THE METHOD OF BALANCING THE PRODUCTION AND CONSUMPTION MODEL IN THE CASE OF INDIVISIBLE GOODS

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Abstract: In this article, a detailed case of the unbalanced production-consumption, namely a model, which is used in the case of indivisible articles, is presented. Also, the method of balancing this model is given, relying on introducing a certain ordering relation in consumers' set. First, three popular models: supply chain equilibrium model, reference model for designing business processes and cobweb model are presented. Based on the theory, authors have proposed a unique production and consumption model, together with the method of balancing in the case of indivisible goods.

Keywords: economy, management, production model, consumption model, balancing method.

Introduction

The issue of a balance in the game of producer - consumer (supplier - receiver) is one of the most important in the economy). This caused, that in the economics and management areas, many models, trying to meet this requirement, have been presented so far. Some of the models deal with the information uncertainty, conflicts between parties, such as: producers, consumers, suppliers or wholesalers. One of the reasons of these models' development might be the fact, that many communication processes fails, which are not intervened at all. And this can cause i.e. conflicts between the above entities (Smarandache and Vladutescu, 2014). One of the most known models is a supply chain network equilibrium model, proposed by J. Dong, D. Zhang and A. Nagurney. This is the first supply chain network equilibrium model with random demands. This model consists of manufacturers and retailers, whose requirements related to the choice of sale points, are random. In this model, assumed that the supply chain is made up of competing manufacturers and retailers, and each of them is trying to achieve the highest profits. The use of this model makes it possible to define a framework of cooperation of manufacturers and retailers based on the principle of balance (Dong et al., 2014). This model can be also used in the case of conflicts, which may appear: between client and supplier (with managing the transfers between them, managing packages or contribution to increase the quality) or manufacturer and wholesaler (keeping control, product choosing, distribution and promotion) (Dima et al., 2010).
Another model, making possible to achieve a balance between demand and supply, is reference model for designing business processes in demand-driven supply

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chains. This model fills the gap between supply chain design process and information systems engineering, throughout development of a coherent set of processes, which on one hand are easy to understand by managers operating within this supply chain, and at the same time, may be grounds for the appropriate information system implementation. The main purpose of this model is balancing the level of supply of goods that are available at the markets, characterized by high dynamics of changes in demand (Verdouw et al., 2010). In the literature we can also find models, used to fix some certain problems, i.e. logistics models for waste management. One of these models is descriptive model of logistics processes, which occur in the management of industrial waste. This model presents all the processes and their linkage, as well as highlight the opportunities to improve flow of industrial waste streams (Dima et al., 2011).
However, when it comes to balance by the price (e.g. Allen's RGB cobwebs model), it must be met stringent conditions of the game - the independence of the participants. As price plays an important role in marketing, and each of the participants, know that price, is very often a key factor in the selection of certain goods or services (Brzeziński et al., 2013). Today, although price is important, not getting it directed mainly about achieving balance. When you can control the producers and consumers, it is interesting to look for such algorithms leading to a compromise through appropriate concessions participants in the game, leading straight into balance (Tsai, 2014). Assuming the deterministic game, proposed a solution resolved balancing demand and supply in the one-dimensional case (one-product).

Production and Consumption Model – main assumptions

In article (Ładyga and Tkacz, 2011a) the unsustainable production and consumption model was defined, in which occur $n \geq 2$ contractors, $m \geq 1$ producers and $n - m \geq 1$ consumers of a certain good.
The model assumes that total demand exceeds total supply in a fixed period of time, what we write in the form of so called uncorrected n-dimensional supply-demand vector, fulfilling the conditions:

1) For every $i \leq m \quad p_i \rangle 0$
2) For every $m+1 \leq i \leq n \; p_i \langle 0$
3) $\sum_{i=1}^{n} p_i \langle 0$

The general case was discussed in (Ładyga and Tkacz, 2011a, 2001b, 2012a, 2012b) articles. A detailed case is so called indivisible articles, for example: wash machine, televisions etc. (you cannot buy 1.5 wash machine).

Proofing the adopted hypothesis

In this case, instead of uncorrected supply-demand vector, we consider so called discrete vector, i.e. uncorrected supply-demand vector, of which every coordinate is an integer.

