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ELASTIC-PLASTIC ANALYSIS FOR CIRCUMFERENTIAL THROUGH CRACK AT BOUNDARY OF SEMI'S BRACE UNDER BEAM WAVE

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ABSTRACT

In order to ensure the safety and reliability of the horizontal brace of semi-submersible platform (SEMI) which functions as the supporting structure in SEMI, this article presents an elastic-plastic method to analyze the variations of the crack tip opening displacement, elastic zone and plastic zone of the cracked section of the horizontal brace under beam wave. The brace of the SEMI was assumed to be located a circumferential through crack at its boundary in this article. In addition, the cracked section of the brace has been divided into crack zone, tensile plastic zone, elastic zone and compressive plastic zone in the presented theoretical model. Moreover, the closed form of the solution has been found in this article which is especially suitable solving complicated problems in practical engineering application. Also, a typical new-generation SEMI that is in practical use was selected to analyze the variation tendency of the cracked brace's parameters using the proposed model which could give good suggestion to semi-submersible platform designers and managers.

Keywords: theoretical model, crack damaged brace, semi-submersible platform, beam wave

INTRODUCTION

Semi-submersible platform (SEMI), one of the most widely used reusable exploitation platform due to their mobility and ability to operate in deep-water, have gained popularity in recent decades with on-going development of deep-water oil and gas exploitation. Although the safety design standards for this kind of structures are quite strict, cracks inescapably initiated during their service life [1]. According to the destruction accident happened before, engineers found that the cracks often occur at the horizontal brace which functions as the supporting structures in SEMI and bear complex loads [2–3]. The presence of such cracks at critical locations can compromise the safety of the braces and then cause serious disaster eventually. Take the accident of *Alexander Keilland* platform as an example, the loads were transferred to the other braces and led them break because of the overload due to the initial fracture of a horizontal brace [4].

As the semi-submersible platform would encounter very harsh marine environment during its service life, it is necessary to analyze the variation tendencies of the crack-damaged brace to ensure the safety and reliability of the SEMI. A number of researchers has investigated such problems [5–7]. In additionally, the crack has been assumed to be on the surface of the brace or lies far away from the boundary section of the brace. However, according to the evaluation of the effects of hydrodynamic loads on the strength of the SEMI's horizontal braces, the high stress almost concentrates on the connection between the horizontal brace and the column of the platform [8]. Moreover, the crack-induced accidents of the SEMIs of *Sedco*, *Alexander Keilland* and *Ocean Ranger* indicate that a circumferential through crack usually lies at the boundaries. Therefore, it is crucial to analyze the mechanical characteristics of these cracks at critical locations that could easily compromise the safety of the semi-submersible platform. Many researchers using finite element method whose effectiveness has been accepted by the engineering community to analyze these crack damaged brace [9–10]. Nevertheless, by using the finite element method, calculations need to be carried out for every specific case and there would be only one calculation for each case. Moreover, the nonlinear calculations will be inefficient and spend significant resources for a structure system with some local defective elements. In this sense, the theoretical analysis on these cracked brace is still necessary.

The present article is aimed to investigate a theoretical analysis method to analyze the variations of the crack tip opening displacement, elastic zone and plastic zone of the cracked section of the horizontal brace under beam wave, which significantly affects the safety and reliability of the brace.

GOVERNING EQUATIONS

A typical new-generation semi-submersible platform is selected to analyze the cracked damaged brace. As can be seen from Fig. 1, the selected platform is mainly composed of the main deck, four columns, two parallel pontoons and the horizontal braces connected to the columns. In this research, the circumferential through crack is assumed to be located at the boundary of the brace. Moreover, the brace of the SEMI mainly subjected to splitting force under beam wave, shown as in Fig. 1 where the coordinate system and direction of the load have been shown.

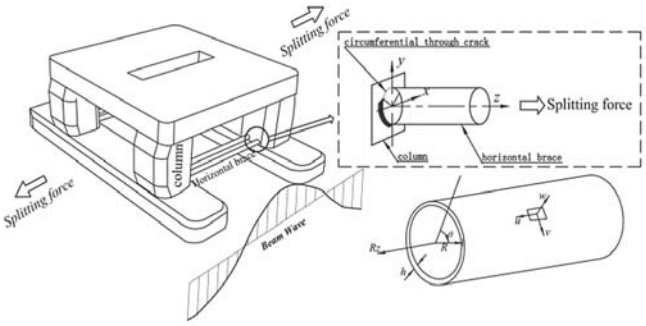

Fig. 1. Beam wave loaded on the brace of the semi-submersible platform

According to the historical records [11–13], the characteristic functions of the cracked damaged brace varies slowly in the z direction but not too rapidly in the θ direction. In the disastrous even of the *Alexander Kielland*, the circumferential through crack at the boundary of brace propagated to almost 67% of circumferential length of the brace before fracture [14]. Additionally, the brace of SEMI belong to the mid-long cylindrical shell category from the viewpoint of the shell theory. Then, the governing equation of the brace can be expressed under a semi-membrane state [5] with the complex-valued variable $\Omega = w + \mathrm{i}\zeta$ as follow,

$$\frac{\partial^2}{\partial\theta^2}\left(\frac{\partial^2\Omega}{\partial\theta^2}+\Omega\right)-\mathrm{i}\varepsilon^{-2}\frac{\partial^2\Omega}{\partial z^2}=0 \quad (1)$$

