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MULTICRITERIA OPTIMIZATION OF MANUFACTURING PROCESSES TAKING INTO ACCOUNT THE VALIDITY CRITERIA

In the paper a method is presented of the best variant selection of a manufacturing process of a rotor to open end spinning machine with use of original optimization proceeding based on adopted criteria, taking into consideration their importance. Unit manufacturing cost and five criteria of manufacturing quality were taken as criteria to the assessment. The assessment criteria resulted from calculations and measurements were normalized. Knowledge of experts was used to determine importance of the criteria taken to the assessment. Each from the experts had built his own importance matrix of the assessment criteria, comparable in pairs, using the Saaty's method. The weights of individual criteria were determined on the basis of cumulative matrix. In the next stage of the proceeding, normalized decisions were created by raising each assessment to a power equal to corresponding weight. In the last stage of the proceeding, a single optimal ordering comprising the smallest s-th components of the individual decisions d_1, d_2, …, d_m was created. A variant which corresponds to the largest component of the optimal ordering is assumed as the best variant.

1. INTRODUCTION

Diversity of means and methods of production and surface treatment lead to a situation when elements identical or similar in shape, dimensions or accuracy, are often produced according to various manufacturing processes, differing from each other in labour consumption and costs, assuring additionally different manufacturing quality of the elements, and as a consequence, better or worse quality performance. In connection with it, emerges a complex multitask of planning and selection of the most rational variant of the manufacturing process of such elements [1].

In course of the optimization of a manufacturing activities, particular position is occupied by optimization of the manufacturing processes. Under concept of optimization of the manufacturing processes should be understood both the optimization of a manufacturing conditions (called as parametric optimization), and optimization of a structure of the processes (called as structural optimization). Optimization of the manufacturing conditions fulfills the task complementary in respect of optimization of the

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structure. These two issues are coupled together: optimization of the structure requires earlier assignment of values, close to the optimal ones, to parameters of the individual treatments making up the operations, and these in turn, making up the process. Optimization of the parameters, however, requires earlier optimization of structure of the process and its individual operations. The iteration proceeding is the solution for the emerged contradictions, in which at beginning usually it is solved the task of selection of structure of the process, and in the successive step, selection of the operation close to the optimal one (assuming typical values of the parameters – recommended by producers of cutting tools), and next, optimization of the parameters is performed, ect. [2].

The objective of the parametric optimization is selection (among possible to usage in a given conditions – in area of allowable solutions restricted by a boundary conditions) of such values of the cutting parameters like: cutting speed v_c, feedrate f and depth of cut a_p, which assure extreme value of assumed criterion of the optimization [3].

The issue of optimization of the manufacturing operations' parameters has achieved extensive bibliography, which has been discussed in detail in the works [3]. However, the issue of optimization and poly-optimization of structure of the manufacturing process has been presented up-to-now in a few publications only [4],[5],[6],[7]. As a starting point to optimization of the structure, designation of a set of solutions (variants) of the process of an analyzed workpiece, assessed in a light of determined criteria, is performed.

In the most general case, except the criteria with deterministic character (explicit, sharp ones) and probabilistic-statistical character, a criteria with fuzzy (subjective) character can be present [8],[9]. In conventional models, the optimization criteria are often treated as the deterministic criteria – e.g. cost, and in many cases, during planning phase of the manufacturing process should be treated as a non-deterministic criteria, and hence, for instance as a subjective point assessments [6],[10], or fuzzy assessments [9]. Generally, however, in majority of cases, during optimization of the manufacturing processes of a similar product, the optimization criteria with probabilistic-statistical character, in order to facilitate the proceeding, are treated as a deterministic ones, e.g. surface roughness parameters.

The objective of the present study is presentation of the multicriteria optimization method of the manufacturing processes in respect of the criteria obtained from calculations and measurements, taking into considerations their importance for selection of the best variant of the manufacturing process of a rotor to open end spinning machine.

2. METHOD OF ASSESSMENT OF THE VARIANTS DUE TO ADOPTED CRITERIA WITH CONSIDERATION OF THEIR IMPORTANCE

As the input data to the developed method were taken:

- number of variants of the manufacturing process – s (s=1, …, n),
- number of the criteria – t (t=1, …, m),
- elements of the importance matrix of individual criteria – $\mathbf{B}=[b_{ij}]$,
- elements of the $C=[c_{st}]$ table, being normalized assessments of the s-th variant according to the t-th criterion.

Let A denotes permissible set of the variants (alternatives) of the manufacturing process:

$$A = \{a_1, a_2, ..., a_n\} \tag{1}$$

and K – set of the assessments received from calculations or measurements:

$$K = \{k_1, k_2, ..., k_m\} \tag{2}$$

Next, the importance matrix of the individual criteria **B** was created:

$$B = [b_{ij}] \quad i = 1, ..., m; \ j = 1, ..., m \tag{3}$$

The matrix **B** is evaluated with use of the Saaty's method [11] consisting in comparison of the successive pairs of the criteria. Individual values b_{ij} of this matrix are taken in the following way:

$b_{ij}=1$; when k_i and k_j are equally important,
$b_{ij}=3$; when k_i is a little bit more important than k_j,
$b_{ij}=5$; when k_i is much more important than k_j,
$b_{ij}=7$; when k_i is distinctly more important than k_j,
$b_{ij}=9$; when k_i is absolutely more important than k_j,
$b_{ij}=2, 4, 6, 8$ – intermediate values between the above situations.

Additionally, it was assumed that $b_{ij}=1/b_{ji}$ and for $i=j$ value $b_{ij}=1$.

In situation of a few experts, creation of the important matrix **B** is performed in the following way:

- each from the experts creates his own matrix **B** individually,
- from the obtained matrices, called as partial matrices, a single collective importance matrix of the criteria is created (any item of the matrix above the main diagonal is calculated as an arithmetic mean from appropriate items of the partial matrices, while the items under the main diagonal are converses of the corresponding items located over the main diagonal).

Since the importance matrix of the criteria is generated in result of comparison of the successive pairs of the criteria, it follows that this matrix is the square matrix having its size equal to the number of criteria taken to the assessment. This matrix should fulfill, at least approximately, the condition of consistence [11]:

$$CI = \frac{\lambda_{max} - m}{m - 1} \leq 0.1 \tag{4}$$

where: λ_{max} – denotes maximal eigenvalue of the matrix **B**; m – number of the criteria, and thus rank of the matrix **B**.

From the Saaty's method is seen, that satisfactory fulfillment of the condition of consistence $CI \leq 0.1$ provides satisfactory adequacy of this method, in which the eigenvalues and eigenvectors of the matrix **B** are present.

