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AN AGENT-BASED MODEL OF CONSUMER CHOICE. AN EVALUATION OF THE STRATEGY OF PRICING AND ADVERTISING

MICHAŁ KOT*

SGH Warsaw School of Economics,
al. Niepodległości 162, 02-554 Warsaw, Poland

The authors develop an agent-based model of the market where firms and consumers exchange products. Consumers in the model are heterogeneous in terms of features, such as risk-aversion or owned assets, which impact their individual decisions. Consumers constantly learn about products' features through personal experience, word-of-mouth, or advertising, update their expectations and share their opinions with others. From the supply-side of the model, firms can influence consumers with two marketing tools: advertising and pricing policy. Series of experiments have been conducted with the model to investigate the relationship between advertising and pricing and to understand the underlying mechanism. Marketing strategies have been evaluated in terms of generated profit and recommendations have been formulated.

Keywords: *consumer choice, agent-based model, learning consumers*

1. Introduction

The discovery of causes and mechanisms behind consumers' purchase decisions is at the heart of economic researchers' interest. The study of observed buyers' behaviour conducted in the field of theory of consumer choice enables us to reveal underlying reasons and understand consumers' thought processes leading to certain decisions [2]. The knowledge about the causes behind consumers' actions is valuable in itself and has real-life implications for firms offering their products. Firms can exploit the knowledge regarding the link between their actions and consumers' decisions by increasing consumers' satisfaction, predicting the results of their marketing strategy and in consequence, optimising strategy to maximise business outcomes [1].

*Email address: mk46032@sgh.waw.pl
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The economic research in the field of consumer choice and optimal firm's marketing decisions can be divided into two trends: one focusing on the investigation of consumer choice on a microscale, and the second on the global optimisation of marketing expenses of the firm.

Economic literature focused on the microscale has two main approaches to modelling consumers' choices: static and dynamic. Static models are often formulated in a form of multinomial logistic regression proposed by McFadden [43] and are commonly used to explain FMCG products' sales [5, 24, 29, 33]. In turn, dynamic models incorporate consumers who learn about products' features and adjust their expectations and are often modelled by the Structural Equations [15–17].

Apart from research on microscale consumer choices, economic researchers since the 1950s concentrate on the optimisation of marketing expenditure. The problem of optimal advertising and pricing has been firstly analysed by Dorfman and Steiner [14]. In their model firm maximises profits if the marginal gain from advertising is equal to price elasticity. Nerlove and Arrow [46] test different specifications of models linking lagged advertising to sales. In the case of the log-log model, they find out that it is optimal to keep fixed advertising to sales ratio.

Since the 1970s, the marketing models have treated advertising and pricing as control variables in the optimisation problems with the finite planning horizon [22, 23, 55]. A detailed summary of these models can be found in Teng and Thompson [54]. In the last 40 years, researchers have focused on determining optimal strategies for pricing and advertising for new product introductions [27, 51] and certain product categories [12, 19, 32, 52]. All mentioned models provide insightful results and useful recommendations for special cases: either new product development or specific product categories. However, there is no generalised, coherent approach that would allow the study of optimal advertising and pricing for firms and products at different stages of development, e.g., well-established products, or a priori test considering variants of marketing strategy.

Although the aforementioned literature focuses on the optimisation of a firm's marketing strategy as a valuable source of knowledge, it does not take a few crucial elements of the problem into account. Firstly, the literature focuses on the research of the aggregated measures and omits the micro-foundations of the problem, e.g., heterogeneous consumers who react differently to the firm's actions. Secondly, one must remember that consumers learn about products and adjust purchase decisions over time [48]. Finally, especially younger, Millennial consumers share their views with others and base their decisions on external reviews [40].

Therefore, in this paper, we want to propose a method allowing for the evaluation of a firm's marketing strategy defined as the 4Ps: product, price, place, and promotion [42] (henceforth marketing strategy) in an environment where heterogeneous consumers communicate one with another, share their opinions and constantly learn about product features. We will focus our research on price and promotion which are firms' decision variables, while product and place will be playing supporting roles. In the case of the price element, we will enable firms to choose the level of a price discount on the product they offer, with the limitation that they must stay profitable. In the case of the promotion element, we would like to emphasise, that firms can use a variety of methods to communicate with consumers. It is especially important in the concept of Integrated Marketing Communication, which aims at delivering a consistent message to the target audience with the use of different communication techniques [30]. Among the possible ways to talk to consumers, the literature points out advertising, direct response, sales promotions, public relations [6], and recently social media [56]. In this article, we will limit the promotion aspect to advertising, by which we assume delivering paid ads to consumers, e.g., TV, radio, display or video online formats, paid results in SEM or advertisements in Social Media [45]. For simplicity, we will assume that advertising contacts impact consumers similarly, regardless of the duration or specific medium. In our model, we will enable firms to choose the level of advertising intensity, that is the volume of advertisements being served in the given unit of time.

Since we want to derive our model from the micro-foundations to implement consumers' heterogeneity and introduce network effects, we will use an agent-based framework to simulate the consumers' responses to the firm's actions. According to Macal and North [39], agent-based modelling (ABM) is the right method to model problems with such two features. What is more, researchers using ABM indicate more advantages of this method: (1) bottom-up approach allowing to analyse of single consumers' actions and total system's behaviour [58], (2) assessment of results' uncertainty [31], and (3) evaluation of counterfactual scenarios [36]. Moreover, the agent-based approach is used to model consumer choices in problems similar to ours: operator choice in the telecommunications industry [21], decoy effect [58], new store openings [38], or new product introduction [28, 49, 50, 53]. An agent-based model used to simulate consumer choice that is the closest to ours is the one of Izquierdo and Izquierdo [26], where consumers learn and communicate one with another. One of the interesting results of their model is a market collapse in the case of a lack of peer-to-peer communication. The authors suggest that a remedy to this problem may be an introduction of additional signals.