The complex characteristic functions Φ and φ both satisfy Eq. (1) here and can be related to each other by

$$\frac{\partial^2\Phi}{\partial z^2}=\varepsilon^2\varphi \quad (2)$$

Here, ε is a small parameter given by $\varepsilon^2 = (h/R)[12(1-v^2)]^{-1/2}$, where h and R is the thickness and radius of the horizontal brace, v is Poisson's ratio. Then, the expression of dimensionless complex displacements u, v, and w, stress functions χ_z, χ_θ, and ζ can be given in terms of Φ and φ by

$$\begin{cases}\varepsilon^2 u=\dfrac{\partial}{\partial z}\left(\dfrac{\partial^2\Phi}{\partial\theta^2}\right) & \varepsilon^2 v=-\dfrac{\partial^3\Phi}{\partial\theta^3} & w=-\dfrac{\partial^2\Phi}{\partial\theta^2}+\mathrm{i}\varphi\\ \varepsilon^2\chi_z=-\mathrm{i}\dfrac{\partial}{\partial z}\left(\dfrac{\partial^2\Phi}{\partial\theta^2}\right) & \varepsilon^2\chi_\theta=-\mathrm{i}\dfrac{\partial^3\Phi}{\partial\theta^3} & \zeta=-\mathrm{i}\dfrac{\partial^2\Phi}{\partial\theta^2}+\varphi\end{cases} \quad (3)$$

Moreover, the dimensionless load parameter corresponding to tension load T can be defined as $\sigma_T = T/(2\pi Rh\sigma_F)$ and the dimensionless characteristic functions mentioned above are given as follows,

$$\begin{cases}\left(\bar{u},\ \bar{v},\ \bar{w}\right)=\dfrac{\sigma_F R}{E}\left(u,\ v,\ \varepsilon^{-2}w\right)\\ \left(\overline{\chi_z},\ \overline{\chi_\vartheta},\ \bar{\zeta}\right)=\sigma_F R^2h^2\varepsilon\left(\chi_z,\ \chi_\theta,\varepsilon^{-2}\zeta\right)\end{cases} \quad (4)$$

Here, E is Young's modulus, σ_F is yield stress of the material and parameters with an overbar denotes dimensional quantity.

According to the research done by Nicholson [15], the complete solution of cracked cylindrical shell composed by the elementary solutions and the solution, Φ_c, incurred by the existence of the crack. Then, the complete solution Φ_c to the problem of brace with a circumferential through crack at the boundary presented in this research can be obtained as

$$\Phi_c = \Phi_b + \Phi_s \quad (5)$$

Here, Φ_b refers to the elementary solutions and Φ_s refers to the solution incurred by the existence of the crack. Here, the elementary solutions to the proposed model in this research can include the splitting force solution, rigid-body motion (for which stresses disappear) solution and null solution (for which displacements disappear). Then, the elementary solution can be expressed as

$$\frac{\partial^2\Phi_b}{\partial\theta^2}=\left\{0.5i\left[1+i\varepsilon^2\left(2+\mu\right)\right]\theta^2+\left(1+i\varepsilon^2\mu\right)\left(0.5\varepsilon^2z^2-i\right)\right\}\sigma_T + ia-\varepsilon bz+iccos\theta+\varepsilon dzcos\theta \quad (6)$$

Here, a, b, c, d are unknown constants.

In order to simplify the treatment of the governing equations of the proposed theoretical model, expressions for the values of the stress functions in terms of prescribed edge load T_z, T_θ, V and M_n acting on the edge $z = 0$ can be obtained by Eq. (7) for there is a static-geometric analogy between displacements and stress functions.

$$\begin{cases}\varepsilon^2\chi_z=sin\theta\int^{\theta}\left(T_\theta sin\eta+\varepsilon^2Vcos\eta\right)d\eta+cos\theta\int^{\theta}\left(T_\theta cos\eta-\varepsilon^2Vsin\eta\right)d\eta-\int^{\theta}T_\theta d\eta\\ \varepsilon^2\chi_\theta=sin\theta\int^{\theta}\left(T_z+\varepsilon^2M_n\right)sin\eta d\eta+cos\theta\int^{\theta}\left(T_z+\varepsilon^2M_n\right)cos\eta d\eta-\int^{\theta}T_z d\eta\end{cases} \quad (7)$$

Here, η refers to the infinitesimal angle of the cracked section. As the circumferential through crack is assumed located at the boundary of the brace, the cross section $z = 0$ is the border form the condition of symmetry. Shown as in Fig. 2, the cracked section has been divided into circumferential through crack zone, tensile plastic zone, elastic zone and compressive plastic zone in this research.

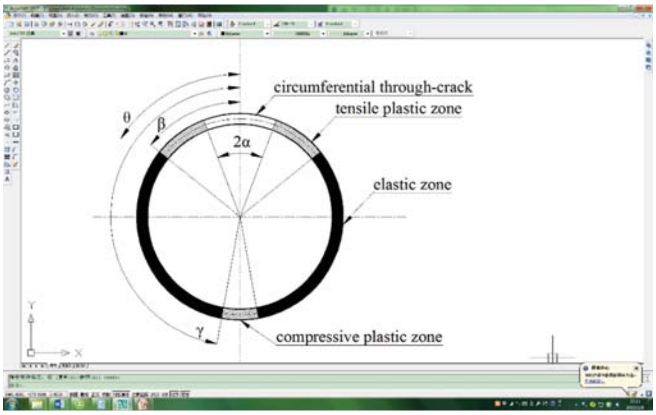

Fig. 2. Cracked section of the brace of SEMI

Here on the cracked section, opening angle of the circumferential through-crack equals to 2α, the tensile zone equals to β-α, the elastic zone equals to γ-β, and the compressive plastic zone equals to π-γ. The boundary conditions can be obtained by means of methods given by Sanders [17]. Those and the displacement conditions are given as follows,

$$\begin{cases} T_z = T_\theta = V = M_n = 0 \quad (0 \le \theta < \alpha) \\ v = \mathrm{V} = M_n = 0;\ T_z = 1 \quad (\alpha \le \theta < \beta) \\ u = v = w = \dfrac{\partial w}{\partial z} = 0 \quad (\beta \le \theta < \gamma) \\ v = \mathrm{V} = M_n = 0; T_z = -1 \quad (\gamma \le \theta \le \pi) \end{cases} \tag{8}$$

