown an increase primarily due to the reduction in the stresses and strains at various locations along the tooth and especially at the tooth tip.

3.2. Modal analysis

Modal analysis is used to find the first 3 modes or natural frequencies of both sets of gears and both sets of assemblies. The obtained results are then used in the harmonic analysis for observing frequency response of different parameters.

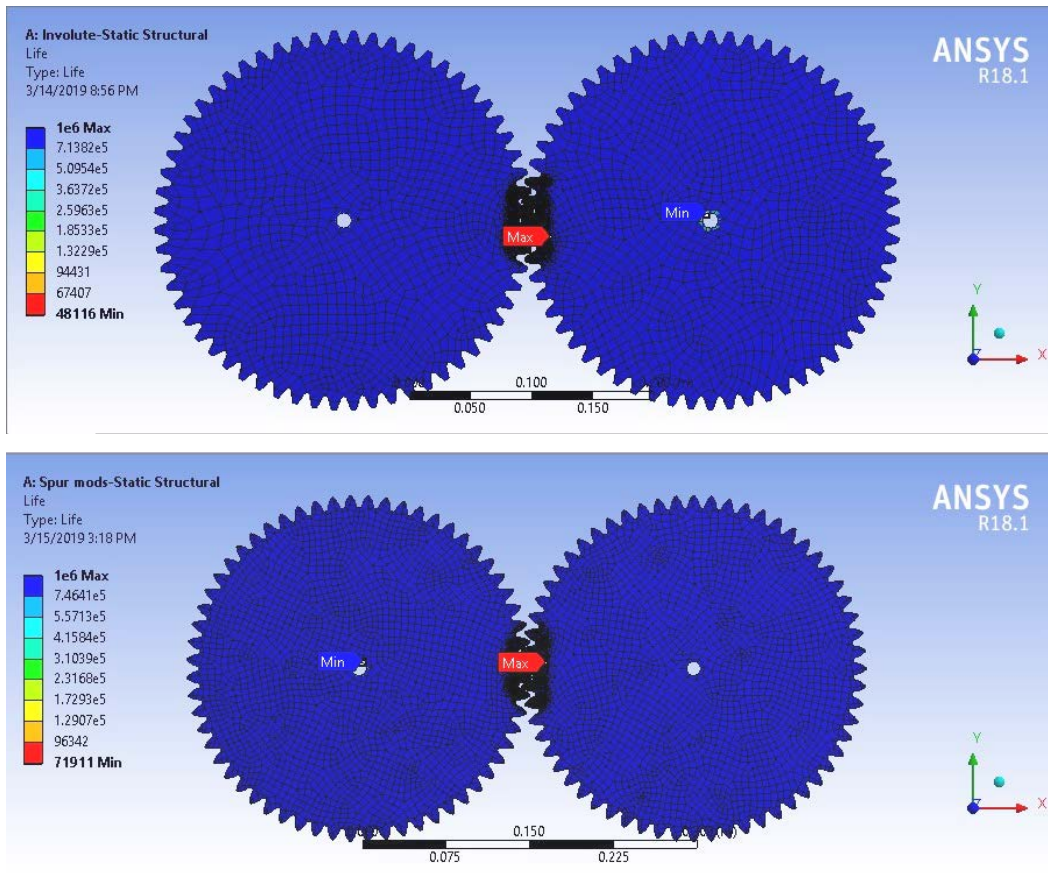


Fig. 11. Life results of involute and modified tooth profile.

Table 5. Involute and modified gear and assembly frequencies.

Mode	Frequency [Hz]			
	Gear		Assembly	
	Involute	Modified	Involute	Modified
1	407.41	424.66	18.845	35.781
2	1442.3	1462.4	1440.3	1442.2
3	2307.2	2326.7	1876	1820.6

3.3. Harmonic analysis

Figures 12–14 show the frequency response of total deformation, equivalent strain, and Von-Mises Stress for involute spur gear assembly. Figures 15–17 show the frequency response of total deformation, equivalent strain, and Von-Mises Stress for involute spur gear assembly, respectively.