The next step of the developed optimization method comprises evaluation of the table $C=[c_{st}]$, performed on the basis of calculations or measurements of the values of the criteria taken to the assessment of the individual variants of the manufacturing process (Table 1).

Values of the criteria c_{st} obtained from calculations or measurements undergo normalization, using the following formula:

$$c_{st}^{*} = 0.1 + \frac{c_{st} - \min\limits_{1\le s\le n}(c_{st})}{\left[\max\limits_{1\le s\le n}(c_{st}) - \min\limits_{1\le s\le n}(c_{st})\right]\cdot 1.25} \qquad (5)$$

where: c_{st} – values of the criteria of analysed variants against individual criteria, s=1, …, n; t=1, …, m; n – number of the variants; m – number of the criteria.

Table 1. Values of the criteria taken to the assessment of the variants of the manufacturing process

		Variants				
		a_1	a_2	a_3	…	a_n
Criteria	k_1	c_{11}	c_{21}	c_{31}	…	c_{n1}
	k_2	c_{12}	c_{22}	c_{32}	…	c_{n2}
	k_3	c_{13}	c_{23}	c_{33}	…	c_{n3}
	…	…	…	…	…	…
	k_m	c_{1m}	c_{2m}	c_{3m}	…	c_{nm}

Obtained normalized assessments c^{*}_{st}, according to the formula (5), are the fractions from interval ≤0.1; 0.9≥. Such method of the normalization eliminates extreme assessments equal to 0 and equal to 1.

Further on, normalized assessments c^{*}_{st} are converted according to a method of the optimization, i.e. depending on a situation when a given criterion should undergo minimization, or maximization according to the following formula:

$$c_{st}^{o} = (1 - k_{\mathrm{rt}})\cdot(1 - c_{st}^{*}) + k_{\mathrm{rt}}\cdot c_{st}^{*} \quad s = 1, ..., n;\, t = 1, ..., m \qquad (6)$$

where: k_{rt} for the t=1, …, m is a scalar value with coordinates 0 or 1.

If k_{rt}=1 – the variant with the highest value of the assessment according to the t-th criterion is taken as the best variant, if k_{rt}=0 – the variant with the lowest value of the assessment according to the t-th criterion is taken as the best variant.

On the basis of the evaluated and transformed values, depending on a method of the optimization, the table with assessments for individual criteria, and for each analysed variant of the planned manufacturing process, is constructed (Table 2).

Table 2. Values of the criteria after normalization and transformation, depending on a method of the optimization, taken to the assessment of the variants

		Variants				
		a_1	a_2	a_3	…	a_n
Criteria	k_1	c^{o}_{11}	c^{o}_{21}	c^{o}_{31}	…	c^{o}_{n1}
	k_2	c^{o}_{12}	c^{o}_{22}	c^{o}_{32}	…	c^{o}_{n2}
	k_3	c^{o}_{13}	c^{o}_{23}	c^{o}_{33}	…	c^{o}_{n3}
	…	…	…	…	…	…
	k_m	c^{o}_{1m}	c^{o}_{2m}	c^{o}_{3m}	…	c^{o}_{nm}

The next step of correct phase of searching after the best (optimal) variant comprises evaluation of the eigenvector **Y**, which fulfills the following matrix equation:

$$\mathbf{B} \cdot \mathbf{Y} = \lambda_{max} \cdot \mathbf{Y} \tag{7}$$

where: **B** – cumulative importance matrix of the criteria, **Y** – eigenvector, which in the above equation creates the column matrix, λ_{max} – scalar value denoting maximal eigenvalue of the matrix **B**.

Therefore, a vector for which the equation $\mathbf{B}\cdot\mathbf{Y}=\lambda_{max}\cdot\mathbf{Y}$ is fulfilled for possibly the highest values of the number $\lambda=\lambda_{max}$ is searched after. This searched vector features as many coordinates as many criteria are present.

These coordinates should satisfy additional condition telling that sum of these coordinates should be equal to the number of the criteria taken to the analysis.

$$\sum_{t=1}^{m} y_t = m \tag{8}$$

where: y_t – t-th coordinate of the eigenvector **Y**.

The coordinates of the eigenvector are also the weights of individual criteria, and are marked with alphanumericals w_1, w_2, …, w_m. Each from these weights expresses importance of the criterion which corresponds to this weight, whereas the higher value of the t-th weight, the higher importance of the t-th criterion.

The next step of the developed method is based on the Yager's method [10], and consists in creation of the normalized decisions by raising each component of the normalized and transformed assessments, depending on a method of the optimization, to a power equal to corresponding weight. In a general form, it can be written in the following way:

$$d_t = \sum_{s=1}^{n} (c_{st}^{o})^{w_t} \tag{9}$$

After transcription, the equation (9) is presented in form of the Table 3.

Table 3. Values of the normalized decisions for each from variants of the manufacturing process, in respect of the criteria taken to the assessment

		Variants				
		a_1	a_2	a_3	…	a_n
Decisions	d_1	$\left(c^o{}_{11}\right)^{w_1}$	$\left(c^o{}_{21}\right)^{w_1}$	$\left(c^o{}_{31}\right)^{w_1}$	…	$\left(c^o{}_{n1}\right)^{w_1}$
	d_2	$\left(c^o{}_{12}\right)^{w_2}$	$\left(c^o{}_{22}\right)^{w_2}$	$\left(c^o{}_{32}\right)^{w_2}$	…	$\left(c^o{}_{n2}\right)^{w_2}$
	d_3	$\left(c^o{}_{13}\right)^{w_3}$	$\left(c^o{}_{23}\right)^{w_3}$	$\left(c^o{}_{33}\right)^{w_3}$	…	$\left(c^o{}_{n3}\right)^{w_3}$
	…	…	…	…	…	…
	d_m	$\left(c^o{}_{1m}\right)^{w_m}$	$\left(c^o{}_{2m}\right)^{w_m}$	$\left(c^o{}_{3m}\right)^{w_m}$	…	$\left(c^o{}_{nm}\right)^{w_m}$

The last step of the developed methodology consists in creation of the optimal ordering of the variants in respect of the criteria taken to the assessment; the optimal variant of the process is selected on the basis of such ordering, i.e. the variant which in the best way complies with all criteria taken to the assessment. The optimal ordering in the developed method, the same as in the Yager's method [10], is the decision of minimum type. The „*s*-th" component of the optimal ordering, (i.e. component corresponding to the „*s*-th" variant of the manufacturing process), is the lowest „*s*-th" component of a particular decisions d_1, d_2, …, d_m. By marking the optimal ordering and its components with capital letters „*D*" it is possible to show them in form of the Table 4.

