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Determination of ship's positions applying the selected M-estimation methods basing on radar observations

Sławomir Świerczyński, Krzysztof Czaplewski

Polish Naval Academy
81-103 Gdynia, ul. Śmidowicza 69, e-mail: krzysztof@czaplewski.pl

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Abstract
The paper presents an analysis of the selected robust adjustment methods applied in geodesy, proving thereby justifiability in choosing the applied attenuation function. With a use of the radar navigation methods, the said methods were applied in the process of determination of the observed vessel's positions.

Introduction

The geometrical measurement structures can be defined in two-dimensional system, for example (X, Y), wherein a parameter to measure are the directions from which the angle is computed. The measurement structure in navigation may reflect the real navigation equipment. It can be a network of coastal radar stations, assigned for collecting navigational data (bearings or distances), necessary for fixing positions of ships in water areas. They are situated along the coast line and they can be formed into different geometrical configurations of radar survey stations. Figure 1 presents an exemplary measurement grid, wherein the coastal radar stations are used for surveying purpose.

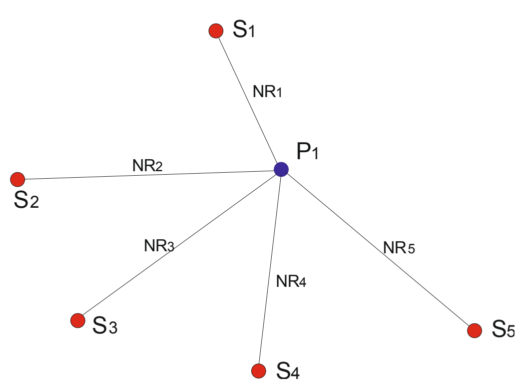

Fig. 1. Measurement structure [own study]

With a use of such a measurement structure, comprising the coastal radar stations, it is feasible to carry out surveys at some time intervals. A bearing is here one of the parameters to measure. On taking measurements we can see that there may come up incidents when there are found gross errors biased survey results. Such errors may result from improper installation or calibration of the measurement equipment, wrong readings from measurement equipment caused by momentary changes of measurement environment parameters etc. The errors of such character are called often gross errors and may considerably affect the measurement results' values. To eliminate a case of determination of false radar echoes, there can be applied the robust estimation methods of properly selected attenuation functions.

In the adjustment calculus there are known several M-estimation methods, which differ in a form of the attenuation function or the weight function. The most popular are the Huber's, Hampel's and the Danish methods. For the purpose of the here presented study, there was carried out analysis of these three functions of attenuation in respect of their influence on the position fixing accuracy in consideration of the gross errors biased measurements.

The Danish attenuation function is characterized with such properties, that beyond the admissible interval $\Delta\bar{v} \in \langle -k{:}k \rangle$, it ex-potentially decreases and takes the following form:

$$t(\bar{v}_n)=\begin{cases}1 & \text{for } \bar{v}_n \in \langle -k:k\rangle \\ \exp\{-l(|\bar{v}_n|-k)^g\} & \text{for } |\bar{v}_n|>k\end{cases} \tag{1}$$

In that case the equivalent values of weights are settled according to the formula:

$$\hat{p}_n=t(\bar{v}_n)p_n=\begin{cases}p_n & \text{for } \bar{v}_n \in \langle -k:k\rangle \\ \exp\{-l(|\bar{v}_n|-k)^g\}p_n & \text{for } |\bar{v}_n|>k\end{cases} \tag{2}$$