The first mode is taken in each assembly from the modal analysis and is used for harmonic analysis. In

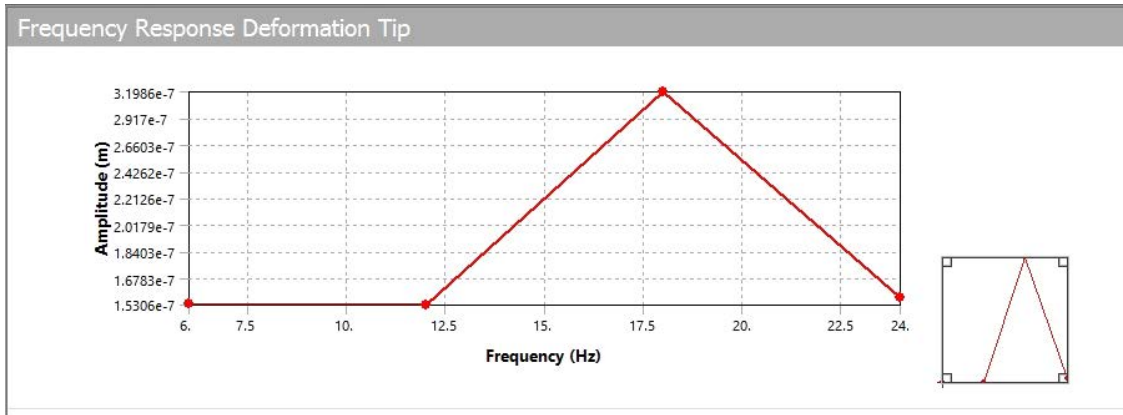


Fig. 12. Frequency response of total deformation for involute profile gear assembly.

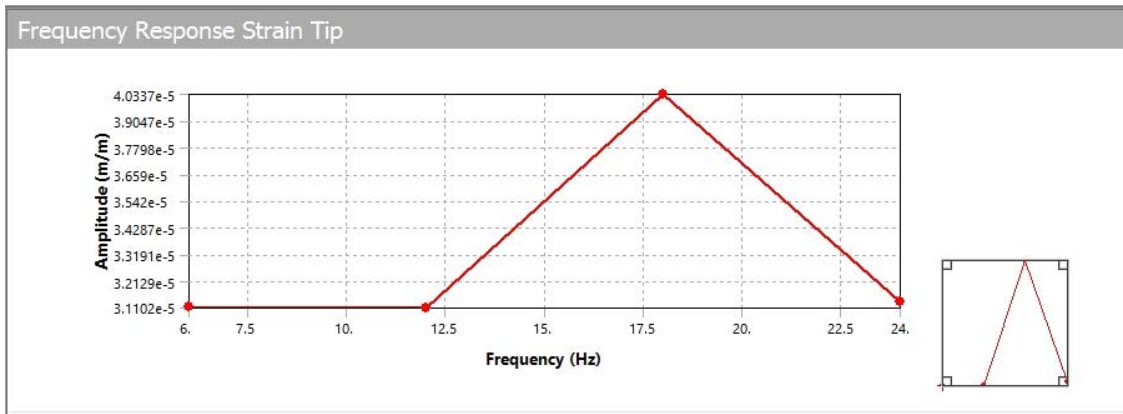


Fig. 13. Frequency response of equivalent strain for involute profile gear assembly.

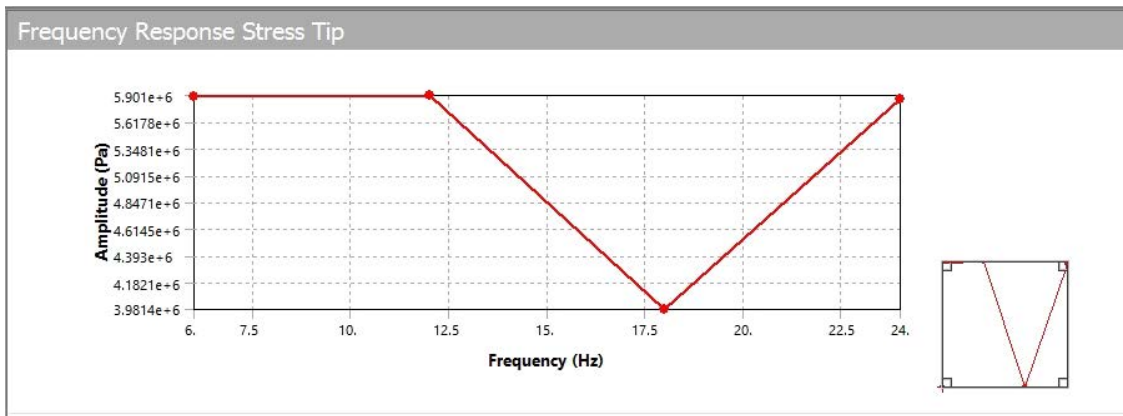


Fig. 14. Frequency response of Von-Mises stress for involute profile gear assembly.