Table 4. Optimal ordering of the variants in respect of the criteria taken to the assessment

	Variants				
	a_1	a_2	a_3	…	a_n
D_s	D_1	D_2	D_3	…	D_n

where:

$$D_s = \min_t (c_{st}^o)^{w_t} \tag{10}$$

The variant corresponding to the highest component of the optimal ordering is considered as the best variant:

$$a_{(opt)} = \max_s D_s \tag{11}$$

3. EXAMPLE OF SELECTION OF THE OPTIMAL VARIANT OF THE MANUFACTURING PROCESS OF A ROTOR TO OPEN END SPINNING MACHINE

In case of the open end spinning machines of the PW12 type, as a raw material to production of their rotors, the aluminum alloy of the *AlCu4Mg1* brand in form of extruded bars after natural precipitation hardening was used, what resulted in high scrap rate (about 60%). Rotors of the spinning machines are operated with rotational speed from 300 to 400 rotations per second (18 000 to 24 000 rpm) and should fulfill predetermined requirements concerning manufacturing quality, i.e. low roughness of internal surfaces R_a=0.08-0.16 μm, very small value of radial and axial run-out of all faces and diameters above 40 mm – $\Delta B \leq 0{,}050$ mm, and high durability. Moreover, assembly of the rotor with the elastic bearing and the runner should be dynamically balanced at the rotational speed of n=200 rps, while values of the unbalance should not exceed value of $e_r \leq 0.05$ μm [7]. Shape of the integral rotor and the rotor combined from two elements is shown in the Fig. 1.

From observations of a spinning machines, performed within industrial conditions, is seen that the rotors belong to the most often replaced parts of the defibering – twisting head, and simultaneously to the most expensive. Considerable wear of the rotors, especially in area of the collective grove, results from nature of the production process of the yarn [12].

To increase operational life of the rotors at a determined manufacturing cost, selection of the material and shape of the semi-finished products, as well as type of surface treatment and finishing treatment, in terms of high qualitative requirements at possibly the lowest manufacturing costs, was performed. Taking into account the design and geometry of the rotors (Fig. 1), seven grades of aluminum alloys with nine different shapes of the semi-finished product were taken to the considerations.

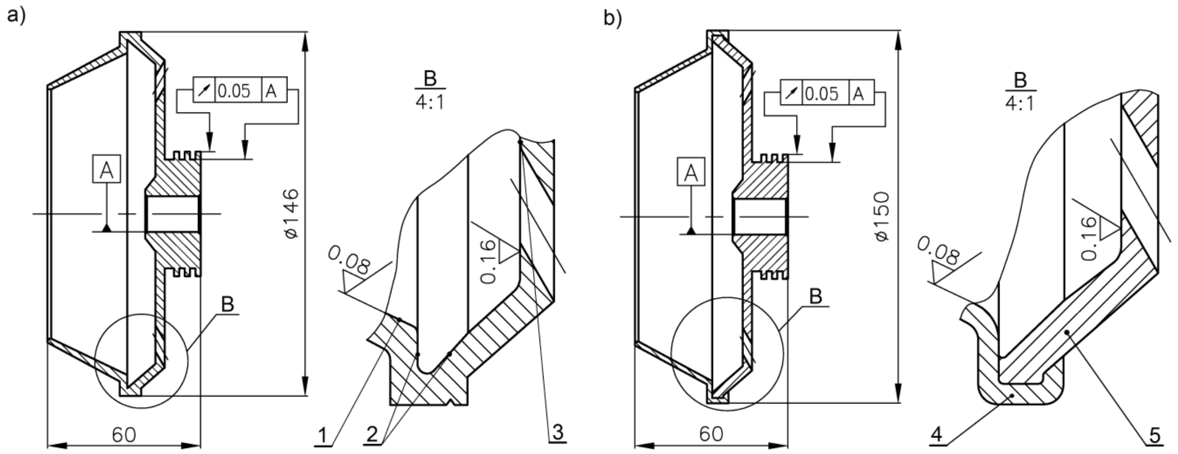


Fig. 1. The rotor: a) integral rotor: 1 – internal, conical surface (element of the cone), 2 – collective groove, 3 – internal face (base of the cone), b) rotor combined from two elements: disc (5), and the cone formed in spinning forming (4) from the AlMg2,5 sheet, connected in lapping process [7]

Table 5. Characteristics of shape of the semi-finished products used for the rotors [7]

Alphanumerical denomination	Variants	Denomination by chemical symbols	Form of the semi-finished product
EN AW-2024	a_1 a_2 a_3	AlCu4Mg1	Extruded bar ϕ150×64.5 mm
EN AW-6082	a_4 a_5 a_6	AlSi1MgMn	Extruded bar ϕ150×64.5 mm
EN AW-2618A	a_7 a_8 a_9	AlCu2Mg1.5Ni	Die forging from die forging hammer ϕ155×70 mm
EN AW-2014	a_{10} a_{11} a_{12}	AlCu4SiMg	Die forging die forging hammer ϕ155×70 mm
EN AW-6082	a_{13} a_{14} a_{15}	AlSi1MgMn	Die forging die forging hammer ϕ155×70 mm
EN AW-45000	a_{16} a_{17} a_{18}	AlSi6Cu4	Casting from sand mould ϕ153×91 mm
EN AW 45000	a_{19} a_{20} a_{21}	AlZn9Si7	Casting from sand mould ϕ153×91 mm
EN AW-2024 EN AW-5052	a_{22} a_{23} a_{24}	AlCu4Mg1 AlMg2.5	Disc produced from extruded rod ϕ155×33 mm, cone formed by spinning operation from metal plate with dimensions 195×195×2 mm
EN AW-2618A EN AA-5052	a_{25} a_{26} a_{27}	AlCu2Mg1.5Ni AlMg2.5	Disc produced from die forging ϕ155×38 mm, cone formed by spinning forming from metal plate with dimensions 195×195×2 mm

In the set of allowable solutions of the manufacturing process of the rotor to open end spinning machine there are distinguished nine sub-groups: *A1*, *A2*, *A3*, *A4*, *A5*, *A6*, *A7*, *A8*, *A9*, differing from each other with shape of the semi-finished product, these sub-groups are presented in the Tables 5 and 6.