The Hampel's attenuation function, the next one applied in the calculations, has two additional intermediate intervals (leftward and rightward from the admissible interval $\Delta\bar{v} \in \langle -k:k\rangle$), where the attenuation function $t(\bar{v})$ linearly reduces its values. The above mentioned function and the weight function resulting therefrom have the forms as follows:

$$t(\bar{v}_n)=\begin{cases}1 & \text{for } \bar{v}_n \in \langle -k:k\rangle \\ \dfrac{|\bar{v}_n|-k_b}{k-k_b} & \text{for } |\bar{v}_n| \in (k:k_b\rangle \\ 0 & \text{for } |\bar{v}_n|>k_b\end{cases} \tag{3}$$

$$\hat{p}_n=t(\bar{v}_n)p_n=\begin{cases}p_n & \text{for } \bar{v}_n \in \langle -k:k\rangle \\ \left(\dfrac{|\bar{v}_n|-k_b}{k-k_b}\right)p_n & \text{for } |\bar{v}_n| \in (k:k_b\rangle \\ 0 & \text{for } |\bar{v}_n|>k_b\end{cases} \tag{4}$$

where k_b is a number which settles the limits of the additional intervals. Generally, it was assumed that $k_b = 4,...,6$.

The last attenuation function taken for analysis, the Huber's function, is the most „radical" one, as it assigns the zero values of weights to all the observations, corrections of which are not within the interval admissible for them and takes the form as follows:

$$t(\bar{v}_n)=\begin{cases}1 & \text{for } \bar{v}_n \in \langle -k:k\rangle \\ 0 & \text{for } \bar{v}_n \notin (-k:k)\end{cases} \tag{5}$$

Therefore, the following weight function results:

$$\hat{p}_n=t(\bar{v}_n)p_n=\begin{cases}p_n & \text{for } \bar{v}_n \in \langle -k:k\rangle \\ 0 & \text{for } \bar{v}_n \notin (-k:k)\end{cases} \tag{6}$$

The equalization problem with application of the gross errors robust method may take the following form:

$$\left.\begin{array}{ll}
\mathrm{V}=\mathrm{A}\hat{\mathrm{d}}_\mathrm{X}+\mathrm{L} & \text{functional model} \\
\mathrm{C_X}=\sigma_0^2\mathrm{Q_X}=\sigma_0^2\mathrm{P}^{-1} & \text{original statistic model} \\
\hat{\mathrm{C}}_\mathrm{X}=\sigma_0^2\hat{\mathrm{Q}}_\mathrm{X}=\sigma_0^2\hat{\mathrm{P}}^{-1} & \text{equivalent statistic model} \\
\hat{\mathrm{P}}=\mathrm{T}(\bar{\mathrm{V}})\mathrm{P} & \text{equivalent weights} \\
\Psi(\hat{\mathrm{d}}_\mathrm{X})=\mathrm{V^T}\hat{\mathrm{P}}\mathrm{V}=\mathrm{V^T}\mathrm{T}(\bar{\mathrm{V}})\mathrm{P}\mathrm{V}=\min & \\
 & \text{equalization criterion} \\
\mathrm{V^T}\mathrm{T}(\bar{\mathrm{V}})\mathrm{P}\mathrm{V}=\hat{\mathrm{V}}^\mathrm{T}\mathrm{T}(\hat{\bar{\mathrm{V}}})\mathrm{P}\hat{\mathrm{V}} & \text{equalization criterion}
\end{array}\right\} \tag{7}$$

where:

$\hat{\mathrm{P}}=\mathrm{T}(\bar{\mathrm{V}})\mathrm{P}$ – equivalent weight matrix;

$\mathrm{C_X}$ – equivalent covariance matrix;

$\hat{\mathrm{Q}}_\mathrm{X}$ – equivalent co-factors matrix;

$\mathrm{T}(\bar{\mathrm{V}})$ – diagonal attenuation matrix.

For this way adopted assumptions the equalization problem solution is of iterative character. To solve the problem there may be adopted an algorithm suggested in [9, 10], where the first stage of the calculation process is equalization applying the classical least squares method. In the equalization process we assume that the observed position forwarded by the watch officer to the traffic supervision system operator is an expected position of the coordinates as follows:

$$\mathrm{P_j^0}=\begin{bmatrix}\mathrm{X_j^0}\\ \mathrm{Y_j^0}\end{bmatrix} \tag{8}$$

for the measuring structure adopted to these considerations the equalization problem's functional model takes the following form:

$$v_{ij}=\left.\frac{\partial NR_{ij}}{\partial X_{P_j}}\hat{d}_{X_{P_j}}+\frac{\partial NR_{ij}}{\partial Y_{P_j}}\hat{d}_{Y_{P_j}}+NR_{ij}^0-NR_{ij}^{obs}\right|_{\substack{i=1,\dots,n\\ j=1,\dots,m}} \tag{9}$$

where:

v_{ij} – corrections to the measured bearing taken from the i-th radar station ($i = 1,\dots,5$);

