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OPTIMISATION OF RECEIVABLES MANAGEMENT IN A MINE, USING LINEAR PROGRAMMING

OPTYMALIZACJA ZARZĄDZANIA NALEŻNOŚCIAMI W KOPALNI Z WYKORZYSTANIEM PROGRAMOWANIA LINIOWEGO

The paper presents an example of a theoretical linear programming model in the management of mine receivables. To this end, an economic production model of linear programming was applied to optimising the revenue of the mine. The amount of product sold by the mine to individual customers was assumed as the decisive variable, and the product price was the parameter of the objective function. As for boundaries, upper receivable limits were assumed for each of the adopted receivable collection cycles. The sequence of collection cycles, and the receivable values assigned to them, were adopted according to the growing probability of overdue and uncollectible receivables.

Two receivables-management optimisation cases were analysed, in which the objective function was to maximise the sales value (revenue) of the Mine.

The first case studied in the model involves application of a discount to reduce the product price, in a mine whose production output is not being used to capacity. To improve cash flow, the mine offers its customers a reduced price and increased purchasing up to the mine's capacity in exchange for shortened receivable collection times. Fixed and variable-cost accounting is applied to determine the relevant price reduction.

In the other case analysed, the mine sells as much as its current output allows, but despite that is still forced to reduce the price of its products. Application of a discount in this case (reducing the product price) inevitably involves shortened receivable collection times and reduced costs of financing trade credit.

Keywords: receivables management, linear programming method, rebate, variable cost account

Artykuł przedstawia przykład teoretycznego modelu programowania liniowego w zarządzaniu należnościami kopalni. Wykorzystano w tym celu model produkcyjno-gospodarczy programowania liniowego do optymalizacji wartości przychodu kopalni. Jako zmienną decyzyjną modelu przyjęto ilość sprzedaży produktu kopalni do poszczególnych odbiorców, natomiast parametrem funkcji celu jest cena sprzedaży produktu. W ograniczeniach brzegowych przyjęto górne dopuszczalne wartości należności dla każdego z przyjętych cykli ściągania należności. Kolejność cykli ściągania należności oraz przypisane

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im wartości należności przyjęto według rosnącego prawdopodobieństwa powstawania należności przeterminowanych i utraconych.

Przeanalizowano dwa przypadki optymalizacji zarządzania należnościami, w których funkcją celu jest maksymalizacja wartości sprzedaży (przychodów) kopalni.

Pierwszy przypadek wzięty pod uwagę do modelu to wykorzystanie skonta do obniżenia ceny produktu, w przypadku kopalni posiadającej niewykorzystaną zdolność wydobywczą. Aby poprawić trudności z płynnością, kopalnia proponuje swoim odbiorcom skrócenie cyklu ściągania należności, w zamian za obniżenie ceny i wzrost zakupów do wartości zdolności wydobywczej. W celu ustalenia tego obniżenia ceny wykorzystuje się rachunek kosztów stałych i zmiennych,

W drugim analizowanym przypadku kopalnia, sprzedaje tyle samo co wynosi jej aktualna zdolność wydobywcza, a mimo to jest zmuszona obniżyć cenę swoich produktów. Wykorzystanie w takim przypadku skonta (obniżenia ceny produktu) musi równocześnie być związane ze skróceniem cyklu ściągania należności i obniżeniem kosztów finansowania kredytu kupieckiego.

Słowa kluczowe: zarządzanie należnościami, metoda programowania liniowego, rabat, rachunek kosztów zmiennych

1. Introduction

A mine operating in a market economy has to keep in mind not only its core business, i.e. mining and processing, but also the current competition and, in the majority of cases, the dominant advantage enjoyed by its customers. Therefore, effective sales are dependent upon many factors, the most important of which are the quality of the product, terms of delivery and timely supply, conditions of guarantee (which is a critical factor in the offered price), and terms of payment. As far as payment terms are concerned, it is known that, in addition to the accustomed difficulties receiving timely payments from customers, the situation has deteriorated thanks to the economic crisis, resulting in increasing payment gridlocks in business and numerous companies going bankrupt. It is estimated that approximately 60% of entities in debt to Polish companies pay their debts up to 3 months beyond the agreed date of payment (Bekas, 2012). This means that companies extend each other mutual credit through failing to pay for purchased products on time, which often results in the above-mentioned payment gridlocks, deterioration in or complete loss of financial liquidity, and possible bankruptcy. Such mutual credit, commonly referred to as trade credit, is accomplished in various ways. It can be said to be done in an individual manner by each enterprise-seller, depending on external conditions and in particular on the specific customer.