Three combinations of the surface treatment and finishing treatment were anticipated for the sub-groups *A1*; *A2*; *A3*; *A4*; *A5*; *A6*; *A7*; *A8*; *A9* of the variants of the manufacturing process of the rotor:

1 – grinding with use of the HTJ-13-3 corundum abrasive cloth, with grain size 150, and next with grain size 220, and polishing with felt wheel impregnated with the Z-50 buffing compound,

2 – grinding with use of the HTJ-13-3 corundum abrasive cloth with grain size 150, and next with grain size of 220, and electrolytic oxidation,

3 – grinding with use of the HTJ-13-3 corundum abrasive cloth with grain size 150, and next with grain size of 220, electrolytic oxidation, grinding with abrasive cloth with grain size of 240, grinding with abrasive cloth with grain size 360, and grinding with corundum abrasive cloth PS20 with grain size 600.

Table 6. Characteristics of the semi-finished materials used for the rotors [7]

Sub-group *A1*, *A2*	The rotors from these sub-groups were produced from semi-finished product in form of rod extruded from the AlCu4Mg1 alloy; i.e. in condition of natural precipitation hardening (ta) and from extruded bar from the AlSi1MgMn aluminum alloy in condition of artificial precipitation hardening (tb).
Sub-group *A3*, *A4*, *A5*	The rotors from these sub-groups were produced from semi-finished product in form of die moulds, produced from the AlCu2Mg1.5Ni*;* AlCu4SiMg*;* AlSi1MgMn aluminum alloys, forged on hammers, and next artificially precipitation hardened (tb).
Sub-group *A6*, *A7*	The rotors from these sub-groups were produced from semi-finished product in form of casting from sand mold from the AlSi6Cu4*;* AlZn9Si7 aluminum alloys.
Sub-group *A8*, *A9*	The rotors from these sub-groups were produced as combined from two elements: the disc and the cone, while the disc was produced in turning operation from extruded bar from the AlCu4Mg1 aluminum alloy (sub-group *A8*) and from die forging made from the AlCu2Mg1,5Ni aluminum alloy, forged on a hammer (sub-group *A9*), while the cone in the both sub-groups was produced in spinning operation from the AlMg2.5 metal plate with thickness of 2 mm.

The hard electrolytic oxidation [6] was performed in the solution of electrolyte having the following composition (gravimetric): sulfuric acid – 6%, sulfuro-salcylic acid – 3%, lactic acid – 2%, glycerol – 2%, aluminum sulfate – 0.1%, and distilled water as remainder. Conditions of the electrolytic oxidation were as follows: DC current + AC current, inclusive of quota of the DC component 85%, anodic density of the electric current 6 A/dm^2, temperature of the electrolyte from – 2°C to + 6°C, time of the oxidation 40 minutes. Position of the cathode: inside the rotor. Rotations of the rotor with rotational speed of about

130 rpm to stir the electrolyte. Prior the electrolytic oxidation, the surfaces destined to the oxidation were degreased in the organic solvent and etched in 5% solution of the sodium hydrate during two minutes, and next were rinsed in water.

Due to high surface roughness obtained from the electrolytic oxidation, before the grinding operations with the abrasive cloth having grain size 240 and 360, and with the PS 20 corundum abrasive paper with grain size of 600, additional grinding operation with abrasive cloth having grain size of 120 was performed for the *A8* and *A9* variants of the manufacturing process of the rotors.

3.1. THE SET OF ALLOWABLE VARIANTS

To increase operational life of the rotors at a predetermined manufacturing costs and maintained functional and operational performance, 27 variants of the manufacturing process of the rotor to open end spinning machine have been elaborated and analyzed, all these variants have been presented on Fig. 2 and described in the Table 7. Conditions of the machining and geometry of the cutting inserts' edges were selected basing on recommendations of the Sandvik Coromant Company. On the other hand, in case of shaped internal surfaces, parameters of the machining and geometry of the cutting edge of a special-type turning tools were determined in respect of the surface roughness, making use of the recommendations of the Sandvik Coromant Company, and basing on a performed experimental tests in manufacturing conditions. It concerned mainly the cutting tool with gooseneck chunk used to machining of the internal face (base of the cone) 3, and the cutter to machining of the internal groove of the rotor 2 (Fig. 1a, Detail A) serving as a functional surfaces. For instance, the parameters of finishing turning of the rotor made from the AlCu4Mg1 alloy were as follows: the internal face 3: n=355 rpm, f=0.2 mm/rotation, a_p=0.5 mm; the collective groove 2: v_c=316.7 m/min, f_w=0.07mm/rotation, a_p=0.5 mm, the internal conical surface 1: n=710 rpm, f=0.07 mm/rotation, a_p=0.3 mm.

Table 7. Operations of variants of the manufacturing process of the rotor [7]

No. of oper.	Name of the operation	Machine
10	Cutting the material to dimension „x”	Saw band of the SBA421/S type
20	Turning the external surfaces and drilling hole ϕ11	Lathe of the TZC-32N1 type
30	Drilling the hole ϕ50, turning the external surfaces, turning the internal surfaces, boring the chamfer and collective groove. Boring the hole ϕ12 and broaching the hole to ϕ12.2 U7	Lathe of the TZC-32N1 type
40	Turning the hub to ϕ47 and grooves to 2.5 mm width	Lathe of the TZC-32N1 type
45	Finish turning the external and internal surfaces, and spot facing the collective groove	Lathe of the TZC-32N1 type
50	Finish turning the external surfaces, and boring the conical internal surface with the collective grove	Lathe of the TZC-32N1 type
60	Inter-operational control	Centering station with gauge
70	Drilling 12 holes ϕ6	Driller of the 2H-125 type