We would like to stress a crucial feature of consumers in our model, namely their ability to learn about the products' characteristics. We assume that consumers are unable to gain perfect information, even by consumption [48], but with time they can learn about each product's quality. Learning in the proposed model is possible because consumers receive biased signals from various sources: their product consumption, social learning, or the firm's advertising. We define consumer learning as an ability to process gathered signals, review initial expectations, update beliefs on products' quality [11], and decrease uncertainty [8]. Our definition of learning is consistent with the literature on Bayesian learning consumers, where one can find the study the variety of different signals that affect consumers: previous product's experience [16], product's price [9], firm's advertising [17], products' reviews [25, 37], and social learning [7, 35]. Based on the received signals, consumers can formulate expectations and optimise future consumption twofold: in the finite or infinite planning horizons, or they can focus only on the next decision [10]. Consumers are assumed to be heterogeneous in terms of risk aversion and the length of the planning horizon and thus their decisions differ [8, 47].

We emphasise that there were two main sources of inspiration for the mechanics of our model. Firstly, it was a model of market trade proposed by Izquierdo and Izquierdo [26], and secondly, it was a mechanism of Bayesian belief updating proposed by Erdem and Keane [16] and subsequent authors. However, our model's main contribution is that it merges the two mentioned approaches into one consistent system that enables one to simulate the result of a given marketing strategy, evaluate what-if scenarios, and find the solution. Moreover, we have further developed each component approach. In comparison to the model of Izquierdo and Izquierdo, we increase the number of possible signals and enrich agents with additional features. In the case of the belief update mechanism, we use the Bayesian learning approach for simulations in an agent-based framework instead of building structural equation models.

We list three objectives of the study presented in this article. Firstly, we want to merge the model of Izquierdo and Izquierdo [26] with the Bayesian belief updating [16] into a coherent framework and implement improvements mentioned in the previous paragraph. Secondly, we want to use our model to evaluate how different combinations of discounts and advertising intensity impact consumers' decisions and the firm's sales and profit. Thirdly, based on the results of the evaluation, we want to solve the firm's problem and recommend an optimal marketing strategy in terms of the intensity of advertising and price level for a given market setup.

The rest of the paper is structured as follows: we provide the specification of the model in Section 2 and set up the experiments in Section 3. In Section 4 we present and discuss obtained results and finally in Section 5 we summarise the paper and propose future developments.

2. Model of consumer choice

In this section, we will discuss the specification of the proposed model. We will separately present (1) entities of supply and demand sides that take part in the simulation, their features and behaviour; (2) necessary simplifying assumptions enabling to perform experiments, and (3) mechanisms governing the whole system's dynamics.

The proposed market model consists of two distinct groups of entities which will be called its sides. The first is the supply side, i.e., firms offering products and the other is the demand side, i.e., the consumers making purchasing decisions. The sizes of both sides of the market are respectively: J of firms indexed by $j \in \{1, 2, ..., J\}$, and N consumers indexed by $i \in \{1, 2, ..., N\}$. The proposed model assumes that the market exists through T successive periods $t \in \{1, 2, ..., T\}$ which correspond to real-world days. In all periods of the market's existence, the simulated process looks the same: consumers make decisions and purchase new products, and afterwards update expectations related to the quality of products. In this model, we assume that consumers learn about products' quality through the processing of various types of signals. All signals are imperfect and biased but a consumer can use them to update his beliefs and thus improve their expectation by increasing accuracy or lowering uncertainty. In this model, we assume that there exist four types of signals: (1) product purchase, (2) product consumption, (3) firm advertising, and (4) communication between consumers. Signals 1 and 3 are driven by firms, while 2 and 4 are related to the market's demand-side, and therefore they will be discussed separately regarding the market's side.

2.1. Model's supply-side

The supply side in the proposed market model consists of competing firms that offer consumers products that are heterogeneous in terms of quality. We make assumptions regarding this side of the model in line with Erdem and Keane [16]. We assume that the quality of the product is the only synthetic metric that aggregates all product features important for the consumer. In addition, we assume that consumers do not know exactly what the quality of a given product is. Contrary, we assume that consumers' knowledge is limited – they know only the expected a priori quality of all products Q which is fixed during simulation. Moreover, each consumer has an individual expectation of quality and uncertainty of the quality of each firm's product which is learned and may vary during simulation. Hence, designating the true product quality level as Q_j for a jth firm, we assume it to be normally distributed $Q_j \sim N(Q, \sigma_{j0}^2)$, where σ_{j0}^2 is a variance of product qualities in the market [16].

Supply side's signals. As already mentioned, there exist signals originating from the supply side which can be used by consumers to update their beliefs. Firstly, consumers can learn about the quality of a product through its purchase. The reason why the purchase of a product does not enable the consumer to obtain unbiased information about the quality of the product results from the manufacturing process. The production process does not guarantee that all units of the product will be of the same quality, and therefore the consumer cannot be sure if the quality of the product is not the result of a production error. Hence, we assume that if the ith consumer purchased the product at time t, the quality of a product unit Q_{ijt}^f is as per equation (1) and the uncertainty resulting from the manufacturing process is given by ξ_{ijt}.

$$Q^f_{ijt} = Q_j + \xi_{ijt},\ \xi_{ijt} \overset{iid}{\sim} N(0, \sigma^2_\xi) \tag{1}$$

Secondly, the company's advertising gives consumers a chance to learn about the quality of the product. The company's message does not provide the consumer with perfect information about the quality of the product, because it may deliberately omit certain product characteristics or be too complex to understand [16]. Let A_{ijt} denote the advertising signal concerning the brand j if the ith consumer received it at the time t. The advertising signal represents the quality of the product to the consumer, according to the equation (2) with the uncertainty term α_{ijt}.

$$A_{ijt} = Q_j + \alpha_{ijt},\ \alpha_{ijt} \overset{iid}{\sim} N(0, \sigma^2_\alpha) \tag{2}$$

Firm's decision variables. In the model, we assume that firms can impact consumers twofold: by advertising or by pricing policy. In the next paragraphs, we will describe the firm's decision process which leads to the creation of a strategy regarding advertising and pricing, but we would like to highlight that these plans are prepared before the simulation starts and cannot be changed during the simulation.