Let there be vectors:

a) discrete vector $\vec{\mathbf{p}} = (p_1, \ldots, p_n)$, where $p_i > 0$ for $i = 1, 2, \ldots m < n$ (p_i - demand of i[th] contractor), $p_j < 0$ for $j = m+1, \ldots n$ (p_j - supply of j[th] contractor), such that $p_1 + p_2 + \ldots + p_n < 0$

b) maximum concession vector $\vec{\mathbf{u}} = (u_1, \ldots u_n)$, where $(u_i \in N \cup \{0\}$, u_i - the size of the maximum possible concession of i[th] contractor), such that $(p_1 + u_1) + (p_2 + u_2) + \ldots + (p_n + u_n) \geq 0$.

c) vector $\mathbf{a} = (a_1, a_2, \ldots, a_n)$, a_i - concession weight of i[th] contractor.

Taking $\mathbf{p}^o = \mathbf{p}$, $\mathbf{a}_o = \mathbf{a}$ for $s \geq 1$ we defining the size of

$$r^s = -\left(p_1^{s-1} + \ldots + p_n^{s-1}\right)$$

$$N^s = \left\{ i \leq n \; : \sum_{j \leq s} a_i^{j-1} r^j \leq u_i \right\}$$

$$\mathbf{a}^s = \left(a_1^s, \; \ldots, a_n^s\right) \quad \text{where}$$

$$a_i^s = \begin{cases} 0 & dla \;\; i \notin N^s \\ a_i^{s-1}\left(B^{s-1}\right)^{-1} & dla \;\; i \in N^s \end{cases}$$

where

$$B^{s-1} = \sum_{j \in N^{s-1}} a_j^{s-1}$$

$$\overline{N}^s = N^{s-1} \setminus N^s \;\; \text{where} \;\; N^o = \{1, \; \ldots, n\}$$

$$\vec{\mathbf{p}}^s = \left(p_1^s, \ldots, p_n^s\right) \text{ where}$$

$$p_i^s = \begin{cases} p_i^{s-1} + r^s \cdot a_i^{s-1} & for \quad i \in N^s \\ p_i + u_i & for \quad i \in \overline{N}^s \\ p_i^{s-1} & for \quad i \in \bigcup_{j < s} \overline{N}^j \end{cases}$$

Iteration process, whose properties was described in [2], we end for *s*-*th* iteration, such that $r^s = 0$. Then vector $\overrightarrow{p^{s-1}}$ balances the model. Let $\vec{x} = (x_1, \dots, x_n)$ be any vector. By $E(\vec{x})$ we understand vector $[E(x_1), \dots, E(x_n)]$ where $E(x_i)$ - integer's total part x_i. Because $\overrightarrow{p^{s-1}} = \overrightarrow{p^s}$, continuing in this article, by overall vector correction vector $\overrightarrow{p^o}$ we adopt vector $\vec{p}^s$. If $E(p_1^s) + E(p_2^s) + \dots + E(p_n^s) = 0$ then vector $E\left(\overrightarrow{p^s}\right)$ balances the model in case of indivisible articles. Assume, that $t = E(p_1^s) + \dots + E(p_n^s) \rangle 0$, what means that supply on certain good exceeds demand. In order to achieve sustainable modem, some producers should reduce the size of production (supply reduction) or, chosen consumers increase demand. In this situation, you may prefer consumer and use a second variant of balancing. It therefore remains to determine which consumer and how much the demand should be increased. The solution can be as follows: add by one unit of good for those consumers, who, the most, gave way.

It is, therefore necessary to prove that:

a) has remained integer of units goods to distribute among consumers,
b) the number of consumers, which can increase demand by one unit of good (in this case, the border in output demand), is not smaller that number of units of goods, what left to distribute.

It will be proved by the statement below.

Statement 1. If:
$M_1 = \{ i \in N^0 : m \langle i \leq n \text{ and } E(p_i^s) - 1 \geq p_i \}$ i $t \rangle 0$ the cardinality of the set M_1 is bigger than t and t is an integer.

The proof.

$t = \sum_{i=1}^{n} E(p_i^s)$ is, obviously, an integer as a sum of integers.