NR_{ij}^0 – radar bearing value for $P_j^0(X_j^0, Y_j^0)$;

P_j – j-th position of the ship ($j = 1,\dots,4$);

Assuming that:

V – corrections vector;

A – matrix of coefficients with unknowns;

$\hat{\mathrm{d}}_\mathrm{X}$ – searched vector of increments in the expected coordinates;

L – free terms matrix.

Thus the matrix system of corrections' equations takes the form as follows:

$$\mathrm{V} = \mathrm{A} \cdot \hat{\mathrm{d}}_{\mathrm{X}} + \mathrm{L} \tag{10}$$

With taking advantage of the indeterminate method, a solution of this equations system is:

$$\hat{\mathrm{d}}_{\mathrm{X}} = -\left(\mathrm{A}^{\mathrm{T}}\mathrm{P}\mathrm{A}\right)^{-1}\mathrm{A}^{\mathrm{T}}\mathrm{P}\mathrm{L} \tag{11}$$

where: P – is the performed observations weights matrix ($i = 1,\ldots,5$), ($j = 1,..,4$)

Therefore, estimators of the equalized coordinates of the ship at sea are as follows:

$$\hat{\mathrm{P}}_{\mathrm{j}} = \mathrm{P}_{\mathrm{j}}^{0} + \hat{\mathrm{d}}_{\mathrm{XY}} = \begin{bmatrix} X_j^0 \\ Y_j^0 \end{bmatrix} + \begin{bmatrix} \hat{d}_{X_{P_j}} \\ \hat{d}_{Y_{P_j}} \end{bmatrix} = \begin{bmatrix} \hat{X}_j \\ \hat{Y}_j \end{bmatrix} \tag{12}$$

where: $\hat{\mathrm{P}}_{\mathrm{j}}\left(\hat{\mathrm{X}}_{\mathrm{j}}, \hat{\mathrm{Y}}_{\mathrm{j}}\right)$ – the estimated position of a vessel at sea.

To find out which of the standardized corrections may represent gross errors (not within $\Delta\bar{v}$), there was determined the corrections vector covariance matrix for $m_0 = 1$.

$$\hat{\mathrm{C}}_{\hat{\mathrm{V}}(\mathrm{m_o}=1)} = \mathrm{P}^{-1} - \mathrm{A}\left(\mathrm{A}^{\mathrm{T}}\mathrm{P}\mathrm{A}\right)^{-1}\mathrm{A}^{\mathbf{T}} \tag{13}$$

and next there was carried out classification, whether for every i – $\bar{v}_i \in \Delta\bar{v}$

In case all $\bar{v}_i \in \Delta\bar{v}$, then we abandon further calculations; otherwise, if any correction $\bar{v}_i \notin \Delta\bar{v}$ there were followed up the next iteration steps:

It is assumed that $j = 0$ then:

$$\mathrm{V}^{(\mathrm{j})} = \mathrm{V}, \quad \mathrm{P}^{(\mathrm{j})} = \mathrm{P}, \quad \mathrm{C}^{(\mathrm{j})}_{\mathrm{V}\,(m_o=1)} = \hat{\mathrm{C}}^{(\mathrm{j})}_{\hat{\mathrm{V}}\,(m_o=1)}$$

and the control parameters for all the functions of attenuation $l = 0.02$, $g = 2$, and for the Huber's function $k_b = 6$ and the function of attenuation and the matrix of attenuation are calculated; afterwards it has to be checked which of the standardized corrections is laid within the admissible interval:

Then iteration is carried out; we increase j by 1, it means: $j: = j + 1$, and calculate the matrix of weights, of increments, so the matrix of corrections:

$$\left.\begin{aligned} &\mathrm{P}^{\mathrm{j}} = \mathrm{T}\left(\overline{\mathrm{V}}^{(\mathrm{j}-1)}\right)\mathrm{P}^{(\mathrm{j}-1)} \\ &\mathrm{d}_{\mathrm{X}}^{(\mathrm{j})} = -\left(\mathrm{A}^{\mathrm{T}}\mathrm{P}^{(\mathrm{j})}\mathrm{A}\right)^{-1}\mathrm{A}^{\mathrm{T}}\mathrm{P}^{(\mathrm{j})}\mathrm{L} \\ &\mathrm{V}^{[\mathrm{j}]} = \mathrm{A} \cdot \mathrm{d}_{\mathrm{X}}^{(\mathrm{j})} + \mathrm{L} \end{aligned}\right\} \tag{14}$$

Having in mind the assumed precision of calculations we check differences between the corrections vectors elements: $\mathrm{v}^{(\mathrm{j})}$ and $\mathrm{v}^{(\mathrm{j}-1)}$.

In case the differences are more essential than the assumed ones, we have to calculate the corrections vector covariance matrix for $m_0 = 1$.

$$\mathbf{C}^{(\mathbf{j})}_{\mathrm{V}\,(m_o=1)} = \left(\mathrm{P}^{(\mathrm{j})}\right)^{-1} - \mathrm{A}\left(\mathrm{A}^{\mathrm{T}}\mathrm{P}^{(\mathrm{j})}\mathrm{A}\right)^{-1}\mathrm{A}^{\mathrm{T}}$$
$$m_{v_i}^{(j)} = \sqrt{\left[\mathbf{C}^{(\mathbf{j})}_{\mathrm{V}\,(m_o=1)}\right]_{ii}} \qquad \bar{v}_i^{(j)} = \frac{v_i^j}{m_{v_i}^{(j)}} \tag{15}$$

Now a value of the attenuation function and the matrix of attenuation are calculated. We increase j by 1, and begin the next step of iteration. The iteration process is finished with such equalization, for which the obtained standardized corrections values are laid within the interval admissible therefor and resulting therefrom the new attenuation matrix would not cause any decrease of the weights matrix value and, in turn, the corrections values (within the assumed limits of calculations precision).

Equalization problem and its solution

The calculations were analyzed taking advantage of the measured navigational observations such as the bearings. There were carried out observations of the ship along the Gulf of Gdańsk; the surveys were performed at five coastal stations, at asymmetrical time intervals. Due to the long measuring sequence, this paper presents the surveys of four ship's positions. Values of the bearings are shown in table 1. For a purpose of this research,

Table 1. The bearings and coordinates of the ship sailing in the Gulf of Gdańsk

Survey point	Coastal radar stations					$\mathrm{P}_{\mathrm{j}}^{0}$
	Hel	Gdynia	Gdańsk North Port	Górki Zachodnie	Krynica Morska	
P_1	140.2°	93.3°	27.6°	29.5°	293.9°	φ = 54° 31.279'N λ = 18° 55.539'E
P_2	156.1°	100.5°	29.3°	27.3°	288.8°	φ = 54° 29.829'N λ = 18° 53.472'E
P_3	172.8°	112.5°	33.3°	21.8°	282.7°	φ = 54° 27.769'N λ = 18° 50.539'E
P_4	180.2°	121°	38.1°	15.1°	278.4°	φ = 54° 26.485'N λ = 18° 48.712'E

one survey was biased with a gross error. In practice, it may happen in case a bearing is taken falsely by the radar operator, improper identification of a surveyed object or an error in indications of the navigational equipment connected with radar. The gross error comes out for the positions P_1, P_2, P_3 and P_4 in all bearings taken at the coastal station in Gdańsk.

To simplify the calculation process there was decided that the further calculations are to be performed applying the rectangular coordinates system, not the terrestrial coordinates one. The calculations were carried out for each location of the ship at the moment of taking a bearing. Due to limits of size of this paper the authors present results of the calculations performed for one position P_2.