80	Blunting the sharp edges	Grinding stand
90	Grinding with abrasive cloth with grain size 50 and 220	Special grinder
100	Polishing with felt polishing wheel impregnated with the Z-50 buffing compound	Special polisher
105	Dynamic balancing	Dynamic balancer
110	Final control	Inspection-measuring stand
120	Electrolytic oxidation with applied DC and AC current	Station to anodic oxidation
130	Final control	Inspection-measuring stand
140	Grinding with abrasive cloth with grain size 240 and 360	Special grinder
150	Grinding with abrasive paper with grain size 600	Special grinder
160	Grinding the face and external diameter of the hub, and face of the disc, drilling the hole ϕ11	Lathe of the TZC-32N1 type
170	Rough and profile boring the external surface, planning the face, rough and profile boring the internal surface of the hole, turning the collective groove, boring the collective groove and other internal surfaces, boring the hole ϕ102, finish boring the internal cone together with collective groove, chamfering the hole and broaching the hole ϕ12.2 U7	Lathe of the TZC-32N1 type
180	Cutting-off the top nob, drilling the hole ϕ11, broaching the hole to ϕ12.2 U7, turning the external diameter of the hub and planning the face of the hub	Lathe of the TZC-32N1 type
190	Boring the internal surface with the collective groove, turning the external surface, finish boring the internal surfaces and the collective groove	Lathe of the TZC-32N1 type
200	Preliminary turning the hub, turning the cone, spotting the hole ϕ20 with rigid drill, drilling the hole ϕ11, turning the hub	Lathe of the TZC-32N1 type
210	Planning the face, turning the internal surface with chamfer, turning the collective groove, broaching the hole ϕ12.2 U7, chamfering the hole	Lathe of the TZC-32N1 type
220	Turning the external surface of the hub and the disc, turning the grooves	Lathe of the TZC-32N1 type
230	Grinding with abrasive papers with grain size 150 and 220	Special grinder
240	Polishing the internal surface of the disc with felt wheel impregnated with the Z-50 polishing compound	Special polisher
250	Cutting the metal plate from the AlMg2.5 alloy, thickness 2 mm, to dimension 195x195 mm	Mechanical cutter Q11 2x2000
260	Marking-off the hole, drilling the hole ϕ6, blunting the sharp edges, turning the disc to size ϕ188, blunting edges	Driller of the 2H-125 type
270	Grinding with abrasive paper with grain size 220	Special grinder
280	Polishing the cut-off disc ϕ188x2mm with the Z-50 polishing compound	Special grinder
290	Attaching on the core and spinning	Lathe of the TUG-56MN type
300	Attaching on the core, cutting-off the flange and bottom of the cone	Lathe of the TUG-56MN type
310	Lapping the disc and the cone	Lathe of the TUG-56MN type
320	Planning the face of cone and boring the hole ϕ 102±0.2	Lathe of the TUG-56MN type
330	Planning the face of the disc, turning face of the disc, turning the hub, spotting the hole ϕ20 with rigid drill, drilling the hole ϕ11, chamfering the hole	Lathe of the TZC-32N1 type
340	Planning the face , turning the external surface, turning the chamfers, turning the collective groove, broaching the hole ϕ12.2 U7	Lathe of the TZC-32N1 type

3.2. THE SET WITH CRITERIA TAKEN TO THE ASSESSMENT

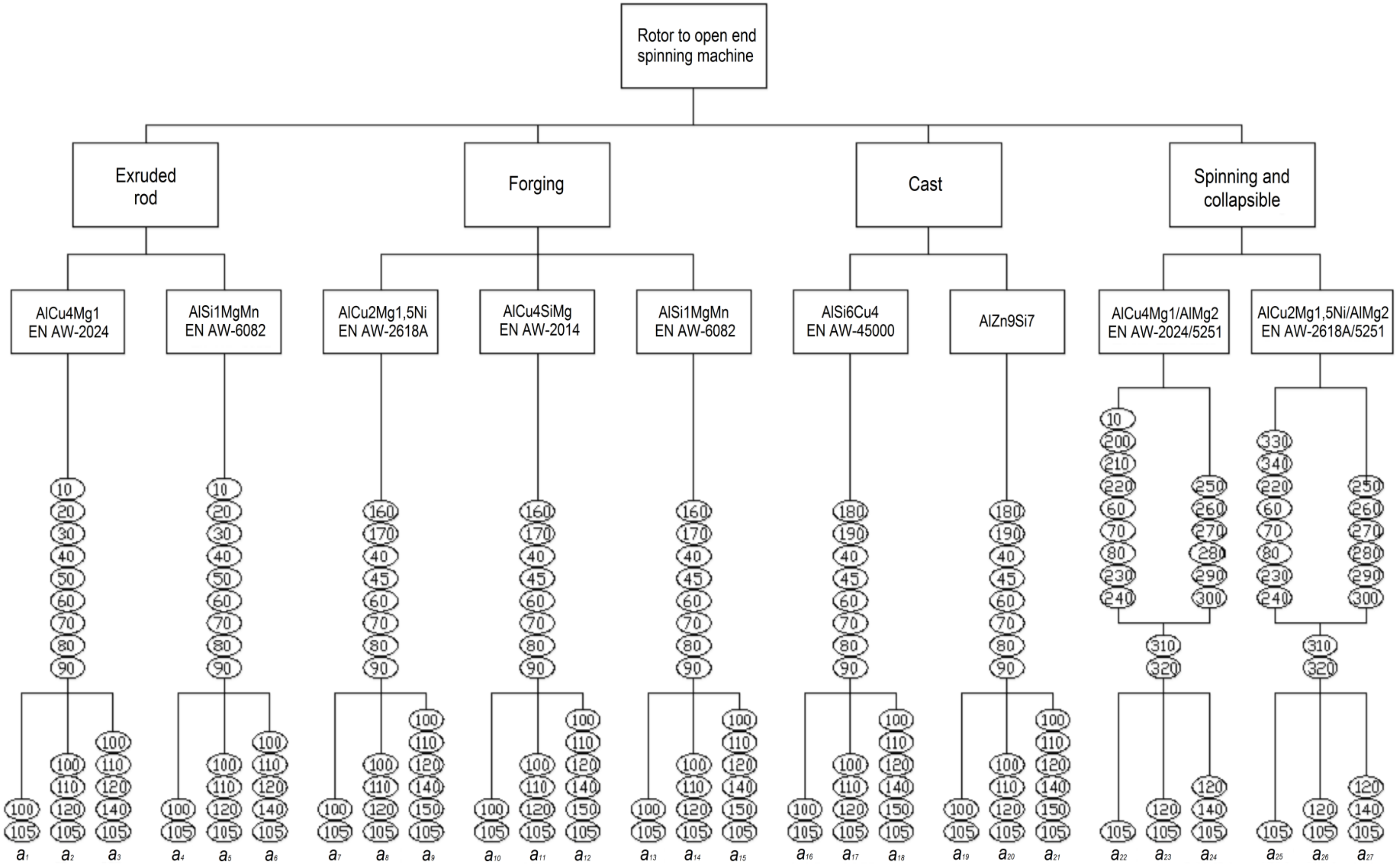
The following six criteria have been taken to assessment of the variants of the manufacturing process of the rotor to open end spinning machine:

- unit manufacturing cost K_w, EUR,
- maximum peak height of the surface S_p, µm,
- root mean square height of the surface S_q, µm,
- maximal hardness on top of surface layer μHV, MPa,
- hardening depth of the surface layer, or depth of the oxide layer g_u, µm,
- spinning efficiency ratio W_{sp}.

