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DYNAMIC STRESS CONCENTRATION AT THE BOUNDARY OF AN INCISION AT THE PLATE UNDER THE ACTION OF WEAK SHOCK WAVES

Olena MIKULICH*, **Vasyl' SHVABYUK***, **Heorhiy SULYM****

*Department of Technical Mechanic, Lutsk National Technical University, 75 Lvivska st., Lutsk, 43018, Ukraine
**Bialystok University of Technology, ul. Wiejska 45C, 15-351 Bialystok, Poland

shypra@ukr.net, shvabyk@lutsk-ntu.com.ua, h.sulym@pb.edu.pl

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Abstract: This paper proposes the novel technique for analysis of dynamic stress state of multi-connected infinite plates under the action of weak shock waves. For solution of the problem it uses the integral and discrete Fourier transforms. Calculation of transformed dynamic stresses at the incisions of plates is held using the boundary-integral equation method and the theory of complex variable functions. The numerical implementation of the developed algorithm is based on the method of mechanical quadratures and collocation technique. For calculation of originals of the dynamic stresses it uses modified discrete Fourier transform. The algorithm is effective in the analysis of the dynamic stress state of defective plates.

Key words: Stress State, Plate, Incision, Weak Shock Wave

1. INTRODUCTION

In metallurgy, machinery, construction and other industries a significant part of processes includes the presence of a variety of dynamic loads, which are caused by technological and mechanical processes. A variety of the dynamic loads are exerted in the structures of buildings by the wind and the seismic forces.

The dynamic loads result in the periodic changes of stresses. They may be within a single cycle, as a vibration or pulse packets, and a combination thereof. Dynamic load of any of these types may have a shock character.

The vibration-shock and shock loads can lead to the damage which cannot be explained by the excess of the yield stress or the accumulation of fatigue phenomena.

The use of the shock load by the ultrasound pulses has been effective in the study of the material defects. These methods are described in the book of Isbell (2005).

Sound pulses are transformed into shock waves, weak due to significant energy dissipation. The transition of shock waves through the domain of any defects, including cracks and pores, is accompanied with dispersing. Scattering of shock pulses gives essential information about the defect condition. Evaluation of diffraction parameters can predict the strength of the solid.

Dynamic stress state, which is occurred by the interaction of a shock wave scattered by the holes and incision in the plates differs from the case of quasi-static deformation.

In the literature, the study of dynamic stress state of the plates with holes was conducted by various methods.

In the works of Guz et al. (1978) and Kubenko (1967) the study of the distribution of dynamic stresses in the plate with a circular hole by the action of the shock load, which is applied to the boundary, is based on the Laplace transform with usage of the series method.

In the works of Benerjee (1994) and Brebbia et al. (1984) for the solving of the problems the singular integral equations are received by using the Laplace transform.

In the work of Popov at al. (2009) by using the Laplace transform it is studied the dynamic stress state of elastic bodies with inclusions.

In the works of Myhas'kiv et al. (2009) for the solution of dynamic problems for the bodies with inclusions and cracks it is used the finite-difference method in time and method of integral equations.

In the works of Ayzenberg-Stepanenko et al. (2012, 2013) with using finite-difference algorithms it is obtained the description of fronts and front zones with a minimal influence of spurious effects of numerical approximation.

By using a modified finite-difference method with respect to time and the method of Fourier series with respect to the angular variable in the works of Onyshko et al. (2015) it is got the time dependence of the dynamic stress concentration on the hole for plane dynamic problem of the theory of elasticity.

In the works of Pasternak et al. (2013) the study of dynamic stresses in the plates due to the antiplane deformation is conducted using the Fourier transform.

In the works of O. Mikulich (Shvabyuk et al., 2015) the method of study the stress concentration in the plates with holes and incisions for time-harmonic case is developed. This method is based on the application of the boundary integral equation method and the theory of function of complex variable. For the numerical calculation the methods of mechanical quadrature and collocation are used. Results of numerical calculations showed that the algorithm is effective for investigation of the action of time-harmonic loads.