Then, by inputting the Eq. (3) and Eq. (7) into the boundary conditions mentioned above, the governing equations can be obtained in terms of the characteristic function Φ and φ as

$$\begin{cases} R\left\{ i\dfrac{\partial}{\partial z}\left(\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right)\right\} = 0 \\ R\left\{ i\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = 0 \end{cases} \quad (0 \le \theta < \alpha) \tag{9}$$

$$\begin{cases} R\left\{ \dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = 0 \\ R\left\{ i\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = -\cos(\theta-\alpha) - 0.5(\theta-\alpha)^2 + 1 \end{cases} \quad (\alpha \le \theta < \beta) \tag{10}$$

$$\begin{cases} R\left\{ \dfrac{\partial}{\partial z}\left(\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right)\right\} = 0 \\ R\left\{ \dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = 0 \end{cases} \quad (\beta \le \theta < \gamma) \tag{11}$$

$$\begin{cases} R\left\{ \dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = 0 \\ R\left\{ i\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = -\pi\sigma_T\theta - cos\theta - G_1 cos\theta + 0.5(\pi-\theta)^2 - 1 + G_2 \end{cases} \quad (\gamma \le \theta \le \pi) \tag{12}$$

The symbol $R\{\}$ denotes the real part of the expression in brackets and the subscript c means that the expression are in terms of the complete characteristic functions.

With Eq. (5–6) and Eq. (9–12), the particular integral Φ_s at the boundary incurred by the existence of the crack turns into

$$\begin{cases} R\left\{ i\dfrac{\partial}{\partial z}\left(\dfrac{\partial^2 \Phi_s}{\partial \theta^2}\right)\right\} = -\varepsilon b_I + \varepsilon d_I \cos\theta \\ R\left\{ i\dfrac{\partial^2 \Phi_s}{\partial \theta^2}\right\} = a_R + c_R \cos\theta + \left(1 - 0.5\theta^2\right)\sigma_T \end{cases} \quad (0 \le \theta < \alpha) \tag{13}$$

$$\begin{cases} R\left\{ \dfrac{\partial^2 \Phi_s}{\partial \theta^2}\right\} = a_I + c_I \cos\theta \\ R\left\{ i\dfrac{\partial^2 \Phi_s}{\partial \theta^2}\right\} = a_R + c_R \cos\theta + \left(1 - 0.5\theta^2\right)\sigma_T - \cos(\theta-\alpha) \quad (\alpha \le \theta < \beta) \\ \qquad - 0.5(\theta-\alpha)^2 + 1 \end{cases} \tag{14}$$

$$\begin{cases} R\left\{ \dfrac{\partial^2 \Phi_s}{\partial \theta^2}\right\} = a_I + c_I \cos\theta \\ R\left\{ \dfrac{\partial}{\partial z}\left(\dfrac{\partial^2 \Phi_s}{\partial \theta^2}\right)\right\} = \varepsilon b_R - \varepsilon d_R \cos\theta \end{cases} \quad (\beta \le \theta < \gamma) \tag{15}$$

$$\begin{cases} R\left\{ \dfrac{\partial^2 \Phi_s}{\partial \theta^2}\right\} = a_I + c_I \cos\theta \\ R\left\{ i\dfrac{\partial^2 \Phi_s}{\partial \theta^2}\right\} = a_R + (c_R - G_1)\cos\theta - \cos\theta + 0.5(\pi-\theta)^2 \quad (\gamma \le \theta \le \pi) \\ \qquad + \left[\pi(\pi-\theta) + \left(0.5\theta^2 - 1\right)\right]\sigma_T + G_2 - 1 \end{cases} \tag{16}$$

Here, the subscripts R and I refer to the real and imaginary parts of these constants. Furthermore, any solution to the Eq. (1) satisfy the conditions [10]

$$\int_0^\pi \left.\frac{\partial^2 \Phi_s}{\partial \theta^2}\right|_{z=0} d\theta = \int_0^\pi \left.\frac{\partial^2 \Phi_s}{\partial \theta^2}\right|_{z=0} \cos\theta d\theta = 0 \tag{17}$$

$$\frac{\partial}{\partial z}\left(\frac{\partial^2 \Phi_s}{\partial \theta^2}\right) = -i^{\frac{3}{2}}\varepsilon \frac{\partial^2}{\partial \theta^2}\left(\frac{\partial^2 \Phi_s}{\partial \theta^2} + \frac{1}{2}\Phi_s\right) \tag{18}$$

Now put

$$\left.\frac{\partial^2 \Phi_s}{\partial \theta^2}\right|_{z=0} = F(\theta) \tag{19}$$

And following the Eq. (17) and Eq. (18) to get

$$\int_0^\pi \left(F'' + F\right) d\theta = \int_0^\pi F'' \cos\theta d\theta = 0 \tag{20}$$

$$\left.\frac{\partial}{\partial z}\left(\frac{\partial^2 \Phi_s}{\partial \theta^2}\right)\right|_{z=0} = -i^{\frac{3}{2}}\varepsilon\left(F'' + \frac{1}{2}F\right) \tag{21}$$