Two parameters: S_p and S_q have been taken to assessment of the 3D surface roughness, because values of calculated coefficient of linear correlation R between these parameters, and coefficient of kinetic friction μ_k of the yarn were the highest [6]. Recording and measurements of the parameters of surface geometrical structure were performed with use of the Perthometer Concept V.700 meter, made by the Mahr Company, using a profile gauge with conical shape and imaging nose radius of r_{os}=2 µm. The measurements were performed on the surface area of 2×2 mm, with measuring pressure of 0.75 mN, feedrate of the measuring gauge of 0.5 mm/s, digitization step of 0.35 µm, distance between individual profiles 5 µm (number of the profiles 401), elementary sector of 0.4 mm and measuring section of 5×0.4 mm=2 mm. At least three measurements, spaced every 120º were performed at each internal surface (vertical and oblique one).

To assess physical properties of the surface layer the following parameters were taken: maximal hardness of the surface layer HV and hardening depth of the surface layer or depth of the oxide layer g_u. For this reason, in result of many years observations and investigations within industrial environment, concerning wear of a components of the ring spinning machines and the open end spinning machines, being in direct contact with the yarn, as well as measurements of hardness distribution in the surface layer, it was confirmed that wear of these components decreases together with increase of hardness on the surface layer and in the surface layer of such components [6]. The measurements of hardness distribution in depth of the surface layer or the oxide layer $HV=f(g_u)$ of the rotors were performed with use of the Vickers method on oblique metallographic specimens cut at the angle of 1°30' (0.026 rad), under intender's load equal to 0.245 N, using the Leitz Wetzlar micro-hardness tester. In course of the measurements at least threefold repeatability was used. All measurement results were verified for statistical homogeneity to eliminate a coarse errors, using the Grubbs's test. Critical value of the test function T_{kr} was read from the Table 51 [13] depending on a number of the tests n_p=5 and n_p=3 and assumed value of the importance level α=0.05 (5%). Mean values of individual criteria of the assessments were calculated after elimination of the gross errors (Fig. 2).

To assess operational quality of the rotors, the criterion of effective spinning W_{sp} was taken and expressed as a number of effective spinning per 10 tests. The tests were performed on the open end spinning machine at constant rotational speed of the rotors n_r=18 000 rpm, and at constant speed of the defibering shafts n_{wl}=3 400 rpm, and output speed v_w=90÷120 m/min for each from variants of the manufacturing process of the rotors.



	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}
K_w, EUR/piece	36,00	64,57	66,14	32,74	61,29	62,69	24,64	53,26	55,25	24,73	53,30	55,15	24,38	52,95	54,74	21,69	50,02	51,82	21,07	49,64	51,50	29,79	58,36	59,43	25,43	54,07	55,29
S_p, µm	2,33	3,51	1,65	0,79	5,96	2,33	2,05	5,67	1,06	2,13	7,10	4,67	0,86	5,68	1,68	2,29	17,01	4,24	5,33	12,32	6,90	1,29	9,74	2,90	2,09	5,73	1,04
S_q, µm	0,19	0,92	0,77	0,19	1,58	0,86	0,39	1,35	0,36	1,01	1,95	1,02	0,18	1,49	0,86	0,20	5,37	1,35	0,34	4,04	2,05	0,17	1,20	0,67	0,37	1,40	0,38
μHV, MPa	1530	2570	2740	1010	6750	6840	890	3870	4560	1510	1920	2120	1080	7080	7280	1140	3150	3440	1360	2940	3390	1480	2470	2640	840	3820	4420
g_u, µm	15	110	95	15	70	65	15	110	105	15	80	75	15	120	110	10	80	70	20	80	70	15	110	105	15	110	105
W_{sp}	1,0	0.9	1.0	1.0	0.9	1.0	1.0	0.7	1.0	1.0	0.9	1.0	1.0	0.9	1.0	1.0	0.5	1.0	1.0	0.6	1.0	1.0	0.6	0.9	1.0	0.6	0,9

Fig. 2 The graph-tree with variants of manufacturing process of the rotor to open end spinning machine [7]

To verify value of effectiveness of the spinning, a feeding tape – viscose 100%, with linear mass 9.0 ktex and yarn with linear mass 300 tex, were used. Value of the coefficient of effective spinning W_{sp} reached value of one, when none (zero) rips of the yarn occurred for ten tests of the spinning, while value of zero when 10 rips of the yarn occurred in 10 tests.

3.3. SELECTION OF THE OPTIMAL (THE BEST) VARIANT IN RESPECT OF UNIT MANUFACTURING COST AND CRITERIA OF MANUFACTURING QUALITY, WITH CONSIDERATION OF THEIR IMPORTANCE

Values of the criteria of the assessment, obtained from calculations and measurements of the analyzed variants of the manufacturing process of the rotors are presented in the Fig. 2. In the next step of the proceeding the normalization of values of the criteria to interval of $\leq 0.1; 0.9 \geq$ was carried out. The first normalization step enables direct reduction of the assessments to the normalized value c_{st}^{*} using the function described by the equation (5). In the second normalization step it is decided if a given criterion of the optimization should be maximized or minimized. To do it, the function presented by the equation (6) is used.

In the analyzed example, manufacturing cost of a single rotor K_w, the maximum peak height of the surface S_p, and the root mean square height of the surface S_q, are the minimized criteria (for which $k_{rt}=0$), while the maximal hardness of surface layer μHV, the hardening depth of surface layer or oxidation layer g_u and the coefficient of effectiveness of the spinning W_{sp} are the maximized criteria ($k_{rt}=1$).

Values of the assessment after the normalization and transformation, depending on a method of the optimization, for individual criteria and for each from variants of manufacturing process of the rotor are presented in the Table 8.

Knowledge of five experts was engaged to determination of importance of the individual criteria used to valuation of the analysed set of variants of the manufacturing process of the rotors; while the expert E1 – was a specialist, design-engineer from area of textile machinery, the expert E2 – was a specialist from area of planning of manufacturing processes, the expert E3 – was a specialist, process engineer from textile industry, the expert E4 – was a specialist from field of operation and reliability of the textile machinery, the expert E5 – was a specialist from field of manufacturing costs and economic analyses. Each from the experts, to determine importance of the assumed criteria, has built his own importance matrix of the assessment criteria, comparable in pairs, using the Saaty’s method [11] (Tables 9-13).