While carrying out equalization of the performed observations in the first step applying the least squares method, for the selected functions of attenuation the following results were obtained:

1) In case of the Danish function of attenuation:

The vector of increments in the expected coordinates is:

$$\hat{d}_X = -\left(A^T P A\right)^{-1} A^T P L = \begin{bmatrix} \hat{d}_{X_{P_1}} \\ \hat{d}_{Y_{P_1}} \end{bmatrix} = \begin{bmatrix} 3059.04 \\ -1193.10 \end{bmatrix}_{[m]}$$

Thus, the correction vector takes the value:

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 1.42 \\ -4.80 \\ 11.05 \\ -7.33 \\ 5.21 \end{bmatrix}$$

Consequently, the estimator of the observed ship's position at sea, with a use of the observations carried out at the coastal station is as follows:

$$\hat{P}_1 = \begin{bmatrix} \hat{X}_1 \\ \hat{Y}_1 \end{bmatrix} = \begin{bmatrix} X_1^0 \\ Y_1^0 \end{bmatrix} + \begin{bmatrix} \hat{d}_{X_{P_1}} \\ \hat{d}_{Y_{P_1}} \end{bmatrix} =$$

$$= \begin{bmatrix} 6040883.49 \\ 363431.40 \end{bmatrix} + \begin{bmatrix} 3059.04 \\ -1193.10 \end{bmatrix} = \begin{bmatrix} 6043942.53 \\ 363590.33 \end{bmatrix}$$

Basing on the obtained results we set out which of the standardized corrections can represent gross errors. Assuming for the calculations $\gamma = 0.95$, wherefrom $k = 2$. The admissible interval $\Delta\bar{v}$ form is as follows: $\Delta\bar{v} \in \langle -k; k \rangle = \langle -2; 2 \rangle$. Then the corrections vector's covariance matrix $\hat{C}$ for $m_o = 1$.

$$\hat{C}_{\hat{V}(m_o=1)} = P^{-1} - A\left(A^T P A\right)^{-1} A^T =$$

$$= \begin{bmatrix} 0.06837 & -0.07554 & 0.00954 & 0.06422 & 0.04995 \\ -0.07554 & 0.17395 & -0.06657 & -0.03361 & 0.04429 \\ 0.00954 & -0.06657 & 0.13804 & -0.09869 & 0.03454 \\ 0.06422 & -0.03361 & -0.09869 & 0.14578 & 0.01412 \\ 0.04995 & 0.04429 & 0.03454 & 0.01412 & 0.22387 \end{bmatrix}$$

then we have to carry out the classification:

$$\bar{\hat{v}}_1 = \frac{(\hat{v}_1)_{(m)}}{(\sqrt{0.06837})_{(m)}} = 5.42 \quad \notin \Delta\bar{v}$$

$$\bar{\hat{v}}_2 = \frac{(\hat{v}_2)_{(m)}}{(\sqrt{0.17395})_{(m)}} = -11.52 \quad \notin \Delta\bar{v}$$

$$\bar{\hat{v}}_3 = \frac{(\hat{v}_3)_{(m)}}{(\sqrt{0.13804})_{(m)}} = 29.73 \quad \notin \Delta\bar{v}$$

$$\bar{\hat{v}}_4 = \frac{(\hat{v}_4)_{(m)}}{(\sqrt{0.14578})_{(m)}} = -19.20 \quad \notin \Delta\bar{v}$$

$$\bar{\hat{v}}_5 = \frac{(\hat{v}_5)_{(m)}}{(\sqrt{0.22387})_{(m)}} = 11.01 \quad \notin \Delta\bar{v}$$

The obtained results prove that none of the standardized corrections' estimators is laid within the admissible interval. If the geodesy methods were applied, it would be necessary to reject any measurements biased with such errors and to repeat the surveys. However, it is difficult for a port approaching ship to turn back to let us perform surveys at the previously measured positions. So, remaining not influenced by such errors we perform equalization of the measurement results, making the observation results robust to gross errors with application of the function of attenuation.