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2. SOLUTION OF THE PROBLEM

2.1. Statement of the problem

In applying of the Fourier transform it is existed the following relation between original and transformed function:

$$\tilde{f}(x,\omega) = \int_{-\infty}^{\infty} f(x,t)e^{-i\omega t}dt,$$
$$f(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(x,\omega)e^{i\omega t}d\omega. \quad (1)$$

This paper applies the Fourier transform (1) to study of the dynamic stress state of infinite plate with an incision.

Consider the equation of the motion of the isotropic plate written for displacements (Brebbia et al., 1984):

$$(c_1^2 - c_2^2)u_{i,ij} - c_2^2 u_{j,ii} + b_j = \frac{\partial^2 u}{\partial t^2}, \quad (2)$$

where: $\mathbf{u}(\mathbf{x},t) = \{u_j(\mathbf{x},t)\}, j = 1;2$ is the vector of the displacement of the point $\mathbf{x} = \{x_1, x_2\}$, $c_1^2 = \frac{E}{\rho(1-\nu^2)}$, $c_2^2 = \frac{E}{2\rho(1+\nu)}$ are speeds of expansion and shear waves, E is the Young's modulus, ρ is the density of the material; ν is the Poisson ratio, $\mathbf{b} = \{b_j\}$ is the vector of mass forces, $i = 1;\ 2$, $(\ )_{,j}$ means for differentiation with respect to x_j, t is time.

Applying transform (1) to equation (2) one obtain

$$(c_1^2 - c_2^2)\tilde{u}_{i,ij} - c_2^2\tilde{u}_{j,ii} + \tilde{b}_j + \omega^2\tilde{u}_j = 0, \quad (3)$$

where:
$\tilde{u}_j = \int_{-\infty}^{\infty} u_j(x,t)e^{-i\omega t}dt$, $\tilde{b}_j = \int_{-\infty}^{\infty} b_j(x,t)e^{-i\omega t}dt$. This equation is equivalent to the equation of time-harmonic motion with a cyclic frequency ω (Brebbia et al., 1984).

This approach allows conducting the research in the field of transforms of displacements; thereby the effect of the time factor is isolated.

Consider the case when incident weak shock wave is scattered by the incision at the plate. This weak shock wave is similar to (Gruber at al., 2013), which is given by the potential in the form:

$$\phi(\mathbf{x},t) = \begin{cases} \phi_0 f\left(\frac{\mathbf{x}}{a} - c_1\frac{t}{a}\right), & t \ge 0, \\ 0, & t < 0; \end{cases} \quad \psi(\mathbf{x},t) = 0.$$

Here ϕ_0 is a constant (Gruber et al., 2013), a is a characteristic of length. Applying transform (1) to this formula leads to

$$\tilde{\phi}(\mathbf{x},t) = \widetilde{\phi_0}\frac{a}{c_1}\tilde{f}\left(\frac{\omega a}{c_1}\right)e^{-k_1\mathbf{x}};\ \tilde{\psi}(\mathbf{x},t) = 0, \quad (4)$$

where: $\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$.

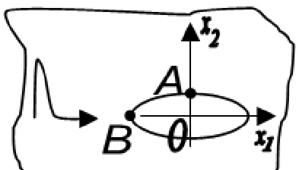

Fig. 1. Model of the plate

In further studies modulation of impulse from the time is chosen in the form of weak shock waves (Pasternak et al., 2013):
$f(t) = p_* t^{n_*} e^{-\alpha_* t}, t > 0, n_* \ge 0,$

where: $\tilde{f}(\omega) = p_*(n_*)!\,(\alpha_* + i\omega)^{-n_*-1}$.

Consider an infinite plate with an incision that is under the weak shock wave (4) (Fig. 1). The problem consists in determination of the dynamic stresses at the boundary of incision at the plate.