An inevitable effect of trade credit are receivables, i.e. amounts to be paid by the customer at some point after the products are delivered. There are, in many cases, receivables which are overdue or even completely lost. The question, then, requires an appropriate receivables management policy, for which there are no specific uniform guidelines. As previously mentioned, each policy is implemented individually and can be (Bekas-Nowak, 2008; Czekaj & Dresler, 1998; Panfil, 2004; Partyka, 1993; Sierpińska, 2005, 2007; Sierpińska & Wędzki, 1998):

– aggressive, when the selling enterprise disregards the potential consequences;
– conservative, without risk, but primarily taking into account the situation of each customer, including external conditions;
– moderate, taking risk into consideration, yet primarily focused on the target, i.e. financial liquidity and profit.

Rebates are commonly applied in receivables management, meaning, in general, a reduction in product price, occurring in various forms (Kłak, 2006; Portalska, 2003; CIT Law (Journal of

Laws no. 54, 2000, item 654 as amended)), of which the law concerning VAT (Journal of Laws no. 54, 2004, item 535 as amended) specifies the following:

– allowance, or a price reduction as a form of compensation for losses incurred;
– discount, meaning a price reduction associated with the purchase of large quantities;
– approved complaint, which is the right of the purchaser of a product or service to a price reduction in the case of a justified claim;
– discount, or reduction of the price of a product or service for a purchaser paying in cash or before the agreed date.

Receivables management, including the application of rebates, consists in achieving various goals of the mine, including mainly (Bekas-Nowak, 2008; Kłak, 2006; Micherda, 2005; Paczuła, 2005; Portalska & Kornatowicz, 2003; Rytko, 2009):

– improvement or maintenance of cash flow;
– increased profit;
– increased use of available mining and processing capacity;
– acquisition of new markets or expansion of existing ones;
– ways to compete more effectively.

Receivables management, in addition to the benefits listed above, also involves the necessity to incur extra cost or even loss. Trade credit involves at the same time extra financial cost and the aforementioned overdue or bad debts, consequently leading to the further cost of monitoring the debtors and/or debt collection, as well as posing the risk of losing some of the receivables.

In this context, receivables management should, therefore, consist of comparing benefits achieved to anticipated or unexpected financial inconveniences. This is precisely how the literature on this subject presents the effects of receivables management (Czekaj & Dresler, 1998; Kłak, 2006; Pluta, 1995; Sierpińska & Wędzki, 1998). One paper (Czekaj & Dresler, 1998) formulates the opinion that the main tool should be, in this case, the incremental analysis method, which consists in comparing increases in profit to the increase in costs resulting from offering more competitive payment terms to customers.

In this paper, however, the question is posed whether receivables management can be optimised, for example by maximising sales or profits, minimising the cost of trade credit, minimising the risk of bad debts etc. To this end, the linear programming method was applied, with the aim of optimising the selected goal in receivables management.

2. Optimisation of receivables management

2.1. Preliminary assumptions

A mine selling a uniform, high-quality product was assumed for this study. The price of the product c is, in fact, acceptable to the customers, but the existing competition in the market forces the mine to consider the potential consequences of price reduction. The mine also carefully monitors its customers and has at its disposal specified probabilities of timely collection of receivables, within five days, for various collection cycles from 30 to 120 days. As a result of monitoring, the mine has also established the unit cost of receivables collection for each of its customers and for each of the determined collection cycles. Research into the monitoring results has enabled the mine to establish that, in order to maintain financial liquidity, the cost of trade credit may not exceed the value determined for each collection cycle individually and for the entire mine.