Finally the following solution is found:

$$\hat{d}_X = -\left(A^T \hat{P} A\right)^{-1} A^T \hat{P} L = \begin{bmatrix} -284.73 \\ -267.45 \end{bmatrix}_{(m)}$$

$$\hat{V} = A \cdot \hat{d}_X + L = \begin{bmatrix} 0.04 \\ 1.28 \\ 21.64 \\ 2.15 \\ 2.08 \end{bmatrix}_{(m)}$$

$$\hat{P}_1 = \begin{bmatrix} \hat{X}_1 \\ \hat{Y}_1 \end{bmatrix} = \begin{bmatrix} X_1^0 \\ Y_1^0 \end{bmatrix} + \begin{bmatrix} \hat{d}_{X_{P_1}} \\ \hat{d}_{Y_{P_1}} \end{bmatrix} =$$

$$= \begin{bmatrix} 6040883.49 \\ 363431.40 \end{bmatrix} + \begin{bmatrix} 3059.04 \\ -1193.10 \end{bmatrix} = \begin{bmatrix} 6043942.53 \\ 362238.30 \end{bmatrix}$$

2) For the Hampel's function:

The vector of increments in the expected coordinates is:

$$\hat{d}_X = -\left(A^T P A\right)^{-1} A^T P L = \begin{bmatrix} \hat{d}_{X_{P_1}} \\ \hat{d}_{Y_{P_1}} \end{bmatrix} = \begin{bmatrix} 3059.04 \\ -1193.10 \end{bmatrix}_{[m]}$$

So, the corrections vector takes the following value:

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 1.42 \\ -4.80 \\ 11.05 \\ -7.33 \\ 5.21 \end{bmatrix}$$

Thus, the following is the estimator of the observed position of the ship at sea, with a use of the observations taken at the coastal stations:

$$\hat{P}_1 = \begin{bmatrix} \hat{X}_1 \\ \hat{Y}_1 \end{bmatrix} = \begin{bmatrix} X_1^0 \\ Y_1^0 \end{bmatrix} + \begin{bmatrix} \hat{d}_{X_{P_1}} \\ \hat{d}_{Y_{P_1}} \end{bmatrix} =$$

$$= \begin{bmatrix} 6040883.49 \\ 363431.40 \end{bmatrix} + \begin{bmatrix} 3059.04 \\ -1193.10 \end{bmatrix} = \begin{bmatrix} 6043942.53 \\ 362238.30 \end{bmatrix}$$

Basing on the obtained results we were capable to define which of the standardized corrections may represent gross errors. Assuming $\gamma = 0.95$ for calculation purpose it is obtained thereby that $k = 2$. The admissible interval $\Delta\bar{v} \in \langle -k;k \rangle = \langle -2;2 \rangle$. Within this function range we also assume a value $k_b = 6$. Then there is calculated the corrections vector's matrix for $m_o = 1$, and next carry out classification, which of the standardized corrections is laid within the admissible interval:

$$\bar{\hat{v}}_1 = 5.42 \quad \notin \Delta\bar{v}, \quad \bar{\hat{v}}_2 = -11.52 \quad \notin \Delta\bar{v}$$
$$\bar{\hat{v}}_3 = 29.73 \notin \Delta\bar{v}, \quad \bar{\hat{v}}_4 = -19.20 \notin \Delta\bar{v}$$
$$\bar{\hat{v}}_5 == 11.01 \notin \Delta\bar{v}$$

Finally, we obtain the following solution:

$$\hat{d}_X = -\left(A^T \hat{P} A\right)^{-1} A^T \hat{P} L = \begin{bmatrix} 3241.08 \\ -932.85 \end{bmatrix}_{(m)}$$

$$\hat{V} = A \cdot \hat{d}_X + L = \begin{bmatrix} -0.01 \\ -5.39 \\ 11.12 \\ -6.83 \\ 5.6 \end{bmatrix}_{(m)}$$

$$\hat{P}_1 = \begin{bmatrix} \hat{X}_1 \\ \hat{Y}_1 \end{bmatrix} = \begin{bmatrix} X_1^0 \\ Y_1^0 \end{bmatrix} + \begin{bmatrix} \hat{d}_{X_{P_1}} \\ \hat{d}_{Y_{P_1}} \end{bmatrix} =$$

$$= \begin{bmatrix} 6040883.49 \\ 363431.40 \end{bmatrix} + \begin{bmatrix} 3241.08 \\ -932.85 \end{bmatrix} = \begin{bmatrix} 6044124.57 \\ 362498.54 \end{bmatrix}$$