Additionally, by using Eq. (20) and Eq. (21), the Eq. (13–16) can be expressed in the equivalent form as follows,

$$\begin{cases} F_R = -\left(a_R + 2\sqrt{2}b_I\right) - \left(c_R + 2\sqrt{2}d_I\right)cos\theta \\ \qquad + Scos\dfrac{\theta}{\sqrt{2}} - \left(0.5\theta^2 - 1\right)\sigma_T \qquad (0 \le \theta < \alpha) \\ F_I = -\left(a_R + c_R cos\theta\right) - \left(0.5\theta^2 - 1\right)\sigma_T \end{cases} \tag{22}$$

$$\begin{cases} F_R = a_I + c_I cos\theta \\ F_I = -\left(a_R + c_R cos\theta\right) - \left(0.5\theta^2 - 1\right)\sigma_T \ (\alpha \le \theta < \beta) \\ \quad + cos\left(\theta - \alpha\right) + 0.5\left(\theta - \alpha\right)^2 - 1 \end{cases} \tag{23}$$

$$\begin{cases} F_R = a_I + c_I cos\theta \\ F_I = 2\sqrt{2}b_R - a_I + \left(2\sqrt{2}b_R - c_I\right)cos\theta \ \ (\beta \le \theta < \gamma) \\ \quad + 2\sqrt{2}Pcos\dfrac{\theta - \beta}{\sqrt{2}} + 2\sqrt{2}Qcos\dfrac{\theta - \beta}{\sqrt{2}} \end{cases} \tag{24}$$

$$\begin{cases} F_R = a_I + c_I cos\theta \\ F_I = -a_R - \left(c_R - G_1\right)cos\theta - \left[\pi\left(\pi - \theta\right) + \left(0.5\theta^2 - 1\right)\right]\sigma_T \ (\gamma \le \theta \le \pi) \\ \quad + cos\theta - 0.5\left(\pi - \theta\right)^2 - G_2 + 1 \end{cases} \tag{25}$$

Then, the governing equations mentioned above got 15 unknown constants including the five real constants of integration of S, P, Q, G_1 and G_2, the two undetermined parameters of the cracked section of brace β and γ, and eight real and imaginary parts of the constants a_R, b_R, c_R, d_R, a_I, b_I, c_I, d_I. Additionally, the continuity of the displacements and stress functions at the cracked section implies that F_R, F_R', F_R'', at α and F_I, F_I', F_I'', F_I''' at β and γ should be continuous (F_I, F_I', F_I'' at α are continuous from simply checking), from which eleven equations can be got. Moreover, four more equations can be found for the real parts and the imaginary parts equal to zero in Eq. (20). There are thus 15 conditions to determine the 15 constants and all the constants can be finally determined by means of algebraic methods. And results for the 15 constants follow from

$$a_I = \frac{1}{\pi}\left[S\left(\frac{\alpha}{2}cos\frac{\alpha}{\sqrt{2}} - \frac{\sqrt{2}}{2}sin\frac{\alpha}{\sqrt{2}}\right) - \frac{1}{3}\alpha^3\sigma_T\right]$$

$$b_I = \frac{1}{2\sqrt{2}}\left[-a_R - a_I + 0.5Scos\frac{\alpha}{\sqrt{2}} - 0.5\alpha^2\sigma_T\right]$$

$$c_I = \frac{1}{2\pi cosa}\left[Scos\frac{\alpha}{\sqrt{2}}\left(\alpha - sin\alpha cos\alpha\right) - 2\left(2\alpha cos^2\alpha - \alpha - sin\alpha cos\alpha\right)\sigma_T\right]$$

$$d_I = \frac{1}{2\sqrt{2}cos\alpha}\left(-c_R cos\alpha - c_I cos\alpha + 0.5Scos\frac{\alpha}{\sqrt{2}} + \sigma_T\right)$$

$$a_R = \frac{\left(\pi - \beta\right)A}{\pi} + \frac{P}{\pi}\left[2sin\frac{\gamma - \beta}{\sqrt{2}} + \sqrt{2}\left(\pi - \gamma\right)\right] + \frac{2Q}{\pi}\left(1 - cos\frac{\gamma - \beta}{\sqrt{2}}\right)$$
$$+ \frac{\sigma_T}{\pi}\left[\frac{1}{6}\left(2\pi^3 - 2\gamma^3 - \beta^3\right) - \pi\gamma\left(\pi - \gamma\right)\right] + \frac{1}{6\pi}\left[\left(\beta - \alpha\right)^3 + 2\left(\pi - \gamma\right)^3\right]$$

$$b_R = \frac{\sqrt{2}}{4}\left(A - a_R + a_I\right)$$

$$c_R = -\left(\beta - \gamma + sin\beta cos\beta - sin\gamma cos\gamma\right)\frac{C}{\pi} + \left(\pi - \gamma - sin\gamma cos\gamma\right)\frac{G_1}{\pi}$$
$$+ \left(\sqrt{2}sin\gamma - \sqrt{2}sin\beta cos\frac{\gamma - \beta}{\sqrt{2}} - cos\beta sin\frac{\gamma - \beta}{\sqrt{2}}\right)\frac{4P}{\pi} + \frac{2}{\pi}\left(sin\alpha - sin\beta - sin\gamma\right)$$
$$- \left(sin\gamma + \sqrt{2}sin\beta sin\frac{\gamma - \beta}{\sqrt{2}} - cos\beta cos\frac{\gamma - \beta}{\sqrt{2}}\right)\frac{4Q}{\pi} - \left(sin\gamma - sin\beta\right)\frac{2\sigma_T}{\pi}$$
$$+ \frac{\left(\beta - \alpha\right)cos\alpha + cos\beta sin\left(\beta - \alpha\right) - sin\gamma cos\gamma - \gamma}{\pi} + 1$$