Considering the above conditions, it was found that, in this case, overall mine sales can be maximised by using the so-called economic production model of linear programming. The appropriate mathematical model, in general form, is presented as follows (Gass, 1963; Łapińska-Sobczak 1998; Manteuffel and Seiffart, 1975; Miszczański, 1997; Nykowski 1984; Radzikowski, 1971; Simmonard, 1969; Czopek, 2001a; Czopek, 2001b; Jaśkowski, 1998):

1) optimise (maximise) the objective function:

$$P = c_1 \cdot x_1 + c_2 \cdot x_2 + \ldots c_n \cdot x_n \tag{1}$$

2) for the following side restrictions:

$$\left.\begin{array}{c} a_{11} \cdot x_1 + a_{12} \cdot x_2 + \ldots a_{1n} \cdot x_n \leq b_1 \\ a_{21} \cdot x_1 + a_{22} \cdot x_2 + \ldots a_{2n} \cdot x_n \leq b_2 \\ \cdot \\ \cdot \\ \cdot \\ a_{m1} \cdot x_1 + a_{m2} \cdot x_2 + \ldots a_{mn} \cdot x_n \leq b_m \end{array}\right\} \tag{2}$$

3) with boundaries:

$$x_j \geq 0 \tag{3}$$

4) assuming:

$$m \leq n \tag{4}$$

2.2. Objective function

In the function expressed by the formula (1), individual symbols are interpreted as:

x_j — the mathematically-sought value of the decisive variable, which in this study is the amount of production sold to customer $j = 1, 2, 3 \ldots, n$:

c_j — objective function coefficient, in this case the price per unit of product delivered to the customer j.

For the amount of product delivered to the customer j, the boundaries (3) must be repeated here, i.e. $x_j \geq 0$. From a practical point of view, this is obvious, as total sales to any customer cannot be a negative value. However, from a practical perspective, these boundaries (3) may not be the only ones, since otherwise the objective function could be infinitely large (in the case of maximisation) or equal to zero (minimisation). One such obvious limitation, in the case of a mine, is the available mining and processing capacity Z_w, so an additional restriction has to be introduced to the objective function:

$$\sum_{j=1}^{n} x_j \leq Z_w \tag{5}$$

If the mine carefully monitors its customers, it can also establish real limits on the quantities of product that can be sold to each customer j, namely:

– for the lower limit of sales $Z_j^{\min} \geq 0$, then the condition is formulated:

$$x_j \geq Z_j^{\min} \tag{6}$$

– for the upper limit of sales $Z_j^{\max} \geq 0$, the resultant condition is:

$$x_j \leq Z_j^{\max} \tag{7}$$

Restrictions (6) and (7) can, of course, be presented in combined form:

$$0 \leq Z_j^{\min} \leq x_j \leq Z_j^{\max} \tag{8}$$

The unit price of the product c_j for customer j depends on a number of factors, including the following:

– the receivables management policy adopted by the mine;
– the goal of receivables management of the mine;
– a reliability assessment for each of the customers;
– detailed procedures governing trade credit granted to each customer, the type of rebate offered to every customer, how much the product price is reduced for a given customer;
– whether the mine sells as much of the product as the available production capacity allows, or whether the current sales figures are below mining and processing capacities.

In the case of product price, it is difficult to imagine that all customers would accept increases, except for a new, better-quality product. Trade credit and discounts are mainly ways of reducing product price in return for increased sales or shortened terms of payment for the delivered product. Given the large number of factors affecting the proposed product price, this study is limited to the last factor from the above list, i.e. mining and processing capacity. The most common allowance, i.e. reducing product price, is the discount, which can be presented as follows:

$$\text{ps/os, net ok.} \tag{9}$$

where:

ps — discount percentage (percentage of price reduction);
os — discount period;
ok. — current credit period for the customer;
net — symbolising the preceding credit period.

