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MULTIFRACTAL SCALING LAW FOR HIGH-CYCLE FATIGUE STRENGTH APPLIED TO ALUMINUM ALLOY

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Abstract

Fatigue strength of the material decreases with an increase in the cross sectional area of the specimen. Such phenomenon can be described phenomenologically applying the fractal approach, describing a change in the strength properties by applying the fractal dimension. With that approach involves modelling the damage of a material ligament. The damage size depends on the defects and microfractures in the object analysed. The effect of the disordered material microstructure on the mechanical properties is decreasing with an increase in the cross sectional area of the object. Bearing that in mind, the description of the size effect should use nonlinear scaling (multifractal scaling law). That approach was used for the results of high-cycle fatigue studies, for various aluminum alloys. Determining the model parameters facilitates the evaluation of fatigue life and strength for the plants with a different geometry than the specimens studied in the lab.

Keywords: *size effect, fractal approach, aluminum alloy*

1. Introduction

Fatigue strength and fatigue life changes depending on the cross-sectional area. This phenomenon was studied and described in more detail as the size effect. The effect of the change in the cross-section on the strength properties of the material depends on the kind and the local properties of the material; including micromechanical material damage which are indispensable mechanism disturbing its structure [3]. The size effect is described with coefficient K_d which is a quotient of the specimen fatigue strength with any cross-section to the normative specimen fatigue strength determined for the same fatigue life [7]. To present the degree of the change in coefficient K_d against the cross sectional area, Fig. 1 presents the results of the experiments reported for selected aluminium alloys.

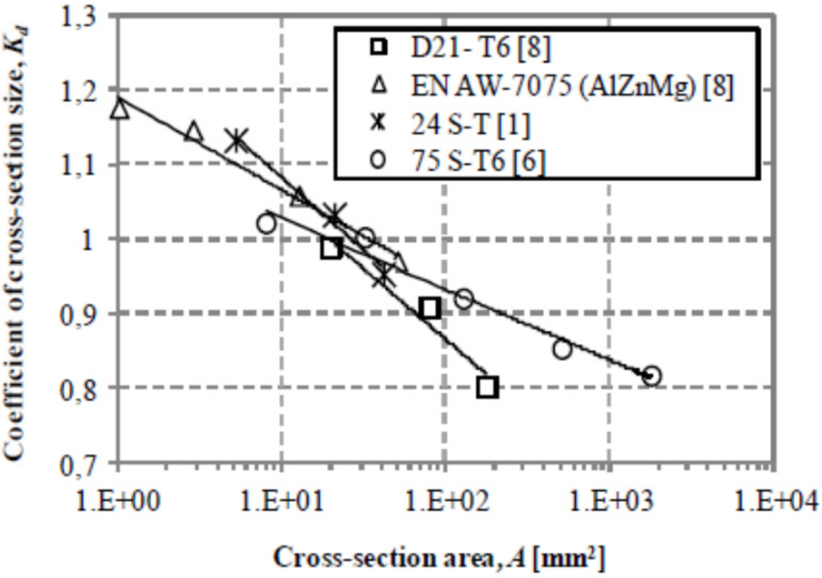

Fig. 1. Dependence of the coefficient of the cross-section (K_d) to the cross sectional area of round bendable specimens – own study based on [1, 6, 8]

To provide a description of the size effect of construction materials, the fractal approaches can be applied [2, 3]. The main assumption is to evaluate the strength properties of the plants different in size than the specimens studied in the lab. The extrapolation of the values can take place towards non-standard specimens (e.g. mini specimens) [7] or real-life plants with a much greater cross sectional area.

The aim of the present paper has been to determine the multifractal model parameters for a given group of construction materials. Additionally the aim of the paper is also to determine the lowest fatigue strength for the infinite value of the cross sectional area and the value transitional for the material structure. The model analysis has been based on the results of experiments reported in literature for various aluminium alloys.

2. Fractal approach

To describe the sensitivity of the material to the change in the size of the cross-section, fractal models are used in the scope of tensile strength [3] and fatigue strength [4, 5]. Applying the fractal approach involves the modelling of a damaged material ligament. Its weakening comes from defects and microfractures all throughout the size scope of the object. In the macroscopic scale, nominal strength σ is defined for nominal area a_0. As for an increase area, there occurs more and more discontinuity resulting in a decrease in nominal area a_1 responsible for fictional micro-stress σ_1. On the border of the microscopic scale there is assumed value a^* for renormalized strength σ^* determined from the dependence [3]:

$$\sigma^* = \frac{F}{A^{-d}} \tag{1}$$

where:
- F – external force,
- A – cross-sectional area,
- d – fractal dimension ($0 < d < 0.5$) of the ligament resulting from disordered material.

External force F does not change against the observation scale, which can be written as $F = \sigma a_0 = \sigma_1 a_1 = ... = \sigma^* a^*$. The cross-sectional area of object A can be compared to nominal areas $a_0 \sim A$, $a^* \sim A^{-d}$. In the bilogarithmic pattern the dependence of the strength to the cross-sectional area has been descried with a straight with a slope equal β_σ (Fig. 2) where the nominal strength decreases with an increase in the cross-sectional area.