In the case of advertising, the first step for the firm is to define the amount of budget B_j to be spent on serving advertisements. In this model, we omit costs related to the creation of advertisements. After the budget has been set, the firm calculates the volume of advertising contacts it can generate c_j by dividing B_j by a cost of a single contact c_k. For the sake of simplicity, it is assumed that c_k is equal for all firms and is not related to the size of the advertising budget. Knowing the number of advertising contacts, the company selects the phasing strategy, i.e., the distribution of advertising contacts between moments in time. Let the number of advertising contacts assigned by the jth company to the period t be given as c_{jt} and the condition $\sum_{t=1}^{T} c_{jt} = c_j$ be fulfilled. It is assumed in this paper that the firm can use only a continuity strategy with a flat allocation of contacts across all periods.

In the case of the pricing policy, the first step for the firm is setting the regular price of the product p^r_{jt}. We assume that all firms have the same fixed regular price. After the regular price has been set, the firm chooses the times of occurrence and depth of the price discounts that temporarily lower the regular price. Let the set P contain the moments in time when the firm lowers the price of the product and $1_p(t)$ be the characteristic function of P. Since each firm produces its products at cost C, we assume that discount D cannot exceed $p^r_{jt} - C$ to prevent the firm from being unprofitable. The price of the product of the *j*th company at time *t* faced by consumers marked as p^t_{jt} is

$$p^t_{jt} = p^r_{jt} - 1_p(t)D \tag{3}$$

2.2. Model's demand-side

This section will outline the demand side of the proposed market model, which consists of heterogeneous consumers making purchasing decisions. The presentation will include: (1) describing the network of connections between consumers, (2) signals about product quality arising on the demand side of the market, and (3) discussing consumer characteristics and the impact of these characteristics on consumer behaviour. In addition, in this section, we will describe two key mechanisms governing the model: the mechanism of updating consumer expectations and the mechanism of consumer choice.

Network of consumers. An important feature of the discussed model is the network aspect, which mirrors the ability of consumers to communicate with each other. To map this phenomenon, all consumers are placed in a network that defines the relationship between them. The network of connections has the form of an undirected graph $G = (V, E)$, where V is the set of the graph's vertices and E is the set of its edges. In G each consumer is a single vertex and the association between two consumers is an edge. The presence of undirected edges in the graph results from the assumption that the familiarity relation is symmetric. Since we do not want to make any assumptions about network structure that may impact the paper's goals, in this paper we will investigate a random network structure known as the Erdos–Renyi model [18]. The purpose of mapping the network of connections with the use of a graph is to place the phenomenon of social learning in the discussed model, i.e., the observation of decisions made by other consumers and their reactions to purchased products. In G we define the neighbourhood of the vertex v denoted as $N_k(v)$, which is the set of all vertices of the graph whose distance from the vertex v is less than or equal to *k*. We assume that the *i*th consumer can observe decisions of consumers from $N_k(i)$ and knows the average quality level of the products they purchase.

Demand side's signals. In the section on the supply side of the market, we discuss two out of four sources of signals available to consumers which were dependent on firms. In the following paragraphs, we will present an extension of the signal related to the purchase of a product unit and will introduce a third possibility of learning, resulting from the exchange of information between consumers.

Firstly, in the presented model we assume that the purchase of the product does not give the consumer clear information about the quality of the product due to the nature of the production process. Here we extend the assumption regarding uncertainty resulting from the manufacturing process and additionally assume that the consumption of the product also does not provide the buyer with full information about the features of the product. Such a situation may result, inter alia, from the fact that certain features of the products are discovered gradually after repeated use or consumption of the product. For this reason, let $\eta_{ijt} \overset{iid}{=\sim} N(0, \sigma_\eta^2)$ denote the randomness associated with the fact that the consumer is not able to correctly recognise the features of the product during its consumption. Then, equation 4 combines the uncertainty of a product unit and consumption. From the construction of δ_{ijt} it follows directly that it is an independent random variable with the distribution $\delta_{ijt} \overset{iid}{\sim} N(0, \sigma_\xi^2 + \sigma_\eta^2)$ which captures the uncertainty resulting from the manufacturing process and consumer's biased assessment of the product's quality [16].

$$Q_{ijt} = Q_{ijt}^f + \eta_{ijt} = Q_j + \xi_{ijt} + \eta_{ijt} = Q_j + \delta_{ijt}, \delta_{ijt} = \xi_{ijt} + \eta_{ijt} \tag{4}$$

Secondly, each consumer can observe the purchasing decisions of other consumers in their neighbourhood $N_k(v)$. In addition, we assume, after Izquierdo and Izquierdo [26], that the consumer communicates with other consumers from their neighbourhood and that they know the quality of the products purchased by them. To simplify the simulation, we assume that other consumers share the quality of their products immediately after purchase and their reviews are treated as equally important. The construction of a synthetic signal W_{ijt} that aggregates the quality of products purchased by consumers from the environment of the ith consumer is presented in equation 5. $\sum_{m \in N_k(i)} Q_{mjt}$ is the sum of independent random variables with the same normal distribution, and hence the distribution of W_{ijt} is given as $W_{ijt} \overset{iid}{=\sim} N\left(Q_j, \frac{\sigma_\delta^2}{|N_k(i)|}\right)$.

$$W_{ijt} = \frac{\sum_{m \in N_{k(i)}} Q_{mjt}}{|N_k(i)|} \tag{5}$$

Belief update mechanism. The key mechanism in the proposed model is the ability of consumers to learn and update their expectations regarding product quality by processing the received signals. Let the expectation of the *i*th consumer regarding the quality of the *j*th product at time $t(Q_{ijt})$ be given as $E(Q_{ijt} \mid I_i(t))$ where $I_i(t)$ defines the information set of the *i*th consumer, i.e., all information available to them that indicates the quality of the *j*th product. At the moment $t = 0$, when $I_i(t) = \varnothing$, $E(Q_{ijt} \mid I_i(t)) = Q$. At any time starting from $t = 1$, provided that at least one signal is received, the information set $I_i(t)$ is no longer empty. The updating mechanism allows the consumer to verify their expectations based on the received signals at the end of a given moment and define a posterior forecast. After moving to the next step, the posterior expectation from the previous period becomes the consumer's prior knowledge and the process is repeated.