The centre of gravity of the plate is placed at the origin of a Cartesian coordinate system Ox_1x_2. Symbol D denotes the domain occupied by the plate, and L is the boundary of the domain D.

The boundary conditions of the problem are written as (Brebbia et al., 1984):

$$\sigma_n|_L = \Phi_1(\mathbf{x},t),\ \tau_{sn}|_L = \Phi_2(\mathbf{x},t), \quad (5)$$

where: $\Phi_1(\mathbf{x},t), \Phi_2(\mathbf{x},t)$ are the predetermined functions. Applying the Fourier transform (1) to the boundary conditions (5) leads to

$$\tilde{\sigma}_n|_L = \widetilde{\Phi_1}(\mathbf{x},\omega),\ \tilde{\tau}_{sn}|_L = \widetilde{\Phi_2}(\mathbf{x},\omega). \quad (6)$$

2.2. Integral equations of the problem

For the plane stress state the potential solution for transforms of displacements is selected as (Bonnet, 1995):

$$\tilde{u}_j(\mathrm{x},\omega) = \int_L p_i(\mathbf{x}^0,\omega)U_{ij}^*(\mathbf{x},\ \mathbf{x}^0,\omega)ds, \quad (7)$$

where $p_1,\ p_2$ are unknown complex potential functions. Integration along the boundary is performed within variables x_1^0, x_2^0, where $\mathbf{x}^0 = \{x_1^0, x_2^0\}$. The representation of the images U_{ij}^* should be choosing with the regard to Zommerfeld radiation condition, since the plate is infinite. It has the form (Brebbia et al., 1984):

$$U_{ij}^* = \frac{1}{2\pi\rho c_2^2}\left(\psi\delta_{ij} - \chi\cdot r_i r_j\right), \quad (8)$$

where:

$$\psi = K_0(k_2 r) + \frac{1}{k_2 r}\left(K_1(k_2 r) - \frac{c_2}{c_1}K_1(k_1 r)\right);$$
$$\chi = K_2(k_2 r) - \left(\frac{c_2}{c_1}\right)^2 K_2(k_1 r),$$

$r_j = \frac{\partial r}{\partial x_j}$, $r = \sqrt{(x_1 - x_1^0)^2 + (x_2 - x_2^0)^2}$, $k_j = \frac{i\omega}{c_j}$, $K_m(r)$ are Bessel functions of the third kinds, $j = 1, 2,\ m = 0, 1, 2$.

For determination of unknown boundary functions $p_1,\ p_2$ the calculation of stresses at an arbitrary point of the plate are performed by the formula (Savin, 1968):

$$\tilde{\sigma}_n = \frac{\sigma_{11}+\sigma_{22}}{2} + \frac{1}{2}\left(e^{-2i\alpha}\left(\frac{\sigma_{11}-\sigma_{22}}{2} + i\sigma_{12}\right) + e^{2i\alpha}\left(\frac{\sigma_{11}-\sigma_{22}}{2} - i\sigma_{12}\right)\right);\ \tilde{\tau}_{sn} = \frac{i}{2}\left(e^{2i\alpha}\left(\frac{\sigma_{11}-\sigma_{22}}{2} - i\sigma_{12}\right) - e^{-2i\alpha}\left(\frac{\sigma_{11}-\sigma_{22}}{2} + i\sigma_{12}\right)\right), \quad (9)$$

where: α is the angle between the normal $\vec{n}$ to the boundary of the plate and the axis Ox_1.

Substituting the representation for displacements (7) in the formulas (9), we obtain formulas for determining stresses at the boundary of the plate:

$$\tilde{\sigma}_n = \int_L f_1(z,\zeta)q d\zeta + \int_L f_2(z,\zeta)\overline{q}d\overline{\zeta};$$
$$\tilde{\tau}_n = \int_L g_1(z,\zeta)q d\zeta + \int_L g_2(z,\zeta)\overline{q}d\overline{\zeta}, \quad (11)$$

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where: $q = \frac{ipds}{dt}$ is an unknown function, which had to be determined, $\zeta = x_1^0 + ix_2^0$, $f_k(z,\zeta)$, $g_k(z,\zeta)$, $k = 1, 2$ are known functions, which contain Bessel functions of third kind and are obtained similarly to those in Mikulich (2016).