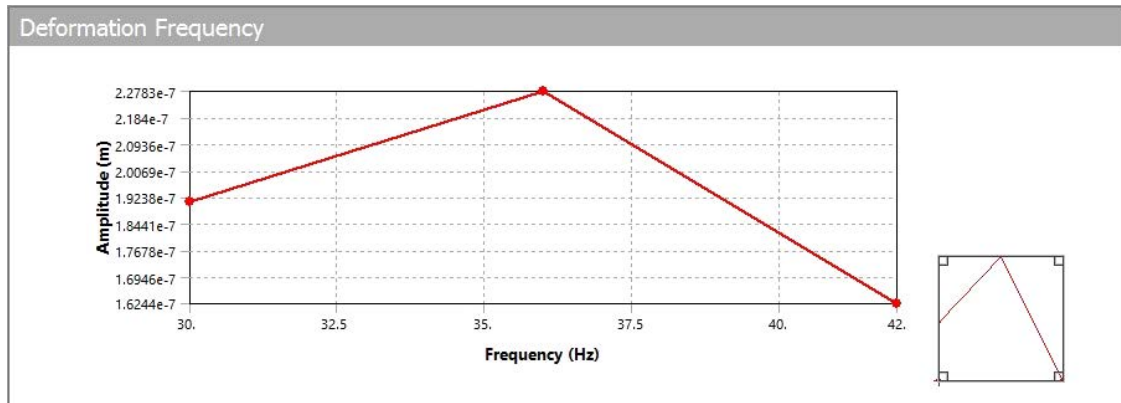


Fig. 15. Frequency response of total deformation for modified profile gear assembly.

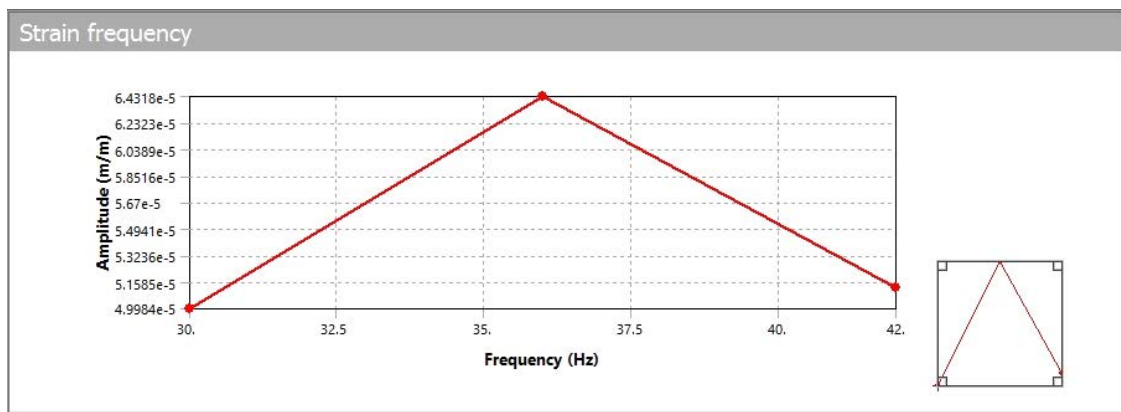


Fig. 16. Frequency response of equivalent strain for modified profile gear assembly.