In each of the T periods, the consumer has a chance to receive three different types of information, signalling the quality of the product which were presented in eqs. (2), (4), and (5). Let the fact that the consumer receives a given signal at moment t be denoted by S^C_{ijt}, S^A_{ijt}, and S^W_{ijt} for the consumption, advertising and social learning signals, respectively. Based on the prior expectation and the signals received in a given iteration, the consumer updates their expectations regarding two indicators: (1) product quality and (2) uncertainty. The expectations update process, which was developed based on [13] and [16], will be discussed in the following paragraphs and broken down into updating the uncertainty related to the quality of a given product and updating the expected quality of the product. This order results from the fact that the updated uncertainty expectation is necessary to update the expected product's quality.

Uncertainty of the consumer regarding the quality of a given product is defined by the parameter σ^2_{ijt}. At the time t = 0, the value of this parameter is $\sigma^2_{ij0} = \sigma^2_{j0}$, which indicates the basic level of consumer uncertainty. At subsequent times, the consumer updates their uncertainty as shown in equation (6). One important extension of our model in comparison to the original model of Erdem and Keane [16] is that consumers may not remember all signals but their memory may be limited to m previous periods. When m = 0, consumers remember only the last period, while when m = T, they have perfect memory as in Erdem and Keane [16].

$$\sigma^2_{ijt} = \left(\frac{1}{\sigma^2_{j0}} + \frac{\sum\limits_{s=\max(1,\,t-m)}^{t} S^C_{ijs}}{\sigma^2_{\delta}} + \frac{\sum\limits_{s=\max(1,\,t-m)}^{t} S^A_{ijs}}{\sigma^2_{\alpha}} + \frac{\sum\limits_{s=\max(1,\,t-m)}^{t} S^W_{ijs}}{\sigma^2_{\delta}} \right)^{-1} \tag{6}$$

Equation 6 shows that the uncertainty is a decreasing function of the number of received signals. Moreover, the lower the uncertainty related to the received signal the more this signal reduces the overall uncertainty related to the quality of the product. The expectation of product quality is constructed based on the consumer's processed signals. To update expectations, we construct Kalman gains β, which can be interpreted as the weight given by the consumer to the signals coming from a given source. The weight of a signal reflects how reliable a given signal source is from the point of view of the consumer. The construction of Kalman gains is presented in equation 7. By estimating the Kalman gains, the consumer can determine the posterior expectation $E(Q_{ijt}^{P} \mid I_i(t))$. Posterior is the outcome of two elements: (1) prior $E(Q_{ijt} \mid I_i(t-1))$ and (2) recent signals Q_{ijt}, R_{ijt} and W_{ijt} which update the information set from $I_i(t-1)$ to $I_i(t)$

$$\beta_{ijt}^{C} = \frac{\sigma_{ijt}^{2}}{\sigma_{ijt}^{2} + \sigma_{\delta}^{2}},\ \beta_{ijt}^{A} = \frac{\sigma_{ijt}^{2}}{\sigma_{ijt}^{2} + \sigma_{\alpha}^{2}},\ \beta_{ijt}^{W} = \frac{\sigma_{ijt}^{2}}{\sigma_{ijt}^{2} + \sigma_{\delta}^{2}} \tag{7}$$

The posterior expectation is represented in equation 8. The consumer updates their expectations based on the surprising elements of the received signals, expressed by the differences between the value of the received signal and the consumer's expectation

$$\begin{aligned} E\left(Q_{ijt}^{P} \mid I_i(t)\right) &= E\left(Q_{ijt} \mid I_i(t-1)\right) + \beta_{ijt}^{C}\left(Q_{ijt} - E(Q_{ijt} \mid I_i(t-1)\right) \\ &+ \beta_{ijt}^{A}\left(A_{ijt} - E(Q_{ijt} \mid I_i(t-1)\right) + \beta_{ijt}^{W}\left(W_{ijt} - E(Q_{ijt} \mid I_i(t-1)\right) \end{aligned} \tag{8}$$

Consumers' features. Consumers have additional features that play a crucial role in their decision-making process. Consumers are described with the same list of features but remain heterogeneous in terms of the specific values assigned to given attributes. The features that describe the consumer are the price the consumer is willing to pay for quality P_i [26], accumulated inventory level S_{it}, and risk-aversion r_i [16]. The listed features will be described in the next paragraphs.

P_i reflects the value that a consumer assigns to each unit of product quality and can be taken as an indication of the wealth of that consumer.

The level of inventory of a consumer impacts the need to purchase a product at a given moment t. We introduce it to improve the model's flexibility and ability to simulate markets of products with different repurchase times. We assume that only when the quantity of the product is depleted, the consumer starts looking for the product and immediately make a purchase decision. If the product was purchased, i.e., the consumer was able to pay the price of $\min_j p_{jt}^{t}$, then the consumer restores the stock for S moments. If the consumer is unable to pay the minimum purchase price required by any firm at time t, they wait for the next moment when they repeat the product search process. As a result, relatively richer consumers will replenish their stocks on an ongoing basis, while others may refuse to consume the product for the entire lifetime of the model.

Risk-aversion of given consumer influences which products they prefer. The consumer may be inclined to make risky choices and prefer less known products. It allows such consumers to learn about the market offer and consciously choose the best product. On the other hand, the consumer may be risk-averse and only repeat purchases of products well known to them.

We assume that among the above-mentioned features the level of inventory S_{it} may change in subsequent periods, while the reservation price and the relation to risk are predetermined at $t = 0$ and remain fixed.