Integration of functions $f_k(z,\zeta)$, $g_k(z,\zeta)$, $k = 1,2$ for small values of the argument leads to singularity. To establish their characteristics we use the asymptotic expressions for the Bessel functions of the third kind for small values of the argument (Cohen, 2007).

Let us perform the limiting transition when $(x_1, x_2) \to L$ in the last formula according to Plemelj-Sokhotski formulas (Savin, 1968). Consequently, integral equations for determination of the unknown functions q and $\overline{q}$ for given loading at the boundary are obtain:

$$Re\left(\frac{q}{2}\right) - Im\left(A_1 \int_L \frac{qd\zeta}{z-\zeta} + \frac{dz}{d\bar{z}}\left(A_2 \int_L \frac{\bar{q}d\bar{\zeta}}{z-\zeta} + A_3 \int_L \frac{(\bar{z}-\bar{\zeta})qd\zeta}{(z-\zeta)^2}\right)\right) +$$

$$+\int_L f_1^R(z,\zeta)qd\zeta + \int_L f_2^R(z,\zeta)\bar{q}d\bar{\zeta} = \widetilde{\Phi_1}(\mathbf{x},\omega); \qquad (12)$$

$$Im\left(\frac{q}{2}\right) - Re\left(\frac{dz}{d\bar{z}}\left(A_2 \int_L \frac{\bar{q}d\bar{\zeta}}{z-\zeta} + A_3 \int_L \frac{(\bar{z}-\bar{\zeta})qd\zeta}{(z-\zeta)^2}\right)\right)$$

$$+\int_L g_1^R(z,\zeta)qd\zeta + \int_L g_2^R(z,\zeta)\bar{q}d\bar{\zeta} = \widetilde{\Phi_2}(x,\omega),$$

where: $A_i, i = 1..3$ are constants, that in the case of plane stress state are as follows $A_1 = \frac{1+\nu}{4\pi\nu}$; $A_2 = \frac{3-\nu}{4\pi}$; $A_1 = \frac{1+\nu}{4\pi}$, ν is a Poisson's ratio, functions $f_1^R, f_2^R, g_1^R, g_2^R$ are known and regular (Mikulich, 2016).

2.3. Numeric solution algorithm

For calculation of the dynamic stresses at the infinite plate with incision the algorithm (Shvabyuk et al., 2016) is applied. The incision in the plates is modeled as an elliptic hole with axis ratio of 10.

Replacing the integrals at the system of integral equations (12) with the specified quadrature formulas (Eshkuvatov et al., 2009), the system of linear algebraic equations for determination of the nodal values of unknown boundary functions q and $\bar{q}$ is obtained:

$$\frac{q_s + \bar{q}_s}{4} + h\sum_{n=1}^{N} f_{1sn} q_n \psi'_n + h\sum_{n=1}^{K} f_{2sn}\bar{q}_n \bar{\psi}'_n = \tilde{\Phi}_{1s},$$

$$\frac{q_s - \bar{q}_s}{4i} + h\sum_{n=1}^{N} g_{1sn} q_n \psi'_n + h\sum_{n=1}^{K} g_{2sn}\bar{q}_n \bar{\psi}'_n = \tilde{\Phi}_{2s}, \quad (13)$$

where: $\psi = \psi(\theta)$, $0 < \theta < 2\pi$ is a set of parametric boundary equations, $f_{jsn} = f_j(\zeta_n, z_s)$, $g_{jsn} = g_j(\zeta_n, z_s)$, $j = 1, 2$; $z_s = \psi(\theta_s)$, $\zeta_n = \psi(\theta_n)$, $\theta_n = nh$, $\theta_s = \theta_n + \frac{h}{2}$, $h = \frac{2\pi}{N}$, N is a set of points of the partition boundaries, $\tilde{\Phi}_{1s}, \tilde{\Phi}_{2s}$ are known functions, which are determined by (4).