$$d_R = \frac{\sqrt{2}}{4}\left(C - c_R + c_I\right)$$

$$A = -\sqrt{2}\left(Pcos\frac{\gamma - \beta}{\sqrt{2}} + Qsin\frac{\gamma - \beta}{\sqrt{2}}\right) - 0.5\beta^2\sigma_T + 0.5\left(\beta - \alpha\right)^2$$

$$C = \frac{-\beta\sigma_T + sin\left(\beta - \alpha\right) + \beta - \alpha}{\sin\beta}$$

$$G_1 = \frac{C\cos\gamma + \sqrt{2}P - \sigma_T - cos\gamma - 1}{\cos\gamma}$$

$$G_2 = -A - \sqrt{2}P - \left[\pi\left(\pi - \gamma\right) + 0.5\gamma^2\right]\sigma_T - \left(\pi - \gamma\right)^2$$

$$P = Q\cot\frac{\gamma - \beta}{\sqrt{2}} - \frac{\beta\sigma_T + \beta - \alpha}{sin\dfrac{\gamma - \beta}{\sqrt{2}}}$$

$$Q = -\left(\pi - \gamma\right)\sigma_T - \pi + \gamma$$

$$S = \frac{2\left(sin\alpha - \alpha cos\alpha\right)\sigma_T}{\sqrt{2}cos\alpha sin\dfrac{\alpha}{\sqrt{2}} - sin\alpha cos\dfrac{\alpha}{\sqrt{2}}}$$

In order to obtain the unknown parameters of the cracked section β and γ which can the tensile plastic zone, elastic zone and compressive plastic zone of the cracked section with given α and σ_T, the following simultaneous transcendental equations should be used.

$$\begin{cases} B_1\sigma_T - N_1 = 0 \\ B_2\sigma_T - N_2 = 0 \end{cases} \tag{26}$$

Where

$$B_1 = \left(sin\beta + \beta cos\beta\right)sin\frac{\gamma - \beta}{\sqrt{2}} + \sqrt{2}\left(\pi - \gamma + \beta cos\beta cos\frac{\gamma - \beta}{\sqrt{2}}\right)sin\beta$$

$$B_2 = \left[\left(\pi - \gamma\right)cos\gamma - sin\gamma\right]sin\frac{\gamma - \beta}{\sqrt{2}} - \sqrt{2}\left[\left(\pi - \gamma\right)cos\frac{\gamma - \beta}{\sqrt{2}} + \beta\right]sin\gamma$$

$$N_1 = \left[sin\beta - sin\alpha + \left(\beta - \alpha\right)cos\beta\right]sin\frac{\gamma - \beta}{\sqrt{2}} - \sqrt{2}\left[\pi - \gamma - \left(\beta - \alpha\right)cos\frac{\gamma - \beta}{\sqrt{2}}\right]sin\beta$$

$$N_2 = \left[sin\gamma - \left(\pi - \gamma\right)cos\gamma\right]sin\frac{\gamma - \beta}{\sqrt{2}} + \sqrt{2}\left[\left(\pi - \gamma\right)cos\frac{\gamma - \beta}{\sqrt{2}} - \beta + \alpha\right]sin\gamma$$

Then, the displacements on the crack are obtained can be obtained from the above results and the dimensionless crack tip opening displacement (CTOD) subjected to the splitting force can be expressed as

$$\delta = \frac{\sqrt{2}}{4}^{-1}\left[-A + C\cos\alpha - \left(0.5\alpha^2 + 1\right)\sigma_T\right] \tag{27}$$

Further, for the present load circumstance the full plastic condition on the cracked section is

$$\sigma_T = \frac{2}{\pi}\left[arccos\left(0.5sin\alpha\right) - 0.5\alpha\right]$$

And

$$\beta = \gamma = 0.5\left(\pi + \alpha + \pi\sigma_T\right)$$

SOLUTIONS

The theoretical model proposed in the research is aiming to calculate the mechanical characteristic of the cracked damaged brace with a circumferential through crack lies at its boundary when the SEMI subjected to beam wave. At the beginning of the solutions, basic parameters such as parameters of the brace of SEMI, angle of the circumferential through crack and the splitting force should be input for the solution for plastic zones, elastic zones and *CTOD* of the cracked section.

As it's hard to find explicit solution from Eq. (26) to obtain the unknown parameters of the cracked section β and γ, numerical method should be used in this research. Firstly, assume an initial β_0 and initial γ_0 to start the calculation. And then, define two more parameters F and λ according to Eq. (26) following from

$$F=\left[B_1(\beta_0,\gamma_0)\sigma_T-N_1(\beta_0,\gamma_0)\right]^2+\left[B_2(\beta_0,\gamma_0)\sigma_T-N_2(\beta_0,\gamma_0)\right]^2$$

$$\lambda=\frac{F(\beta_0,\gamma_0)}{\left[\frac{F(\beta_0+0.01,\gamma_0)-F(\beta_0,\gamma_0)}{0.01\beta_0}\right]^2+\left[\frac{F(\beta_0,\gamma_0+0.01)-F(\beta_0,\gamma_0)}{0.01\gamma_0}\right]^2}$$