2.3. Consumer's decision mechanism

In this part, we will describe the exchange mechanism linking the supply and demand sides together in each simulation's step. The purpose of the exchange mechanism is to match willing buyers with firms and perform as many transactions as possible. From the supply side, firms offer their products at p^t_{jt}, which reflect their cost of production and margin. From the demand side, consumers treat products' prices as given. If in a given moment consumer's stock has depleted, they seek the product maximising their utility function U_{ijt}, as in equation (9).

$$\begin{aligned} &\max_j E\left(U_{ijt} \mid I_i(t)\right) = E\left(Q_{ijt} \mid I_i(t)\right) - r_i\sigma^2_{ijt} \\ &\text{s.t.} \\ &P_i E\left(Q_{ijt} \mid I_i(t)\right) - p^t_{jt} \geq 0 \\ &S_{it} \leq 0 \end{aligned} \tag{9}$$

The form of the consumer's utility function was inspired by Markowitz's construction of an effective frontier in modern portfolio analysis [41]. In equation (9), the portfolio's expected return has been changed with the consumer's expectation of the product's quality. The second term of the utility function reflects consumers' uncertainty of products' quality modified by personal risk aversion factor. Our approach to the problem of the consumer is presented in Fig. 1.

When two or more products have the same $E(U_{ijt} \mid I_i(t))$, we assume that the consumer will choose the product which maximises their surplus $P_i E(Q_{ijt} \mid I_i(t)) - p^t_{jt}$. If still, two or more products will meet the conditions, the consumer will randomly pick one. In equation (9), if the consumer is risk-seeking and r_i is close to 0, he will pick the product of the highest expected quality regardless of the risk. Otherwise, he will seek an optimal trade-off between expected quality and risk.

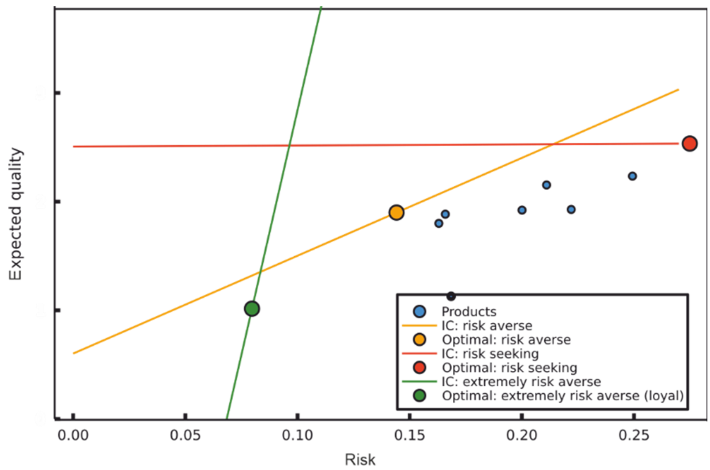

Fig. 1. Optimal consumer choice for various levels of risk-aversion

To sum up the presentation of the model, we present its mechanics, observed from a point of view of the ith consumer in a single moment of the model life, in a simplified form a list below.

- If the reservation price and expected quality allow purchasing of any product $P_i E(Q_{ijt} \mid I_i(t)) - p^t_{jt}$, choose the product that maximises expected utility. If more than one product is chosen, the consumer will pick the one which maximises their surplus. If still more than one product can be chosen, the consumer will choose a product at random;
- If any product was purchased, evaluate its quality.
- If any consumer from $N_k(i)$ has bought the product, observe the quality of their product.
- If an advertisement is received, process the information about the product's quality.
- Recalculate the uncertainty as per equation (6).
- Update prior belief with the recent signals as per equations (7) and (8).

3. Simulation setup

In this section, we will discuss the setup of conducted experiments. We will present the parameters that control the dynamics of the simulation and the setting of the firm's decision variables concerning marketing plans. We will present all variables controlling the system along with the reference to their counterparts discussed in the previous sections of this article and feasible ranges of variation in Tables 1 and 2.

Table 1. Single-valued, fixed simulation parameters

Definition	Variable in GitHub code	Symbol in the paper	Range
Number of firms	num_firms	J	[1, ..., ∞)
Number of consumers	num_consumers	N	1, ..., ∞)
Number of links between consumers in the network	link_vol	$\|\|E\|\|$	1, ..., ∞)
Agent's neighbourhood	neigh_dist	K	1, ..., ∞)
Market's average quality of products	q	Q	[0, 2]
Uncertainty of products' quality	σ2_v0	σ^2_{v0}	[0, 1]
Uncertainty of experience signal	σ2_∈	σ^2_{ε}	[0, 1]
Uncertainty of advertising signal	σ2_α	σ^2_{α}	[0, 1]
Duration of stock/repurchase time	max_stock_period	S	[0, T]
Memory of an agent	buyer_memory	M	[0, T]
Price of product	p	p^r_{jt}	[0, 100]
Cost of production	c	C	[0, p]
Level of discount	d	D	[0, p-c]
Cost of serving advertisements	cpp	c_k	[0, 100]
Share of population served advertisements	ad	c_j	[0, 1]
Additional share of population contacted by advertisements of Firm #1	ad_i		[–ad, 1 – ad]
Simulation's length	max_iter	T	1, ..., ∞)

Table 2. Random simulation parameters

Definition	Variable in GitHub code	Symbol in the paper	Distribution
Wealth of agent	wealth	P_i	Tri (0, W, 100)
Risk aversion of agent	risk	r_i	Tri (0, R, 100)

In Table 1, we present fixed, single-valued parameters which control the global behaviour of the simulation, while in Table 2 the probability distributions that control the behaviour of a single consumer. In this model, we have two parameters that control the behaviour of a single consumer: reservation price P_i and risk-aversion r_i. In both cases, we assume that the distribution of the parameters in the consumer population is of a triangular distribution. We do so because in this application the triangular distribution has two advantages: firstly, it allows one to control minimum and maximum values, and secondly, it allows one to conduct experiments for different levels of dominant values.

We assume that the distribution of the reservation price has a minimum value of 0, a maximum value of 100, and a dominant $W \in [0, 100]$. Similarly, in the case of risk-aversion, we assume that in a population of consumers, its distribution has a minimum value equal to 0 (risk-seeking), the maximum value equal to 100 (risk-averse) and the dominant $R \in [0, 100]$.