After determination of the unknown functions, dynamic stresses of the plate are calculated by dependencies, which are obtained in accordance with representation (7) by providing singular components in the kernels of equations and consequently using Plemelj-Sokhotski formulas:

$$\tilde{\sigma}_{\theta s} = \frac{q_s + \bar{q}_s}{4} + h\sum_{n=1}^{N} y_{1sn} q_n \psi'_n + h\sum_{n=1}^{K} y_{2sn}\bar{q}_n \bar{\psi}'_n +$$
$$+G_{2s}, \qquad (14)$$

where: $y_{jsn} = y_j(\zeta_n, z_s)$, $j = 1, 2$; G_{2s} is a known function, which is determined as a potential of reflected wave by (4).

For determination of the original of calculated dynamic stresses it is used the type of representation received from (1):

$$\sigma_\theta(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \widetilde{\sigma_\theta}(x,\omega) e^{i\omega t} d\omega.$$

At numeric calculations of unknown quantities it can be implemented based on the discrete Fourier transform, which proved effectiveness in the calculations for antiplane deformation (Pasternak at al., 2013). A transforms of mentioned hoop stresses are complex. For calculation of originals of the dynamic stresses it is used a modified discrete Fourier transform:

$$\sigma_\theta(t_k) = \frac{1}{T}\left(\sum_{n=-K}^{K-1} \tilde{\sigma}_\theta(\omega_n) \exp\left(2\pi i \frac{nk}{K}\right) - \sum_{n=-K}^{K-1} \tilde{\sigma}_\theta(\omega_n)\right), \qquad (15)$$

where: $t_k = \frac{kT}{K}$, $k = -K..K-1$, T is time of weak shock wave propagation, K is a number of elements of discrete sample, $\omega_n = \frac{2\pi n}{T}$ are frequency of sample. To optimize the numerical calculations by the formula (15) fast discrete Fourier transform (Ramamohan at al., 2010) is used, where the number of elements is chosen as follows $K = 2^m$, m is a positive integer. The calculations are performed for dimensionless time parameter $\mathcal{T} = t\frac{c_1}{a}$.

2.4. Numeric calculation of stresses in the plate

Let's study the dynamic stress state of the plate with incision from the actions of a weak shock wave. Potential of the waves is written as (4) with pulse duration $t = 1\ s$. At numerical calculations the parameters of the shock wave at formula (4) are chosen as $\alpha^* = 10, p^* = 185, n^* = 2$.

The incision is modelled as an elliptical hole with axes a and $0.1a$. At the calculations, it was assumed that the wave front is directed along the axis Ox_1. The study is conducted for the dimensionless time parameter $\mathcal{T} \in \left(0, \frac{Tc_1}{a}\right), T = 4\ s$. For additional testing of the algorithm the center of incision is shifted along the axis at a distance $\mathcal{T}_0 = T_0\frac{c_1}{a}, T_0 = 2\ s$. In this case, before the wave reaches the boundary of incision ($\mathcal{T}_* = T_*\frac{c_1}{a}, T_* = 1\ s$) dynamic stresses should be zero.

Since derived from the formula (15) the values of the originals of dynamic stresses σ_θ are complex in general, the results of numerical calculations graphically presents both real and imaginary values of dynamic circular stresses $\sigma_\theta^R = Re(\sigma_\theta)$, $\sigma_\theta^I = \mathrm{Im}(\sigma_\theta)$. Since the potential (4) is selected to be a complex one, real and imaginary parts of stresses correspond to the real and imaginary parts of complex loading given by (4).

Fig. 2 shows the change of the real and imaginary values of the dynamic stresses related to σ_0, which are calculated at the points A and B of the boundary. Here and in further calculations the value of σ_0 is chosen to be 1.