Table 8. Values of the criteria after normalization and transformation, depending on a method of the optimization

		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
Criteria	k_1	0.6350	0.1279	0.1000	0.6929	0.1862	0.1613	0.8366	0.3286	0.2933
	k_2	0.8240	0.7658	0.8576	0.9000	0.6450	0.8240	0.8379	0.6593	0.8867
	k_3	0.8969	0.7846	0.8077	0.8969	0.6831	0.7938	0.8662	0.7185	0.8708
	k_4	0.1857	0.3149	0.3360	0.1211	0.8342	0.8453	0.1062	0.4764	0.5621
	k_5	0.1364	0.8273	0.7182	0.1364	0.5364	0.5000	0.1364	0.8273	0.7909
	k_6	0.9000	0.7400	0.9000	0.9000	0.7400	0.9000	0.9000	0.4200	0.9000

		a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}
Criteria	k_1	0.8351	0.3280	0.2950	0.8413	0.3341	0.3024	0.8890	0.3861	0.3542
	k_2	0.8339	0.5888	0.7086	0.8965	0.6588	0.8561	0.8260	0.1000	0.7298
	k_3	0.7708	0.6262	0.7692	0.8985	0.6969	0.7938	0.8954	0.1000	0.7185
	k_4	0.1832	0.2342	0.2590	0.1298	0.8752	0.9000	0.1373	0.3870	0.4230
	k_5	0.1364	0.6091	0.5727	0.1364	0.9000	0.8273	0.1000	0.6091	0.5364
	k_6	0.9000	0.7400	0.9000	0.9000	0.7400	0.9000	0.9000	0.1000	0.9000
		a_{19}	a_{20}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}
Criteria	k_1	0.9000	0.3929	0.3599	0.7453	0.2382	0.2192	0.8227	0.3143	0.2927
	k_2	0.6761	0.3313	0.5986	0.8753	0.4586	0.7959	0.8359	0.6564	0.8877
	k_3	0.8738	0.3046	0.6108	0.9000	0.7415	0.8231	0.8692	0.7108	0.8677
	k_4	0.1646	0.3609	0.4168	0.1795	0.3025	0.3236	0.1000	0.4702	0.5447
	k_5	0.1727	0.6091	0.5364	0.1364	0.8273	0.7909	0.1364	0.8273	0.7909
	k_6	0.9000	0.2600	0.9000	0.9000	0.2600	0.7400	0.9000	0.2600	0.7400

Table 9. The partial matrices of importance of the criteria for the E1

k_j k_i	k_1	k_2	k_3	k_4	k_5	k_6
k_1	1	1/3	2	1/7	4	1/7
k_2	3	1	4	1/5	6	1/5
k_3	1/2	1/4	1	1/8	3	1/10
k_4	7	5	8	1	10	1
k_5	1/4	1/6	1/3	1/10	1	1/10
k_6	7	5	10	1	10	1

Table 10. The partial matrices of importance of the criteria for the E2

k_i	k_1	k_2	k_3	k_4	k_5	k_6
k_1	1	1/6	1/4	1/8	3	1/9
k_2	6	1	3	1/3	8	1/3
k_3	4	1/3	1	1/5	6	1/5
k_4	8	3	5	1	10	1
k_5	1/3	1/8	1/6	1/10	1	1/10
k_6	9	3	5	1	10	1

Table 11. The partial matrices of importance of the criteria for the E3

k_j k_i	k_1	k_2	k_3	k_4	k_5	k_6
k_1	1	1/7	1/4	1/7	5	1/9
k_2	7	1	4	1	7	1/3
k_3	4	1/4	1	1/4	4	1/6
k_4	7	1	4	1	7	1/3
k_5	1/5	1/7	1/4	1/7	1	1/9
k_6	9	3	6	3	9	1

Table 12. The partial matrices of importance of the criteria for the E4

k_j k_i	k_1	k_2	k_3	k_4	k_5	k_6
k_1	1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{9}$	7	$\frac{1}{9}$
k_2	6	1	4	$\frac{1}{4}$	6	$\frac{1}{4}$
k_3	6	$\frac{1}{4}$	1	$\frac{1}{7}$	3	$\frac{1}{7}$
k_4	9	4	7	1	9	1
k_5	$\frac{1}{7}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{9}$	1	$\frac{1}{9}$
k_6	9	4	7	1	9	1

On the basis of constructed matrices, called as a partial matrices, a cumulative matrix (Table 14) was created, its items above the main diagonal are the arithmetic means of the corresponding items of the individual partial matrices. In turn, elements of the matrix under the main diagonal are the inverses of the values corresponding to the items over the main diagonal.

Table 13. The partial matrix of importance of the criteria for the E5

k_j k_i	k_1	k_2	k_3	k_4	k_5	k_6
k_1	1	2	$\frac{1}{2}$	$\frac{1}{3}$	6	$\frac{1}{5}$
k_2	$\frac{1}{2}$	1	3	$\frac{1}{4}$	5	$\frac{1}{6}$
k_3	2	$\frac{1}{3}$	1	$\frac{1}{6}$	3	$\frac{3}{5}$
k_4	3	4	6	1	8	$\frac{1}{3}$
k_5	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{8}$	1	$\frac{1}{10}$
k_6	5	6	$\frac{5}{3}$	3	10	1

Table 14. The cumulative matrix of importance of the criteria

k_j k_i	k_1	k_2	k_3	k_4	k_5	k_6
k_1	1	0.5619	0.6333	0.1710	5.0000	0.1352
k_2	1.7795	1	3.6000	0.4067	6.4000	0.2567
k_3	1.5790	0.2778	1	0.1769	3.8000	0.2419
k_4	5.8468	2.4591	5.6529	1	8.8000	0.7333
k_5	0.2000	0.1563	0.2632	0.1136	1	0.1044
k_6	7.3948	3.8962	4.1339	1.3636	9.5749	1

The cumulative matrix was used as the basis to evaluation of the importance (weights) of the individual criteria taken to the assessment of the analysed variants of the manufacturing process of the rotor.

In the next step of the proceeding, using the Power's method [14], the eigenvalues of the cumulative matrix of importance of the criteria **B** were calculated, comparing its determinant to zero and solving the equation of the n=6-th degree with respect to λ:

$$\begin{vmatrix} 1.0000-\lambda & 0.5619 & 0.6333 & 0.1710 & 5.0000 & 0.1352 \\ 1.7795 & 1.0000-\lambda & 3.6000 & 0.4067 & 6.4000 & 0.2567 \\ 1.5790 & 0.2778 & 1.0000-\lambda & 0.1769 & 3.8000 & 0.2419 \\ 5.8468 & 2.4591 & 5.6529 & 1.0000-\lambda & 8.8000 & 0.7333 \\ 0.2000 & 0.1563 & 0.2632 & 0.1136 & 1.0000-\lambda & 0.1044 \\ 7.3948 & 3.8962 & 4.1339 & 1.3636 & 9.5749 & 1.0000-\lambda \end{vmatrix} = 0 \tag{12}$$

Solution of the equation (12) are the eigenvalues λ of the matrix **B**:

6.3005; 0.0380; –0.0115+1.0661i; –0.0115–1.0661i; –0.1541+0.8578i; –0.1541–0.8578i.