3) For the Huber's function:

The following is the vector of increments in the expected coordinates:

$$\hat{d}_X = -\left(A^T P A\right)^{-1} A^T P L = \begin{bmatrix} \hat{d}_{X_{P_1}} \\ \hat{d}_{Y_{P_1}} \end{bmatrix} = \begin{bmatrix} 3059.04 \\ -1193.10 \end{bmatrix}_{[m]}$$

Thus, the corrections vector takes the value:

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 1.42 \\ -4.80 \\ 11.05 \\ -7.33 \\ 5.21 \end{bmatrix}$$

Consequently, the following is the estimator of the observed position of a ship at sea, with a use of the observations taken at the coastal stations:

$$\hat{P}_1 = \begin{bmatrix} \hat{X}_1 \\ \hat{Y}_1 \end{bmatrix} = \begin{bmatrix} X_1^0 \\ Y_1^0 \end{bmatrix} + \begin{bmatrix} \hat{d}_{X_{P_1}} \\ \hat{d}_{Y_{P_1}} \end{bmatrix} =$$

$$= \begin{bmatrix} 6040883.49 \\ 363431.40 \end{bmatrix} + \begin{bmatrix} 3059.04 \\ -1193.10 \end{bmatrix} = \begin{bmatrix} 6043942.53 \\ 363590.33 \end{bmatrix}$$

On the basis of the obtained results it was defined which of the standardized corrections may represent gross errors. Adopting to the calculations $\gamma = 0{,}95$, we find that $k = 2$. The admissible interval $\Delta\bar{v}$ is as follows: $\Delta\bar{v} \in \langle -k;k \rangle = \langle -2;2 \rangle$. A this point there is calculated the corrections vector's covariance matrix for $m_o = 1$, and then carried out classification, which of the standardized corrections is laid within the limits if the admissible interval:

$$\bar{\hat{v}}_1 = 5.42 \quad \notin \Delta\bar{v}, \quad \bar{\hat{v}}_2 = -11.52 \quad \notin \Delta\bar{v}$$
$$\bar{\hat{v}}_3 = 29.73 \notin \Delta\bar{v}, \quad \bar{\hat{v}}_4 = -19.20 \notin \Delta\bar{v}$$
$$\bar{\hat{v}}_5 == 11.01 \notin \Delta\bar{v}$$

Ultimately, the following solution is obtained:

$$\hat{d}_X = -\left(A^T \hat{P} A\right)^{-1} A^T \hat{P} L = \begin{bmatrix} 3059.04 \\ -1193.10 \end{bmatrix}_{(m)}$$

Table 2. Ship's position, sizes of increments and estimated coordinates

Survey point	Reckoned ship coordinates	Estimated coordinates	Coordinates after applying the Danish function	Coordinates after applying the Hampel's function	Coordinates after applying the Huber's function
P_1	X = 6043505.62 Y = 365741.39	X = 6044148.28 Y = 365520.70	X = 6043729.09 Y = 365941.95	X = 6046533.702 Y = 363772.566	X = 6046396.96 Y = 363590.33
P_2	X = 6040883.49 Y = 363431.40	X = 6041537.69 Y = 363401.64	X = 6041168.22 Y = 363698.85	X = 6044124.571 Y = 362498.544	X = 6043942.53 Y = 362238.30
P_3	X = 6037160.10 Y = 360149.01	X = 6037735.18 Y = 360230.87	X = 6037090.70 Y = 360606.77	X = 6038780.569 Y = 360472.854	X = 6040363.66 Y = 359950.24
P_4	X = 6034840.44 Y = 358101.69	X = 6035222.02 Y = 358103.18	X = 6036314.75 Y = 358474.98	X = 6037828.994 Y = 358733.697	X = 6037916.67 Y = 358267.96