Each conducted experiment consists of three phases: setting of parameters, simulation execution, and result collection. Firstly, in the parameters' setting phase we prepare a Cartesian product of all parameters' values so that each combination be tested. Secondly, we simulate the system with a given combination of parameters' values 50 times. Finally, we gather the results of each simulation run and conduct a quantitative analysis of them. The agent-based model and simulation environment is built from scratch in Julia 1.6.0 programming language [3] and the code can be found on GitHub repo[1]. We run three experiments with different parameters' settings which enables us to present the capabilities of our model sufficiently and find answers to the research questions proposed in the first part of this article. Objectives for the experiments are listed below. Corresponding parameters can be found in Table 3.

Table 3. Values of parameters used in experiments

Variable in GitHub code	Experiment 1	Experiment 2	Experiment 3
num_firms	4	4	4
num_consumers	1000	200	200
link_vol	1000	200	{0, 40, 80, 120, 160, 200}
neigh_dist	1	1	1
q	1	1	1
$\sigma 2 v 0$	0.5	0.5	0.5
$\sigma 2 \varepsilon$	0.5	0.5	0.5
$\sigma 2 \alpha$	0.5	0.5	0.5
max_stock_period	10	{10, 20, 30}	20
buyer_memory	365	365	365
p	60	60	60
c	30	30	30
d	10	{0, 2.5, 5, 7.5, 10, 12.5, 15, 17.5, 20, 22.5, 25, 27.5, 30}	{0, 2.5, 5, 7.5, 10, 12.5, 15, 17.5, 20, 22.5, 25, 27.5, 30}
cpp	3	3	3
ad	0.05	0.05	{0, 0.025, 0.050, 0.075, 0.10}
ad_i	0.025	{0, 0.025, 0.05, 0.075, 0.10}	0
max_iter	365	365	365
wealth	50	{25, 50, 75}	50
risk	50	{25, 50, 75}	50

[1]https://github.com/Michal-Kot/ABM_Consumer_Choice

The first experiment is designed to assess the dynamics of a single simulation run, test the model's behaviour for different combinations of parameters, and confirm the model's ability to return reasonable results. The second experiment intends to analyse the complexity of the relationship between advertising and price promotions and to investigate whether the synergy between the two marketing activities exists. The third and final experiment is created to evaluate the company's marketing strategies in terms of planning price and advertising and create a recommendation.

We would like to emphasise a few assumptions we had made when we planned the simulations:

- All simulation runs to cover the distance of 365 steps with each step reflecting one day.
- The simulation environment in Experiments 2 and 3 consists of 200 consumers that form an Erdos–Renyi's random graph and each consumer can observe only the actions of their direct neighbours $(N_1(i))$.
- Each firm's expected quality and quality uncertainty, along with a variance of experience (σ_ε^2) and advertising (σ_α^2) signals are fixed and equal.
- Regular prices, costs and base advertising intensities of each firm are fixed and equal. Firm 1, which we consider as our firm for which we solve the problem, can modify the price and advertising intensity by changing discount (*D*) and the increasing additional intensity of advertising (ad_i), over the market's average level (ad) respectively.
- The product offered to the consumers is an FMCG good. In each experiment, the repurchase times are different to simulate outcomes of strategies for products of short purchase journey, e.g., fruits [34] or dairy products, and a long one, e.g., detergents [4].

4. The results of the experiments with the model

In Experiment 1, we record detailed outcomes of a single simulation run. Figure 2 presents the dynamics of an average quality expectancy and uncertainty registered for all steps of a simulation. Firm 1 uses the price reduction by 10 units (1/6 of the regular price of 60) and extends its advertising intensity over the market's average by 50%. As a result, it increases the volume of signals sent to consumers and hence final uncertainty for this firm is lower than for its competitors, which corresponds with the results of Erdem and Keane [16]. The reason is that consumers are more likely to choose a product whose price has been reduced (both, consumers who changed their preferred product or those who otherwise would be not able to afford to buy the product) and about which they have received more advertising signals. To underline the trajectory of firms' quality-uncertainty tuple, we also provide a chart with snapshots made after 10, 100 and 365 steps shown in Fig. 3.