For every $i \leq m$ there is an inequality $p_i^s \geq 0$, or $E(p_i^s) - p_i^s \leq 0$, and for every $i \rangle m$ there i san inequality $p_i^s \leq 0$, or $0 \leq E(p_i^s) - p_i^s \langle 1$. Therefore:

$$t = \sum_{i=1}^{n} E(p_i^s) = \sum_{i=1}^{n} E(p_i^s) - \sum_{i=1}^{n} p_i^s = \sum_{i=1}^{m} \left(E(p_i^s) - p_i^s\right) + \sum_{i=m+1}^{n} \left(E(p_i^s) - p_i^s\right) \langle \sum_{i=m+1}^{n} \left(E(p_i^s) - p_i^s\right) \leq$$

$\# \overline{M}_1$, where $\overline{M}_1 = \{ i \in N^0 : m \langle i \leq n \text{ and } E(p_i^s) \neq p_i^s \}$.

It remains to present, that $\overline{M}_1 \subset M_1$.

Let $i \in \overline{M}_1$, or $E(p_i^s) \neq p_i^s$, $p_i^s \geq p_i^o$, therefore:

$p_i^o \leq p_i^s \langle E(p_i^s)$, or $p_i^o \langle E(p_i^s)$, but p_i^o, $E(p_i^s)$ belong to the set of integers, so there is inequality $p_i^o \leq E(p_i^s) - 1$,therefore $i \in M_1$.

If $\overline{M}_1 \subset M_1$, that $\# \overline{M}_1 \leq \# M_1$, but $\# \overline{M}_1 \rangle t$, hence $\# M_1 \rangle t$, what ends the proof of above statement.

Statement 1 shows that if $t \rangle 0$, then deducting t coordinates from the M_1 set of vector $[E(p_1^s), \ldots, E(p_n^s)]$ by one, then a vector, which balances the model, will be obtained. It arises a question, which of t coordinates from the M_1 set, should deducted by one, in other words, to which of the consumers, an additional unit of good should be assigned. The following solution is being proposed: increase the demand by one unit for those consumers, who are the closest to its limits. This distance is determined by number of $x_i = (p_i + u_i - E(p_i^s) + 1)\ u_i^{-1}$.

The $100x_i$ number, expresses the percentage of concession remaining i^{th} consumer, if his demand will verify to the amount equal to $E(p_i^s) - 1$. In order to indicate a concrete consumer, to which an additional unit of good in M_1 set will be assigned, an ordering relations is introduced.

Statement 2. If for every i, $j \in M_1$, $i\ R\ j \Leftrightarrow \begin{cases} x_i \langle\ x_j, & x_i \neq x_j \\ i \geq j, & x_i = x_j \end{cases}$

then R relations is ordering relations in M_1 set.

The proof. It should present that relation R is:

a) reversible, ie. for every $i \in M_1$ $i\ R_i$

b) weakly symmetric, ie. for every $i, j \in M_1$ iRj i $jRi \Leftrightarrow i = j$,

c) transitive, ie. for every $i, j, k \in M_1$ iRj i $jRk \Leftrightarrow iRk$,

d) consistent, ie. for every $i, j \in M_1$ iRj lub jRi.

ad a) iRi, becuase $i \geq i$ and $x_i = x_i$.

ad b) iRj, ie. $x_i \langle\ x_j$ or $(x_i = x_j\ \text{ oraz }\ i \geq j)$,

jRi, ie. $x_j \langle\ x_i$ or $(x_j = x_i\ \text{ oraz }\ j \geq i)$.

If iRj and jRi, then: $x_i = x_j$, $i \geq j$, $j \geq i$, therefore $x_i = x_j$ and $i = j$.

ad c) iRj therefore $x_i \langle x_j$ or $(x_i = x_j\ \text{ i }\ i \geq j)$,

jRk therefore $x_j \langle x_k$ or $(x_j = x_k\ \text{ i }\ j \geq k)$.

If $x_i \neq x_j \neq x_k$, then $x_i \langle\ x_j\ \langle\ x_k$ or $x_i \langle\ x_k$ therefore iRk.

If $x_i = x_j \neq x_k$, then $x_i =\ x_j\ \langle\ x_k$ or $x_i \langle\ x_k$ therefore iRk.

If $x_i \neq x_j = x_k$, then $x_i \langle\ x_j\ =\ x_k$ or $x_i \langle\ x_k$ therefore iRk.

If $x_i = x_j = x_k$, then $i \geq j$ and or $j \geq k$ or $i \geq k$.

Therefore iRk.

ad d) Proof by contradiction.

If iRj does not occur; then $x_j\ \rangle\ x_i$ or $\left(x_j = x_i \text{ and } i\ \rangle\ j\right)$.