Hence, searched maximal eigenvalue of the matrix **B** amounts to: λ_{max}=6.3005
Verification of the condition of consistence of the matrix **B**:

$$CI = \frac{\lambda_{max} - m}{m-1} = \frac{6.3005 - 6}{6-1} = 0.0601 < 0.1 \tag{13}$$

Therefore, the condition of consistence is fulfilled, approximately, because CI=0.0601<0.1. Next, for the maximal eigenvalue λ_{max}=6.3005 of the matrix **B** and for the condition telling that sum of coordinates of the eigenvector **Y** should be equal to the number of criteria (equation 8), the values of these coordinates y_t (t=1, …, m) were evaluated, solving the following system of the equations:

$$\begin{cases} (1-6.3005)y_1 + 0.5619y_2 + 0.6333y_3 + 0.1710y_4 + 5.0000y_5 + 0.1352y_6 = 0 \\ 1.7795y_1 + (1-6.3005)y_2 + 3.6000y_3 + 0.4067y_4 + 6.4000y_5 + 0.2567y_6 = 0 \\ 1.5790y_1 + 0.2778y_2 + (1-6.3005)y_3 + 0.1769y_4 + 3.8000y_5 + 0.2419y_6 = 0 \\ 5.8468y_1 + 2.4591y_2 + 5.6529y_3 + (1-6.3005)y_4 + 8.8000y_5 + 0.7333y_6 = 0 \\ 0.2000y_1 + 0.1563y_2 + 0.2632y_3 + 0.1136y_4 + (1-6.3005)y_5 + 0.1044y_6 = 0 \\ 7.3948y_1 + 3.8962y_2 + 4.1339y_3 + 1.3636y_4 + 9.5749y_5 + (1-6.3005)y_6 = 0 \end{cases} \tag{14}$$

Solution of the system of the equations (14) are the values:
y_1=0.4011; y_2=0.8634; y_3=0.4369; y_4=1.8693; y_5=0.1473; y_6=2.2820, satisfying the equity:

$$0.4011 + 0.8634 + 0.4369 + 1.8693 + 0.1473 + 2.2820 = 6 \tag{15}$$

The coordinates y_t are simultaneously the weights w_t of the individual criteria.

The next step of the developed method consists in creation of the normalized decisions, raising each component of the successive assessments to a power equal to the corresponding weight, according with the formula (9). Values of the normalized decisions for each from the variants in respect of the individual criteria are presented in the Table 15.

Table 15. Values of the normalized decisions

		a_1	a_2	a_3	a_4	a_k	a_{15}	a_k	a_{24}	a_{25}	a_{26}	a_{27}
Decisions	d_1	0.8335	0.4383	0.3971	0.8632	……	0.6190	……	0.5440	0.9247	0.6286	0.6109
	d_2	0.8461	0.7943	0.8758	0.9130	……	0.8745	……	0.8211	0.8566	0.6952	0.9022
	d_3	0.9536	0.8994	0.9109	0.9536	……	0.9041	……	0.9185	0.9406	0.8614	0.9399
	d_4	0.0430	0.1153	0.1302	0.0193	……	0.8212	……	0.1214	0.0135	0.2440	0.3212
	d_5	0.7457	0.9725	0.9524	0.7457	……	0.9725	……	0.9660	0.7457	0.9725	0.9660
	d_6	0.7863	0.5030	0.7863	0.7863	……	0.7863	……	0.5030	0.7863	0.0462	0.5030

The last stage of the developed method consists in creation of a single optimal ordering used to selection of the best manufacturing variant of the rotor, i.e. the variant which in the best way meets all criteria taken to the assessment. The optimal ordering in this method is the decision minimum. The s-th component of the optimal ordering, i.e. the component corresponding to the s-th variant of the manufacturing process, is the lowest s-th component of the individual decisions d_1, d_2,…, d_m (the formula 10). Values of the optimal ordering for the individual variants are depicted in the Table 16.

Table 16. The optimal ordering of the variants in respect of criteria taken to the assessment

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
D_s	0.0430	0.1153	0.1302	0.0193	0.5030	0.4810	0.0151	0.1381	0.3407
	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}
D_s	0.0419	0.0663	0.0800	0.0220	0.5030	0.6190	0.0244	0.0052	0.2002
	a_{19}	a_{20}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}
D_s	0.0343	0.0462	0.1947	0.0403	0.0462	0.1214	0.0135	0.0462	0.3212

The best variant (the optimal variant) is the variant corresponding to the highest component of the optimal ordering (formula 11):

$$a_{(opt)} = \max_s D_s = 0.6190 / a_{15} \tag{16}$$

The optimal variant is, therefore, the variant a_{15}, because maximal value of the optimal ordering, equal to 0.6190, corresponds to this variant. In this variant of the manufacturing process, the rotor was produced from the semi-finished material in form of hammer forged die forging, made from the AlSi1MgMn alloy. The semi-finished material had undergone the roughing, profiling and finishing operations. Next, the rotor underwent operations of the electrolytic oxidation, grinding with abrasive cloth having grain size of 240 and 360, and grinding with abrasive paper with grain size of 600.

In case of the optimal variant, values of the criteria to the assessment are as follows: K_w=54.74 EUR/piece; S_p=1.68 µm; S_q=0.86 µm; μHV=7280 MPa; g_u=110 µm; W_{sp}=1.

4. SUMMARY

In course of the multivariant planning of the manufacturing processes of a products similar to a products being already in production, where generally it is possible to define with a sufficient accuracy, the values of criteria taken to the assessment; good results in selection of the best variant are offered by the easy to use modified Yager's method. In this method, criteria of the assessment obtained from calculations and measurements were used instead of the subjective point criteria. In the next stage, these criteria were normalized to interval of <0.1; 0.9>. The first step of the normalization procedure enables direct reduction of the assessments to the normalized value $c_{st}*$, using the developed function described by the formula (5). The second step of the normalization comprises consideration if a given criterion from the normalization task should be maximized or minimized. To do it, the function described by the formula (6) is used. Such method of the normalization eliminates extreme assessments equal to 0 and equal to 1. Further procedures is the same as in the case of classical Yager method. The optimal variant obtained according to this method is compatible with the best variant obtained with use of two-stage method, comprising determination of the set of the Pareto-optimal variants during in its first stage, while in the second stage – using the distance function [7], selection of the best variant from the above mentioned set.

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