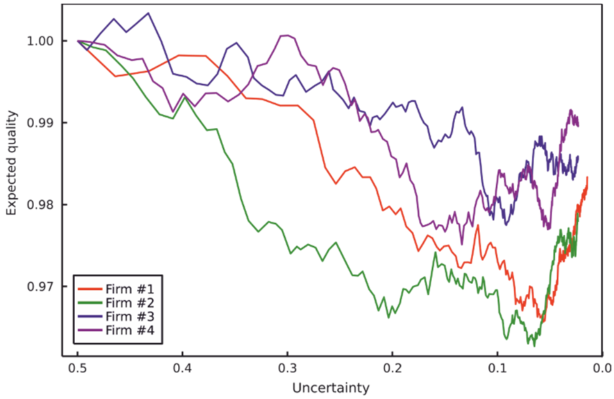

Fig. 2. Phase diagram of expectation and uncertainty regarding products' quality

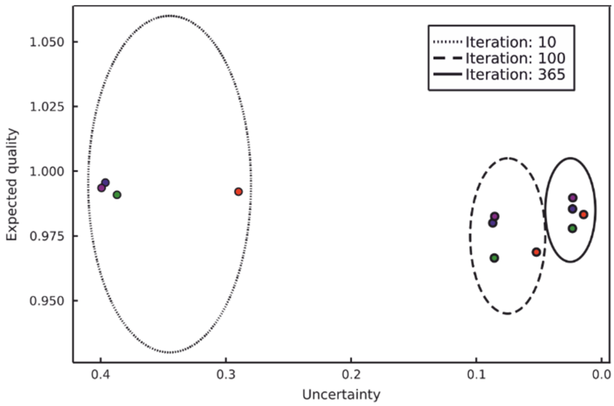

Fig. 3. Snapshots of phase diagram presented in Fig. 2

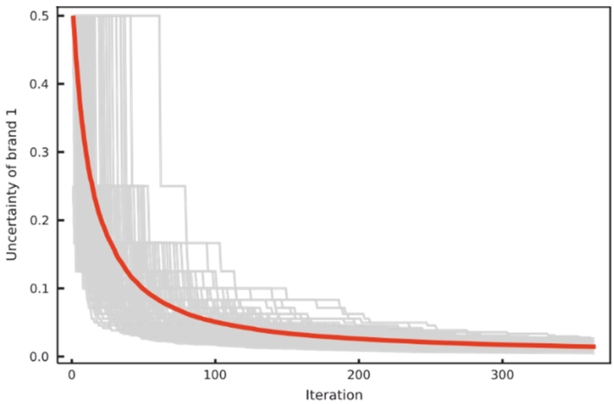

Fig. 4. Dynamics of individual uncertainty and population uncertainty

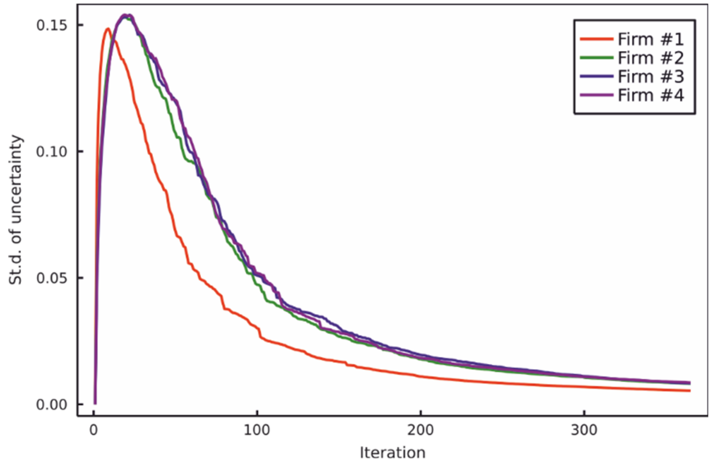

Fig. 5. Dynamics of std. deviation of average uncertainty

Two important issues connected with the presented results have to be highlighted. Firstly, the presented results are a population's average which is derived from individual agents' decisions. As an example, refer to Fig. 4 where the dynamics of each agent's uncertainty has been presented and in a form of a red line, the population's average has been shown. Secondly, since advertising contacts are distributed to the population at random, at the beginning only a small proportion of consumers receive the signal. As a result, during the first moments of a simulation run, the variability of an uncertainty measure skyrockets. After 30 steps it starts to diminish when the fraction of the agents' population which received advertising increases (Fig. 5).

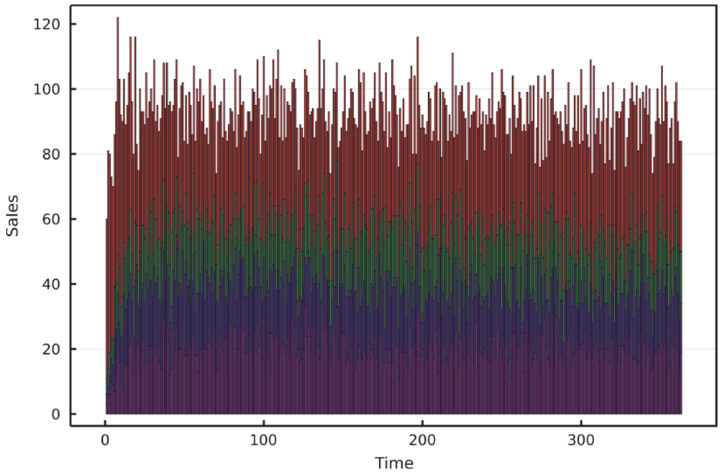

Fig. 6. Dynamics of sales, Firm 1 at the top is red

Finally, Figure 6 presents the dynamics of purchases made by consumers over the total period of a single simulation run. As a result of extended advertising intensity and lowered prices, Firm 1 is the market leader in terms of generated demand with 12 953 pieces sold (42% of the market share), in comparison to 6088 (20%), 5954 (19%) and 5969 (19%) for Firms 2, 3, and 4, respectively.

In Experiment 2, we investigate the relationship between different levels of price discount and intensity of advertising on the demand measured as a total quantity of purchased products over the whole simulation horizon.