To guarantee to precision of the solution, verification is needed to see if $|F|<\varsigma$ is satisfied, where ς is a small specified quantity. If $|F|<\varsigma$ does not appear, the assumed initial β_0 and initial γ_0 should be modified as

$$\beta=\beta_0-\lambda\frac{F(\beta_0+0.01,\ \gamma_0)-F\ (\beta_0,\ \gamma_0)}{0.01\beta_0}$$

$$\gamma=\gamma_0-\lambda\frac{F(\beta_0,\gamma_0+0.01)-F(\beta_0,\gamma_0)}{0.01\gamma_0}$$

After $|F|<\varsigma$ is satisfied, the solution for plastic zones of the cracked horizontal brace is done. Then, the output of β and γ could be used to determine the 15 unknown parameters a_I, b_I, c_I, d_I, a_R, b_R, c_R, d_R, A, C, G_1, G_2, P, Q, S and the crack tip opening displacement (CTOD) can be obtained from Eq. (27) too. Until now, the solutions of the mechanical characteristics of the cracked damaged brace of SEMI with a circumferential through crack that lies at the boundary are done.

EXAMPLES

To illustrate the proposed model with examples, a typical new-generation SEMI that suitable for drilling operations worldwide has been selected to analyze the variation tendency of the cracked brace's parameters. The geometrical dimensions of the main components of the SEMI are listed in Table 1.

As the ocean wave vary greatly under different wave frequencies. The wave frequency was considered as varying from 0.1 to 1.5 rad/s to reflect the actual wave frequencies most commonly seen in ocean areas, and an increment of

Tab. 1. Main geometrical dimensions of the SEMI

Component	Dimension(m)
Pontoon Length	114.07
Pontoon width	20.12
Pontoon Height	8.54
Distance between pontoons	58.6
Column length	17.4
Column width	17.4
Column height	21.4
Distance between longitudinal columns	58.6
Distance between transversal columns	58.6
Deck length	114.4
Deck width	74.4
Deck height	38.6

0.05 rad/s was used in the numerical simulations. In addition, the ocean wave was assumed to be a combination of infinite regular waves in this research, and the Jonswap spectrum was used to characterize random waves in the ocean.

Based on the numerical calculation, the response amplitude operator (RAO) of the swaying, surging and rolling of the SEMI under beam wave have been obtained, shown as in Fig. 3, Fig. 4 and Fig. 5. The RAO results obtained by the numerical calculation coincide with the white noise test and regular wave test under the beam wave.

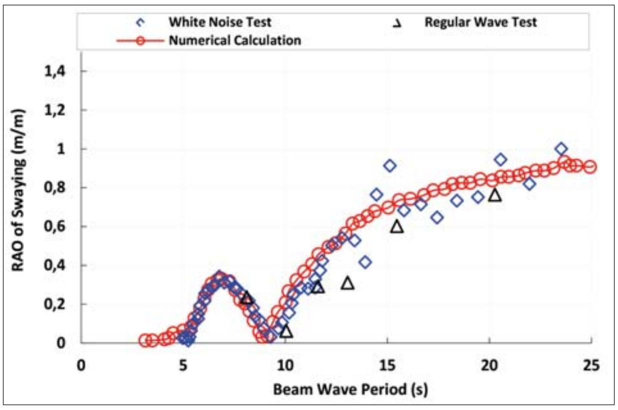

Fig. 3. RAO of swaying of the SEMI under beam wave

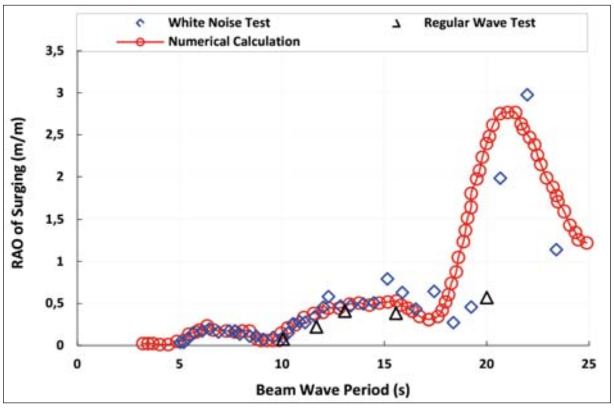

Fig. 4. RAO of surging of the SEMI under beam wave

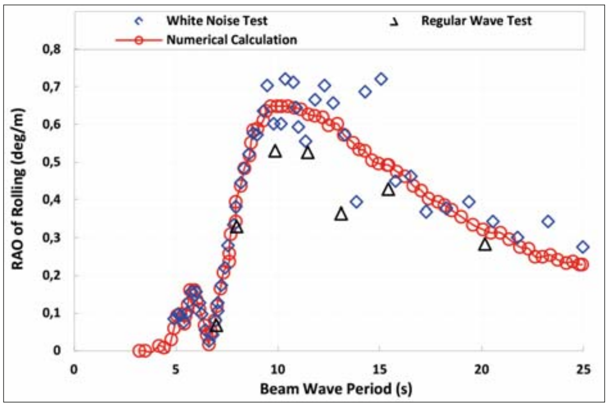

Fig. 5. RAO of rolling of the SEMI under beam wave

The circumferential through crack is assumed to be located at the boundary of the selected SEMI's brace and geometrical dimensions of the brace are listed in Table 2.

Tab. 2. Main geometrical dimensions of the brace

Component	Dimension
Quantity of the brace	4
Diameter of the brace	2.45m
Thickness of the brace	0.1m
Poisson's ratio	0.3
Young's modulus	210GPa
Yield stress of the material	400MPa

Different splitting forces could be loaded on the brace when the SEMI encounter different wave frequencies. According to the response amplitude operator (RAO) and long-term response (LTR) of the splitting force loaded on the chosen SEMI, shown as in Fig. 5 and Fig. 6, the largest splitting force loaded on the brace is 65×10^6N when the beam wave frequency comes to 0.66rad/s.