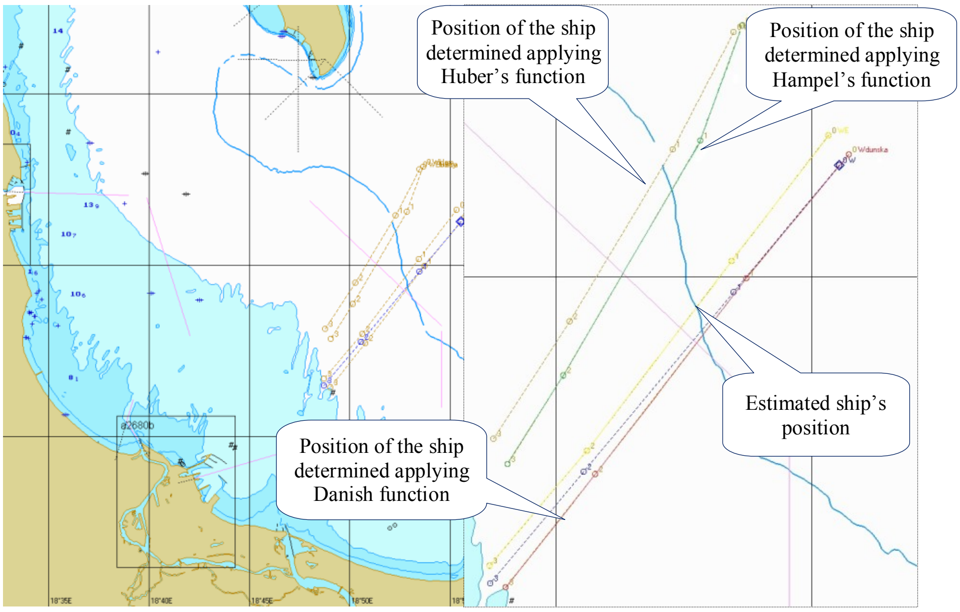

Fig. 2. Graphical presentation of the calculation results [own study]

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 1.42 \\ -4.80 \\ 11.05 \\ -7.33 \\ 5.21 \end{bmatrix}$$

$$\hat{P}_1 = \begin{bmatrix} \hat{X}_1 \\ \hat{Y}_1 \end{bmatrix} = \begin{bmatrix} X_1^0 \\ Y_1^0 \end{bmatrix} + \begin{bmatrix} \hat{d}_{X_{P_1}} \\ \hat{d}_{Y_{P_1}} \end{bmatrix} =$$

$$= \begin{bmatrix} 6040883.49 \\ 363431.40 \end{bmatrix} + \begin{bmatrix} 3059.04 \\ -1193.10 \end{bmatrix} = \begin{bmatrix} 6043942.53 \\ 362238.30 \end{bmatrix}$$

Having finished the final calculations the increments sizes and the estimated ship coordinates were displayed in table 2, and the graphical interpretation of the calculation results in figure 2.

Conclusions

On taking measurement of parameters necessary for determination of vessels positions it has to be taken into consideration that in effect of any disturbances in the survey process the measurement results may be biased with serious errors; at the extreme situation the errors can considerably deviate from the expected results. In case a gross error occurs it is advised to repeat the survey and correct the results. For a ship sailing in water area re-survey is impossible. Within each time interval the ship travels along a certain route distance and turning back to the last measuring point is unfeasible. In consequence, incorrect determination of the ship's position may cause mistaken chart navigation and finally appear a threat to navigation safety. Applying the M-estimation methods can considerably correct such errors and reduce their influence on marking out the plot of route on the chart.

A key problem in robust estimation is selection of the proper attenuation function, affecting the position fixing accuracy. In the paper three methods of robust equalization are presented. The performed analysis proved that for the presented case it appears the best to apply the Danish attenuation function. When the other functions – of Hampel or of Hubert are applied, in case of the serious errors, the results show displacing the vessel's position to an unexpected point.

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