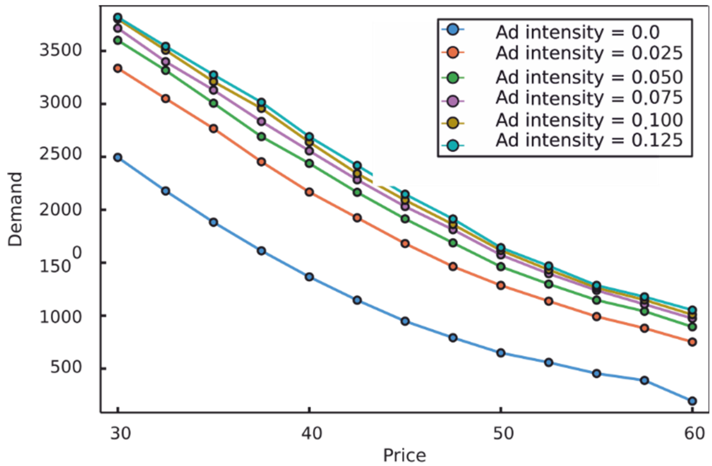

Fig. 7. Price and demand curves for various levels of advertising intensity

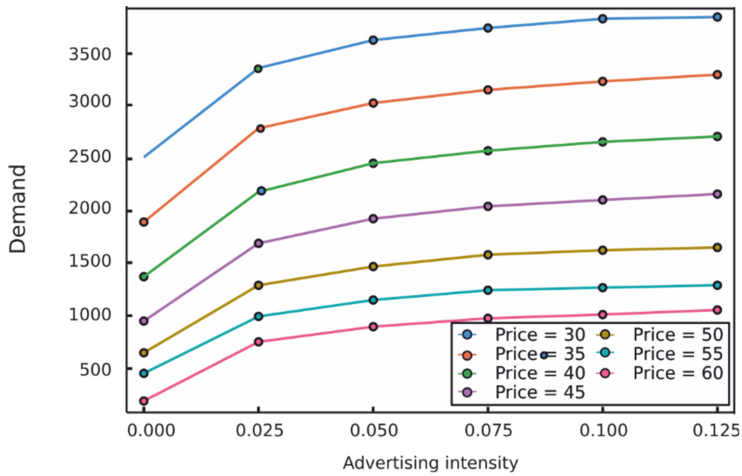

Fig. 8. Advertising and demand curves for various price levels

Obtained results can be found in Figs. 7 and 8 for price and advertising, respectively. Our results confirm, in line with the law of demand [57], that higher price discounts result in better outcomes of advertising campaigns of steady intensity. On the other hand, higher advertising intensity results in higher demand for a given price level. What is more, we investigate how the change in advertising's intensity influences the price elasticity of demand. Our results, which can be found in Table 4, confirm that more intense advertising campaigns result in lower price elasticity of demand, which is coherent with the results obtained by Mela, Gupta and Lehman [44].

Table 4. Price elasticity of demand

Advertising intensity	Price elasticity of demand
0.025	–6.88
0.050	–6.05
0.075	–5.63
0.100	–5.53
0.125	–5.25

In Experiment 3, we investigate if there exists an optimal strategy for Firm 1 in terms of budget allocation between price reduction and advertising. We choose profit as a measure of allocations' effectiveness, which is formulated as the total incremental margin generated minus the cost of marketing tools used. We define total incremental margin as a product of the incremental quantity of sales and regular price diminished by a given discount and cost of production. Incremental quantity is a difference between generated quantity and regular quantity of sales which occurs when there is no advertising or price discount. In terms of costs, advertising's cost is a product of the volume of served ads and the cost of single ad delivery. Our results confirm that under chosen parameter's setting there exists an optimal level of price discount and advertising intensity which maximises profit. We find an optimum for a discount level of 12.5 (20.8% of the regular price) and an advertising intensity of 0.075. Such a combination of parameters generates an average profit of 24 233 which is 2.1% better than the second-best combination (12.5; 0.100). We present the obtained results in a form of a heatmap in Fig. 9 and the five best combinations of parameters in Table 5.

Table 5. Five best combinations of parameters and the corresponding firm's profit

Price discount	Advertising intensity	Generated profit	% of best strategy's profit
12.5	0.075	24 233	100.0
12.5	0.100	23 727	97.9
10.0	0.075	23 509	97.0
15.0	0.075	23 460	96.8
15.0	0.050	23 084	96.6

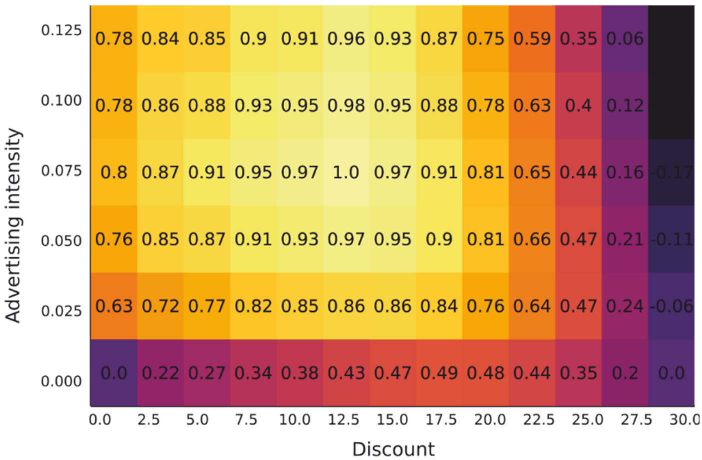

Fig. 9. Heatmap of strategies' profits, relative to
different price levels optimal strategy (12.5, 0.075)

5. Conclusions

We develop an agent-based model of the market where goods are traded and consumers update their beliefs about products' features. We use the model to evaluate the strategy of a firm in terms of two tools the firm can use to influence consumer decisions: advertising intensity and price policy. We conduct a series of experiments to investigate the relationship between the aforementioned tools and provide a solution to the problem of choosing the best marketing strategy.

Our results confirm the possibility to solve the firm's problem to seek an optimal strategy in terms of profit by choosing the proper intensity of advertising and level of price discount. Under the setting of parameters chosen in Experiment 3, an optimal strategy for Firm 1 is to decrease price by 20.8% of the regular price and serve advertisements to 7.5% of the consumer's population in each step of a simulation. We conclude that advertising and price policy have an impact one on another: higher intensity of advertising increases the effectiveness of price discounts and vice versa, and lower prices of products result in more effective advertising campaigns in terms of generated sales. The obtained result where optimal levels of both tools are non-zero proves that the population of consumers in a given moment is diverse in terms of the stage in the purchase journey: either seeking the best product to buy or still consuming recently purchased products. If they are actively seeking the appropriate product then the firm can use price discounts to effectively induce them to buy one. Otherwise, if they are still consuming the previously bought product, then the firm's advertising lowers consumers' uncertainty and prepares them for the next moment of buying. Being active or passive in the purchase journey refers not only to different consumers, but a single consumer may also switch their stage after the product's purchase or after the stock has depleted.

We make a few assumptions regarding the model's structure and mechanics presented in this paper. Our purpose is to simplify the model and put focus on the research goal formulated in the first part of this article. However, one can easily remove or change certain assumptions to adjust the presented model for other research purposes. Moreover, if the firm's problem is complex one can use sophisticated simulation optimisation methods [20]. We plan to further develop the proposed model in the following aspects: (1) to add the loyalty attribute to consumers and restrict only loyal consumers to be sources of the word-of-mouth signals, (2) to allow firms to have different initial levels of expected quality and uncertainty to, e.g., test new product development, and (3) to allow firms to plan their advertising and price promotions in more detail.

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