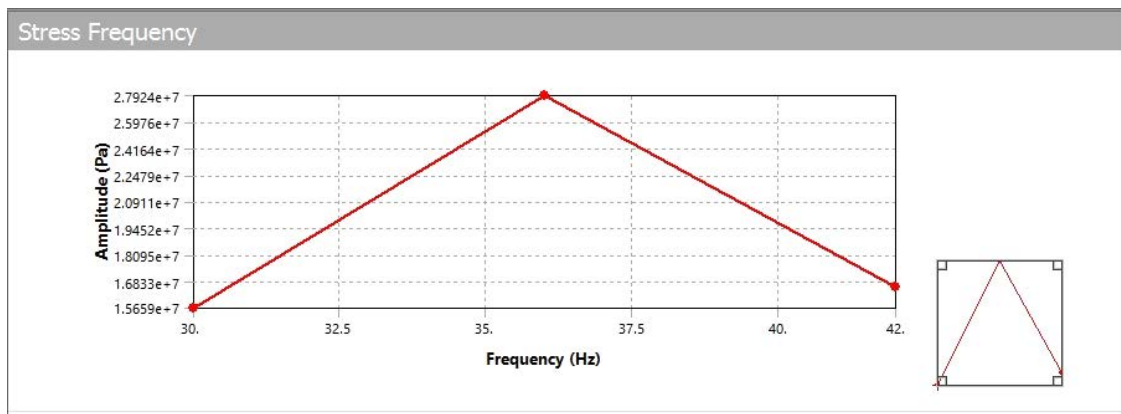


Fig. 17. Frequency response of Von-Mises stress for modified profile gear assembly.

the case of involute spur gear assembly, it is 18.845 Hz. For this the range is taken from 6 to 24 Hz and it is divided into 4 intervals. In the case of modified spur gear assembly, it is 35.781 Hz. For this the range is taken from 30 to 42 Hz and it is divided into 2 intervals. In both cases the solution method used is ‘Full’.

3.4. Power transmission efficiency calculations

Using the change in deformation of the tip of the gear tooth from the static structural analysis it was

determined approximately how much additional power would be required. Since the analysis was done in a frictionless environment this change in power transmission efficiency can be purely attributed to the profile change and not to any other losses.

The calculations done are as follows:

- deformation at tip of involute gear = 0.050278 mm,
- deformation at tip of modified gear = 0.065301 mm,
- difference in deformation, $\Delta = 0.01523$ mm,

$$\theta = r \Delta, \tag{17}$$

where θ is the angle in radians, r is the radius of the major diameter of the gear and Δ is the difference in deformation $\theta = 1.068 \cdot 10^{-4}$ rad.

Now in static structural analysis the analysis is done in one second. Therefore the above angular deformation can be considered as $\frac{d\theta}{dt} = d\omega$, i.e. angular velocity.

Change in power:

$$dP = \tau d\omega = 0.17088 \text{ W.} \tag{18}$$

Input power:

$$P = \tau\omega, \tag{19}$$

where τ is the input torque and ω is the input angular velocity,

$$\omega = \frac{2\pi N}{60}, \tag{20}$$

where N is the input rpm, $N = 6700$ rpm, which implies $\omega = 701.6223593$ rad/s.

Therefore,

$$P = 1122595.775 \text{ W.}$$

Change in power transmission efficiency:

$$dP [\%] = 1.5219 \cdot 10^{-5}.$$

4. Conclusions

Involute spur gear model and modified spur gear model was made using modelling software SolidWorks and the preceding analysis was carried out using ANSYS 18.0. The analysis included static structural, modal, and harmonic aspects for each set of gear assembly. MATLAB was used to compute values of various relief modifications that were provided for the involute tooth profile along the flank in the model designed in SolidWorks.

Static structural analysis gave us the following results:

- It is observed that the modification of profile decreases the stress at the tip, pitch, and root portion of the gear tooth.
- An improvement of about 49.453% in the minimum life of the assembly as compared to involute profile is observed in the modified profile. An improvement in the safety factor is also observed.

From the deformation results obtained via static structural analysis, change in power transmission efficiency is calculated. It is observed that there is insignificant change in power transmission efficiency due to linear profile modification.

From the modal results of the assembly and the pinion gears it is observed that in both cases the natural frequency changes in the case of modified assembly and gear. Table 6 presents the percentage change in the first, second, and third natural frequencies of

Table 6. Percentage change in frequency results of modal analysis of gears and gear assemblies.

Sr. No.	Mode	Percentage change in assembly [%]	Percentage change in pinion [%]
1	Mode 1	89.870	4.234
2	Mode 2	0.132	1.394
3	Mode 3	-2.953	0.845

both the gears and assemblies (positive indicating an increase and negative indicating a decrease).

Harmonic analysis shows us how the deformation, stress, and strain vary when the system is subjected to different frequencies of vibration. It is observed that when each of the assemblies resonate with the first natural frequency obtained from modal analysis, maximum values are observed for deformation and stress.

From the above results it is indicated that the linear tooth profile change does not interfere with the transmission performance of the gear but improves the overall quality and life of the gear assembly.

The results of this paper present a possible scope in applications such as designing of transmission systems for automobiles where the results of the current work can be used to increase the quality of the gears and improve the life of the transmission.

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