The discount to reduce the product price is a beneficial solution in the case of a mine whose mining capacity is not fully utilised, i.e. selling below potential capacity. Considering, for example, a mine with a capacity of 120,000 Mg. Sales in a given period are x_p = 100,000 Mg. In the period analysed, the mine achieved the following results:

Revenue on sales P_o = PLN 5 M, operating cost K_o = PLN 4.5 M, including fixed cost K_s = PLN 3.0 M and variable cost K_z = PLN 1.5, including in turn unit variable cost k_{jz} = PLN 15 /Mg. The profit was therefore PLN $Z_o = P_o - K_o$ = 500,000. Despite its apparent profitability, the mine has problems maintaining financial liquidity because of the high cost of trade credit financing, which results from the long receivable collection period of 45 days. To improve the situation, the mine offers its customers a reduced price and increased purchasing up to mining capacity, in return for reducing the receivable payment term to 30 days. Fixed and variable-cost accounting is applied to determine the reduction, as presented in Fig. 1 (Czopek, 2003; Kłak, 2006):

$$K_o = K_s + k_{jz} \cdot x_p \tag{10}$$

$$P_o = c \cdot x_p \tag{11}$$

$$Z_o = P_o - K_o \tag{12}$$

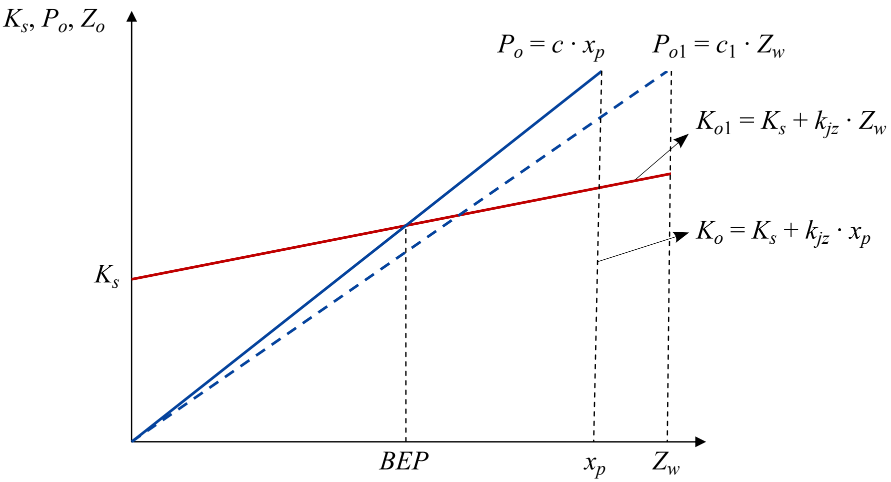

Fig. 1. Increase in sales using discount

The mine has assumed that the operating profit has to remain the same, i.e. PLN 500,000; then the lower limit of price reduction c_1, with an acceptable increase in sales to Z_w = 120,000 Mg, is obtained from the formula:

$$Z_o = c_1 \cdot Z_w - (K_s + k_{jz} \cdot Z_w) \tag{13}$$

Substituting the aforementioned data, the lower price limit, i.e. the discount limit, is:

$$c_1 = \frac{z_o + (K_s + k_{jz} \cdot Z_w)}{Z_w} \tag{14}$$

or finally:

c_1 = 40.00 PLN/Mg

This means that for the sample data the percent of the discount granted would be ps = 20%.

The situation of the mine will, of course, be different when it sells as much as its current capacity, but nevertheless is forced to offer a reduced price compared to current to its customers. This is possible and applies to a mine which has already offered extended payment terms in exchange for purchasing more product. Extended payment terms force the mine to take loans for financing the trade credit granted to the customers.

Application of a discount in this case (reducing the product price) inevitably involves reduced collection times and a reduced cost of financing trade credit. In such a case, price reduction has to result from the comparison:

$$\textbf{reduced revenue} \leq \textbf{reduced cost of running the trade credit} \tag{15}$$

whereby the cost of trade credit K_k is obtained from the formula:

$$K_k = \sum_{j=1}^{n} \frac{r}{100} \cdot \frac{t_j}{360} \cdot N_j \tag{16}$$

where:

- r — interest rate on the bank loan, %;
- t_j — collection time for the receivable in days (e.g. $t_1 = 30$, $t_2 = 45$, …, $t_{12} = 360$);
- N_j — value of the receivable at collection date t_s.