If $x_i\ \rangle\ x_j$, a contradiction that $x_j\ \rangle\ x_i$ or $x_i = x_j$.

If $x_j\ \rangle\ x_i$, a contradiction that $x_i\ \rangle\ x_j$ or $x_j = x_i$.

If $x_i\ =\ x_j$, a contradiction that $i = j$.

It must, therefore, one of iRj or jRi relations occur, what ends the proof of the above statement.

Ordered M_1 set, according R relations, determines by M_{1R}. Increasing demand to the first t^th consumers from the M_{1R} set, by one unit, it leads to sustainable the model. In addition, supply and demand of all contractors, will be expressed by integers. It will be presented in the statement below.

Statement 3. If $t\ \rangle\ 0$,

$$M_1^+ = \left\{\ i \in M_{1R}\ :\ i \leq s\ \right\},$$

$$d_i = \begin{cases} E\ \left(p_i^s\right) - 1 & for\ i \in M_1^+ \\ E\ \left(p_i^s\right) & for\ i \notin M_1^+ \end{cases},$$

$\vec{d} = (d_1, \ldots, d_n)$ then: for $i \leq n$ $\quad d_i$ is an integer and $d_1 + d_2 + \ldots + d_n = 0$.

The proof.

For $i \in M_1^+$ d_i is an integer as a difference of integers. For $i \notin M_1^+$ d_i is also an integer.

$$\sum_{i=1}^{n} d_i = \sum_{i \in M_1^+} d_i + \sum_{i \notin M_1^+} d_i = \sum_{i \in M_1^+} \left(E\left(p_i^s\right) - 1\right) + \sum_{i \notin M_1^+} E\left(p_i^s\right) = \sum_{i=1}^{n} E\left(p_i^s\right) + \sum_{i \in M_1^+} (-1) = t - t = 0$$

Q.E.D

Conclusions

Presented, in the above statement, the method of discrete vector correction it is not just one, obviously, verification method of contractors' supply and demand. Assuming egalitarian concessions, to all players a balancing algorithm model was proposed, in which demand exceeds supply. Algorithm is numerically simple and convergent. The algorithm can be used for many real economic situations such as coordination plans within large business organizations, coordination of international exchange, etc. In today's modern economy, characterized by high dynamics of changes, occurring in the markets, this model can be seen as another tool for verification of supply and demand, occurring in selected markets. Apart from the model mentioned in the introduction, in fields of economy and management there are many other models apllied to balance demand or supply, take actions with regards to rapid changes appearing on the markets or to determine the framework of co-operation between separate cells so that it allows the achievement of the assumed goals by all the participants. Model, proposed in the article is characterized by the fact, that having in mind various economic factors; user can conduct a correction among any relation ordering the set of contractors, and among any assignment. Can, also, use, for this purpose, different measures of "distance" between vectors. What causes that, presented model might be used as a universal tool, used by the economic science.

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METODA RÓWNOWAŻENIA MODELU PRODUKCJI I KONSUMPCJI W PRZYPADKU TOWARÓW NIEPODZIELNYCH

Streszczenie: W niniejszym artykule zaprezentowano szczegółowy przypadek niezrównoważonej produkcji-konsumpcji, model, który używany jest w przypadku artykułów niepodzielnych. Ponadto podana jest metoda równoważenia tego modelu, opierając się na wprowadzeniu pewnej relacji zamówień w zestawie konsumentów. Po pierwsze przedstawione zostały, trzy popularne modele: model równowagi łańcucha dostaw, model projektowania procesów biznesowych i model pajęczyny. Na bazie teorii, autorzy zaproponowali unikalny model produkcji i konsumpcji, wraz z metodą równoważenia w przypadku towarów niepodzielnych.

Słowa kluczowe: ekonomia, zarządzanie, model produkcyjny, model konsumpcji, metoda równoważenia.

平衡生產和消費模式的研究不可分割的商品的情況下的方法

摘要：在這篇文章中，不平衡生產-消費，即一個模型，它採用的是不可分割的物品的情況下的詳細情況，提出了。此外，平衡了該模型的方法給出，依靠引進的消費者集中一定的排序關係。首先，三款熱門機型：供應鏈均衡模型，設計業務流程和蛛網模型參考模型呈現。理論的基礎上，作者提出了一個獨特的生產和消費模式，用在商品不可分割的情況下平衡的方法在一起
關鍵詞：經濟，管理，生產模式，消費模式，平衡法