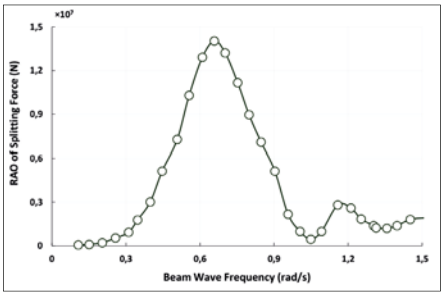

Fig. 6. RAO of splitting force loaded on the brace under beam wave

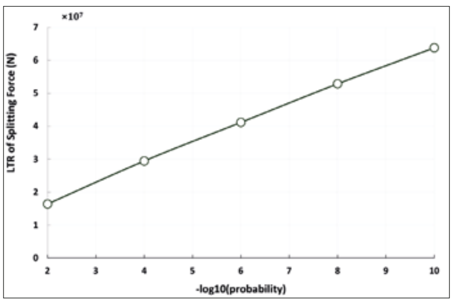

Fig. 7. LTR of splitting force loaded on the brace under beam wave

In order to estimate the variation tendencies of the crack tip opening displacement of the cracked section of brace, the crack tip opening displacement (CTOD) of crack under different splitting forces have been obtained by the method proposed in this research. Additionally, the initial circumferential through cracks have been assumed to be varying from $\pi/24$ to 10/24 with incensement of $\pi/24$ in this research to estimate the crack tip opening displacement of the cracked section under different circumferential through cracks. Shown as in Fig. 8, the values of the CTOD on the cracked section of brace increase smoothly when the cracked damaged brace loaded small splitting force and then increase dramatically when the brace loaded larger splitting forces. In addition, the cracked section of the brace has a larger *CTOD* while the brace locates a greater circumferential through crack at its boundary.

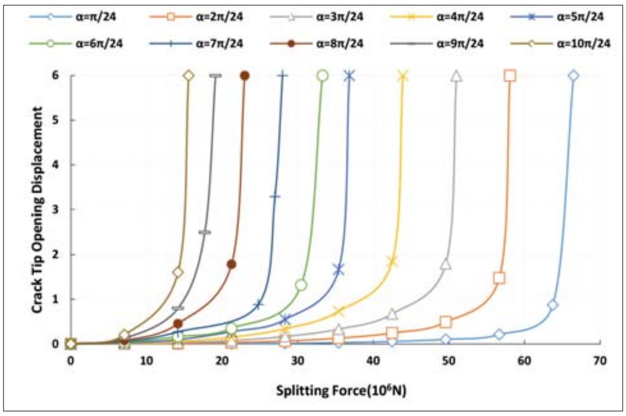

Fig. 8. Variations of crack tip opening displacement of the cracked section of brace

The variation tendency of the tensile plastic zone ($\beta-\alpha$) on the cracked section of MESI's brace is shown in Fig. 9. The tensile plastic zone on the cracked section increase with the splitting force loaded on the circumferential through crack-damaged brace and the values of crack's angle smoothly. Moreover, the cracked section of the brace with greater circumferential through crack has earlier appearance of the compressive plastic zone (parameter $\gamma<\pi$).

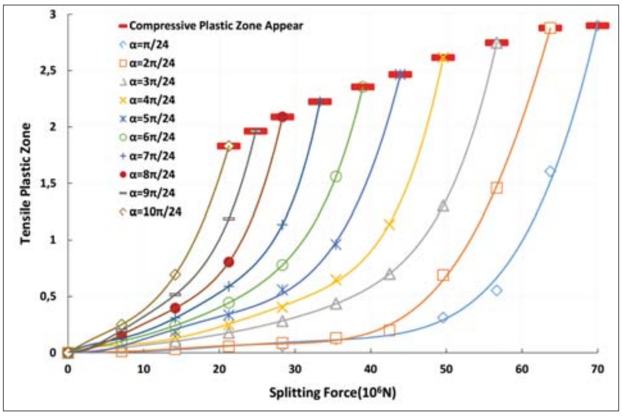

Fig. 9. Variations of tensile plastic zone on the cracked section of brace

The variation tendency of the elastic zone ($\gamma-\beta$) on the cracked section of MESI's brace is shown in Fig. 9. Contrary to the variation tendency of the tensile plastic zone on the cracked section, the elastic zone on the cracked section decrease with the splitting force loaded on the brace and the values of crack's angle. In addition, the cracked section of the brace with greater circumferential through crack has sharper decrease with the splitting force.

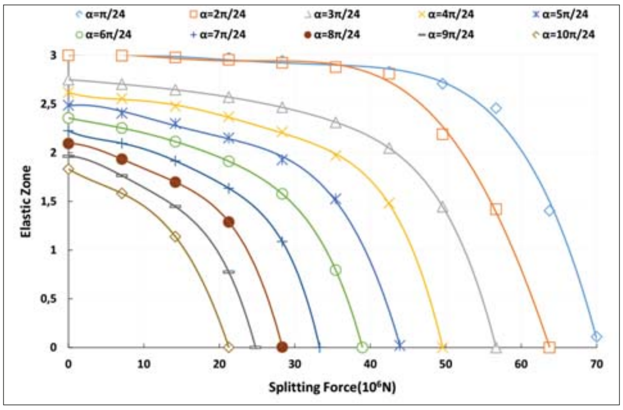

Fig. 10. Variations of elastic zone on the cracked section of brace

The cracked section of the brace would be in full plastic condition when the brace loaded a larger splitting force. Shown as in Fig. 11, the tensile plastic zone of the cracked section is much larger than the compressive plastic zone. However, the tensile plastic zone decrease with the angle of the circumferential through crack while the compressive plastic zone increase with the angle of the circumferential through crack of the brace smoothly.