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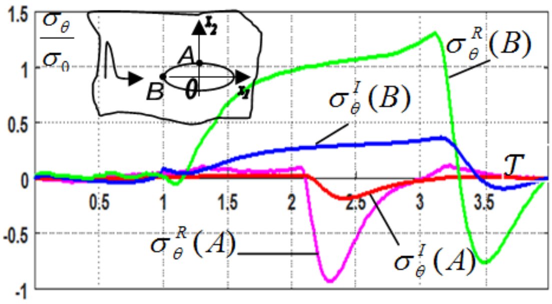

Fig. 2. Effects of weak shock waves on the incision at the plate

Fig. 2 shows that the dynamic stresses at the point B start growing at time $\mathcal{T}_* = T_* \frac{c_1}{a}, T_* = 1\,s$, when the shock wave reaches the semi-major axis of the incision. This is because the incision itself becomes the stress concentrator due the waves reflected from its boundary. This phenomenon is consistent with the principle of Huygens-Fresnel. Wave dissected by the heterogeneity propagates along the boundary of the incision, and the reflected wave portion extends from the boundary of the incision. At time $\mathcal{T}_{**} = T_{**} \frac{c_1}{a}, T_{**} = 3\,s$ the shock wave reaches the right border of the incision. Increasing of stresses at the boundary of incision at time $\mathcal{T}_r = T_r \frac{c_1}{a}, T_r = 3.5\,s$ is associated with the effect of the reflected wave. The intensity of the dynamic stresses decreases at $\mathcal{T}_e = T \frac{c_1}{a}, T = 4\,s$, when shock wave passes through the incision and is damped by reflected waves.

The reliability of the calculations is confirmed with the fact that the dynamic stresses are close to zero until the weak shock wave achieve the boundary of incision.

For a complete study of the dynamic stress state of the plate with an incision the time distributions of the absolute value of stresses for the interval of dimensionless time parameter $\mathcal{T}_* \in \left(2\frac{c_1}{a};\ 3\frac{c_1}{a}\right)$ are studied. The calculation results for the step $\Delta\mathcal{T}_* = 0.2\frac{c_1}{a}$ are shown in Fig. 3.

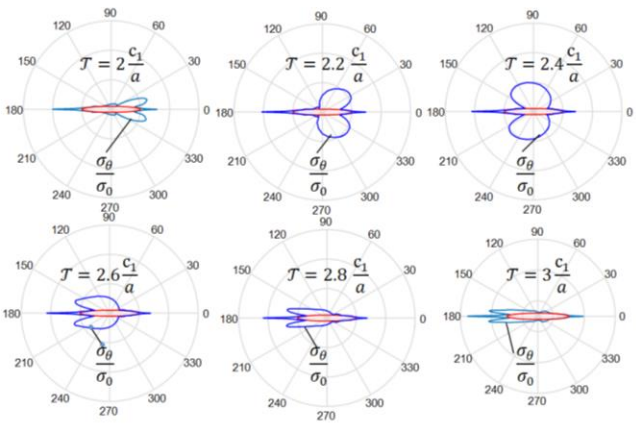

Fig. 3. Time slices of dynamic stresses distributions in the plate

Fig. 3 shows the effect of a weak shock wave at the incision accompanied by a reflected wave that moves along the boundary of the incision and causes a local increase in stresses in the plate. This reflected wave fades quickly and reduces its intensity.

Let's study the influence of weak shock waves with two successive pulses with duration $t = 1\,s$ on the stress state of the plate with the incision. The study is held for the case when the first shock impulse is induced at time parameter $\mathcal{T} = 0$, and the second is at $\mathcal{T}_1 = T_1 \frac{c_1}{a}, T_1 = 1\,s$. Shock pulses have the same intensity. Research is conducted for the dimensionless time parameter $\mathcal{T} \in \left(0,\ T\frac{c_1}{a}\right), T = 4\,s$.