2.3. Side restrictions

In the case of side restrictions described by formula (2), in addition to the already mentioned decisive variable x_j, i.e. sales to customer j, the two remaining inequality factors require explanation, namely, the a_{ij} and b_i factors. Those factors are, to some extent, interrelated, and they mean:

- a_{ij} — the amount of the i factor "used" per unit of product sold to customer j, in this case, a value (b_i) of receivables corresponding to 1 Mg of product sold to the customer j;
- b_i — limited value of the i factor.

As previously assumed, in the problem analysed, the intention is to maximise the revenue of the mine resulting from sales of its product, under the restrictions given in 2.2. These are not the only restrictions, as there are also the b_i values, which – in this case – mean the upper limit of the N_i receivable value fixed by the mine for the cycles of C_i receivables conversion determined and accepted by the customer. The receivables conversion cycle is interpreted, in this case, as the period that elapses from the moment the debt is generated through sale of the product to payment by the customer, i.e. until conversion to cash.

The conversion cycle is obtained from the formula:

$$C_i = \frac{N_i \cdot d_i}{S_i} \quad \text{[days]} \tag{17}$$

where:

- N_i — the value of the receivable for the i cycle, ($i = 1, 2, \ldots, m$);
- d_i — number of days in the i cycle;
- S_i — value of sales in the i cycle.

The following receivable conversion cycles mean, in this case, the periods applied by the mine for which trade credit was granted to individual customers. Those periods may, of course, in practice consist of a different number of days. In this paper, the following numbers were assumed, from the shortest to the longest period: 30, 45, 60, 75, 90, 105, 120 days (i.e., in this paper, $m = 7$).

If the mine manages its receivables effectively, then it incurs a fixed cost for financing trade credit for all values of the cycle i. There are three causes for incurring such a cost:

1) administrative costs resulting from receivables management;
2) cost resulting from overdue, expired and uncollectible receivables;
3) the cost of trade credit granted (k_k) for each cycle i;

$$K_{ki} = \frac{d_i}{360} \cdot \frac{r}{100} \cdot N_i \tag{18}$$

where:

r — interest rate on the bank loan, %.

The b_i value is the sum of the trade credit cost due to these three reasons. Knowing individual b_i values, the mine will introduce another restriction:

$$\sum_{i=1}^{n} b_i \leq B \tag{19}$$

where:

B — the upper limit of the total trade credit cost allowed by the mine for all conversion cycles and for all customers.

Determination of individual values of b_i remains to be done. It may equal any of the b_i values, or the probability of overdue and uncollectible receivables – p_i, e.g. according to the relationship:

$$p_1 > \mathrm{p}_2 > p_3 > p_4 > p_5 > p_6 > p_7 \tag{20}$$

The assumption made in formula 4, i.e. the $m \leq n$ relationship, has to be preserved, so that the solution of the linear programming model will have an unambiguous solution.

Bearing in mind the above, it can be stated that the a_{ij} factors stand for the unit cost of crediting the receivable in the case of each conversion cycle of the receivable i, and each customer j, per 1 PLN of sales of the product to each of the customers j.

3. Summary and final conclusions

Receivables management, in fact, depends finally on the goal to be achieved. Despite the apparent differences among such goals, they are largely interrelated. They may be pursued separately or, as in many cases, all of them can be achieved simultaneously.

Granting trade credit in such cases, and particularly a discount, is intended to improve (increase or decrease) the required parameter.

The paper presents an expanding method to enable the optimisation of the decisions in receivables management. To this end, a theoretical example of mine revenue optimisation was presented, as the application of the method requires the possession of data which mines are reluctant to reveal to third parties. The mines themselves, however, can apply the method to optimise all other important values in receivables management, for example trade credit, probability of receivable collection cycles, optimisation of product prices, etc.

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