Additionally, the larger circumferential through crack of the brace locates, the smaller splitting force has been loaded on the crack-damaged brace when the cracked section of brace is in full plastic condition.

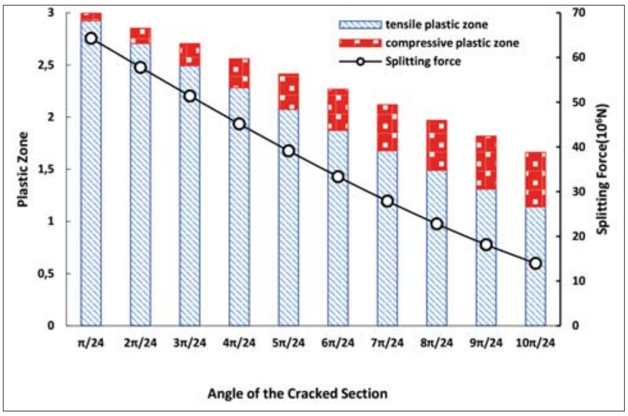

Fig. 11. Variations of tensile plastic zone and compressive plastic zone when the cracked section is in full plastic condition

CONCLUSIONS

An elastic-plastic method to analyze the variations of the crack tip opening displacement, elastic zone and plastic zone of the cracked section of the horizontal brace under beam wave has been proposed in this article. The brace of SEMI is assumed to be located a circumferential through crack at its boundary in this research. In addition, a typical new-generation SEMI that is in practical use was selected to analyze the variation tendency of the cracked brace's parameters using the proposed model and some variation tendency has been got:

(1) The values of the CTOD and tensile plastic zone on the cracked section of brace increase dramatically after a smoothly increase with the splitting force and circumferential through crack. In addition, the cracked section of the brace with greater circumferential through crack has earlier appearance of the compressive plastic zone.

(2) The elastic zone on the cracked section decrease with the splitting force loaded on the brace and the values of crack's angle. In addition, the cracked section of the brace with greater circumferential through crack has sharper decrease with the splitting force.

(3) The tensile plastic zone has a contrary variation tendency to the compressive plastic zone that increase with the angle of the crack of the brace smoothly when the cracked section is in full plastic condition. Moreover, the larger circumferential through crack of the brace locates, the smaller splitting force has been loaded on the crack-damaged brace.

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REFERENCES

1. Colin, H., Espen, F., Martyn, T. (2014). *Worldwide Offshore Accident Databank*, Det Norske Veritas, Oslo.

2. Moan, T. (2009). *Development of accidental collapse limit state for offshore structures.* Structural Safety, 31, 124–135.

3. Zaron, E., Fitzpatrick, P., Patrick J. (2015). *Initial evaluations of a Gulf of Mexico/Caribbean ocean forecast system in the context of the deep-water horizontal disaster.* Frontiers of Earth Science, 9, 605–636.

4. Giovani, D., Mariana, S. (2013). *Risk Based in Inspection Applied to a Semi-Submersible Platform*, Offshore Technology Conference, Rio de Janeiro, Brazil.

5. Sanders, J. L. (1987). *Dugdale model for circumferential through-cracks in pipes loaded by bending.* International Journal of Fracture, 34(1), 71–78.

6. Fei, W., Zheng, L. (2016). *Analytical solution for crack growthing of semi-submersible platform's horizontal brace.* Journal of Engineering Research. 4(1), 146–158.

7. Brighenti, R. (2000). *Surface cracks in shells under different hoop stress distributions*, International Journal of Pressure Vessels and Piping, 77(9), 503–509.

8. Fei, W., Zheng, L. (2017). *Effects of wave loads on the strength of SEMI's horizontal brace*. Proceedings of the Institution of Civil Engineers: Maritime Engineering. 170(2), 163–172.

9. Alexandrov, S., Zerbst, U., Schwalbe, H. (1998). *Limit load solution for cracked tubular T-joints loaded in tension*. Fatigue & Fracture of Engineering Materials & Structures, 21(10), 1249–1257.

10. Lie, S., Chiew, S., Lee, C. (2004). *Fatigue Performance of Cracked Tubular T Joints under Combined Loads*. Journal of Structure Engineering, 130(4), 572–581.

11. Maier, G. (1985). *Case Histories in Offshore Engineering*, Springer Vienna Publishers.

12. Reason, J. (997). *Managing the Risks of Organizational Accidents*, Ashgate Publishers.

13. Inge, L., Odd, O. (2005) *Risk assessment of loss of structural integrity of a floating production platform due to gross errors*, Marine Structures, 17(7), 551–573.

14. Moan, T., Berge, S., Holthe, K. (1981). *Analysis of the fatigue failure of the Alexander L. Kielland*. ASME Winter Annual Meeting, Washington, DC.

15. Nicholson, J.W., Weidman, S.T., Simmonds, J.G. (1983). *Sanders' energy-release rate integral for a circumferentially cracked cylindrical shell.* Journal of Applied Mechanics, 50(2), 373–378.

16. Sanders, J. L. (1972). *Closed form solution to the semi-infinite cylindrical shell problem*. Rotterdam Dam: Delft University Press.

17. Sanders, J. L. (1980). *On stress boundary conditions in shell theory.* Journal of Applied Mechanics, 47(1), 202–204.

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