For the mathematical setting of shock loads one can apply the time-shifting theorem and linearity theorem for the Transform (4) (Ramamohan at al., 2010). At numerical calculations the parameters of both pulses of the shock wave are chosen as $\alpha^* = 10$, $p^* = 185$, $n^* = 2$.

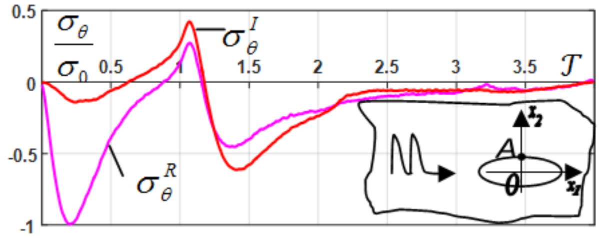

Fig. 4. The action of the weak shock waves with two pulses on the incision in the plate

Fig. 4 shows that dynamic stresses at the point A start growing at time $\mathcal{T} = 0$ due to the effects of the first shock wave pulse. The action of the second pulse at the moment of time parameter $\mathcal{T}_1 = 1\frac{c_1}{a}$ is imposing forward and backward from the right side of the boundary of incision of the waves. This leads to decrease of the intensity of dynamic stresses. Further action of reflected waves doesn't cause a significant increase in dynamic stresses.

Let's study the influence of intensity of pulses of weak shock wave to dynamic stress state of the plate with the incision. Calculations are held for the case of the weak shock wave in the form of two pulses of varying intensity and duration.

The first pulse lasts $t = 1\,s$. The second pulse is twice weaker than the first and lasts twice longer. The study is held for the case when the first shock pulse is induced at time $\mathcal{T}_0 = 0$, and the second is at $\mathcal{T}_1 = T_1 \frac{c_1}{a}, T_1 = 2\,s$. For the mathematical setting of shock loads one can apply the time-shifting theorem and linearity theorem of the transform (4) (Ramamohan at al., 2010). In numerical calculations the parameters of the shock waves have been identified as (4) at $\alpha * = 10, p * = 185, n * = 2$ for the first pulse and $\alpha * = 5, p * = 23, n * = 2$ for the second. Research is conducted for the time parameter $\mathcal{T}_* \in \left(0;\ 8\frac{c_1}{a}\right)$.

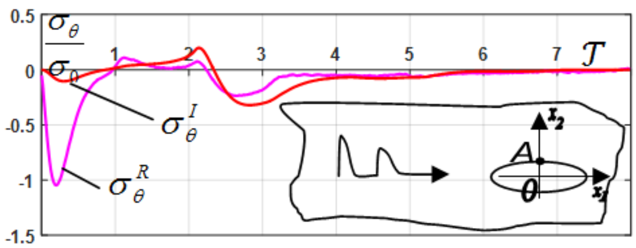

Fig. 5. Effects of weak shock waves with two pulses of varying intensity on incision at the plate

Fig. 5 shows that dynamic stresses at the point A start growing at time $\mathcal{T} = 0$ due to the effects of the first shock wave pulse. The action of the second pulse at $\mathcal{T}_1 = 2\frac{c_1}{a}$ is the imposition of the incident and reflected waves. This leads to increase in dynamic stresses $\mathcal{T} \in \left(2.2\frac{c_1}{a}, 4\frac{c_1}{a}\right)$. Further action of reflected waves doesn't cause a significant dynamic growth.

3. SUMMARY

The technique developed in this paper allows studying the dynamic stresses at the boundary of incisions in plates under the action of weak shock waves. Using Fourier transform of time-variable the dynamic problem reduces to a finite number of time-harmonic problems. To determine the original of dynamic stresses the formula of numerical Fourier inverse transform is modified.

Based on the numerical algorithm the dynamic stresses in the plate with incision for different cases of shock loading are calculated.

The advantage of the proposed algorithm is the ability of determination of the dynamic stresses along the entire boundary, and not at the only specific point. This makes it possible to investigate in details the dynamic stress state of defective plates.

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