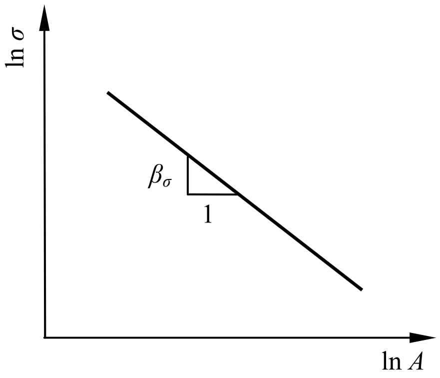

Fig. 2. Monofractal scaling for strength σ against cross-sectional area A [3]

The monofractal approach descried is a linear fractal scaling law. It is true only for a narrow range of sizes in which the fractal size is a constant value. The monofractal approach does not have the upper and lower limit for the scaling range. Cross-sectional area A is tending to infinity, which leads to unrealistic values for strength σ tending to zero. For the objects with a large cross-sectional area, it is false since they have a finite strength value.

Besides, there has been suggested the multifractal approach [3] (nonlinear model) which assumes a gradual disappearance of fractality (d tending to zero). Disordered material microstructure does not depend on the size of the object. However, the effect of disorder on the mechanical properties depends on the ratio of characteristic internal length l_{ch} to external specimen cross-sectional area A. It leads to a decrease in the effect of material microstructure together with an increase in the object size. Drawing on that assumption, multifractal scaling law (MFSL) for strength assumes the following form [3]:

$$\sigma(A) = f_t[1 + l_{ch} / A]^{1/2} \tag{2}$$

where:
- f_t – the lowest value of nominal strength,
- l_{ch} – characteristic internal length (material constant dependent on the cross-sectional area).

The graphic representation of dependence (2) is given in Fig. 3. Asymptotic value f_t corresponds to the lowest nominal strength for infinite cross-sectional area A. Characteristic value of internal length l_{ch} represents the variable of the effect of disorder on mechanical properties.

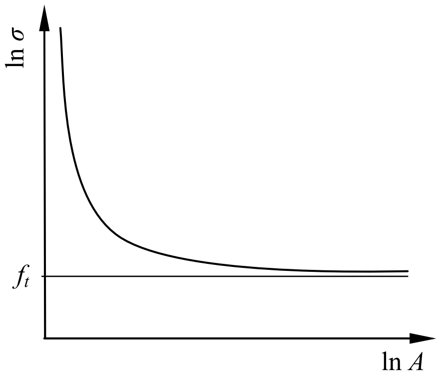

Fig. 3. Multifractal scaling law for strength against cross-sectional area A [3]

3. Multifractal scaling law

Multifractal scaling law (MFSL) can be applied to describe high-cycle fatigue strength. The main relation of MFSL is based on parameter C_r depends on the cross-sectional area A. This relation assumes the form of (original relation [4] depends on the diameter D):

$$C_r = C(A)^{\frac{1}{100}} = \left[\frac{a}{f \cdot (1/A) + 1} + b \cdot (1/A) + c \right]^{\frac{1}{2}} \tag{3}$$

where:

- C – constant parameter of fatigue characteristics σ_a-N with the Basquin equation $C = N(\sigma_a)^k$,
- $a = y_Q - y_P$,
- b – asymptote slope,
- $c = y_P$,
- f – value for the best fitting of the model.

Besides, parameters a, b, c are closely dependent on the material investigated. Exponential equation for C_r^2 is a graphic representation in Fig. 4 considering characteristic points y_P, y_Q.

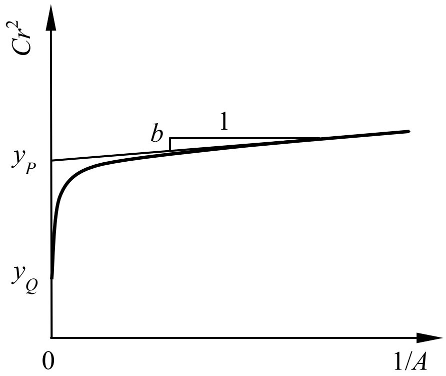

Fig.4. Dependence of the second power of parameter C_r^2 for cross-sectional area 1/A considering characteristic points y_P, y_Q [4]

The model facilitates determining the values characteristic for the material analysed. Fig. 5 provides the curve determined from relationship (3) and coordinates of characteristic point T. The first coordinate is the lowest fatigue strength $\ln C_{r\infty}$ for infinitely large cross-sectional area A, determined from the relationship [4]:

$$C_{r\infty} = (a + c)^{1/2} \tag{4}$$

Yet another one is parameter A_T treated as transitional value for the material structure size. It is determined from the dependence [4]:

$$A_T = \frac{b}{a + c} \tag{5}$$

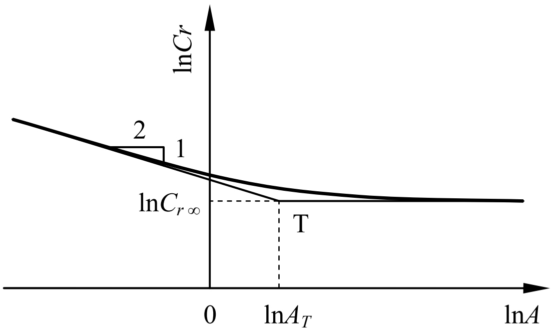

Fig. 5. Dependence of parameter lnCr to cross-sectional area lnA considering intersection T [4]

4. MFSL Application

Multifractal scaling laws have been applied for the results of the tests of high-cycle fatigue testing for aluminum alloy D21-T6 [8], EN AW-7075 (AlZnMg) [8], 24 S-T [1], 75 S-T6 [6] (original material markings following the authors of papers have been applied). The analyses facilitated determining model parameters (Table 1) described with the relationship (3) and served for nonlinear description of parameter C_r^2 for *1/A* (Fig. 6). For parameters *a*, *b*, *c*, recorded, there was determined parameter *f* responsible for the best fitting of the model for the results of the experimental studies. The parameters should be treated as similar. As for the case increase in the scope of geometries analysed (greater *A* scope), the model received will be more detailed.

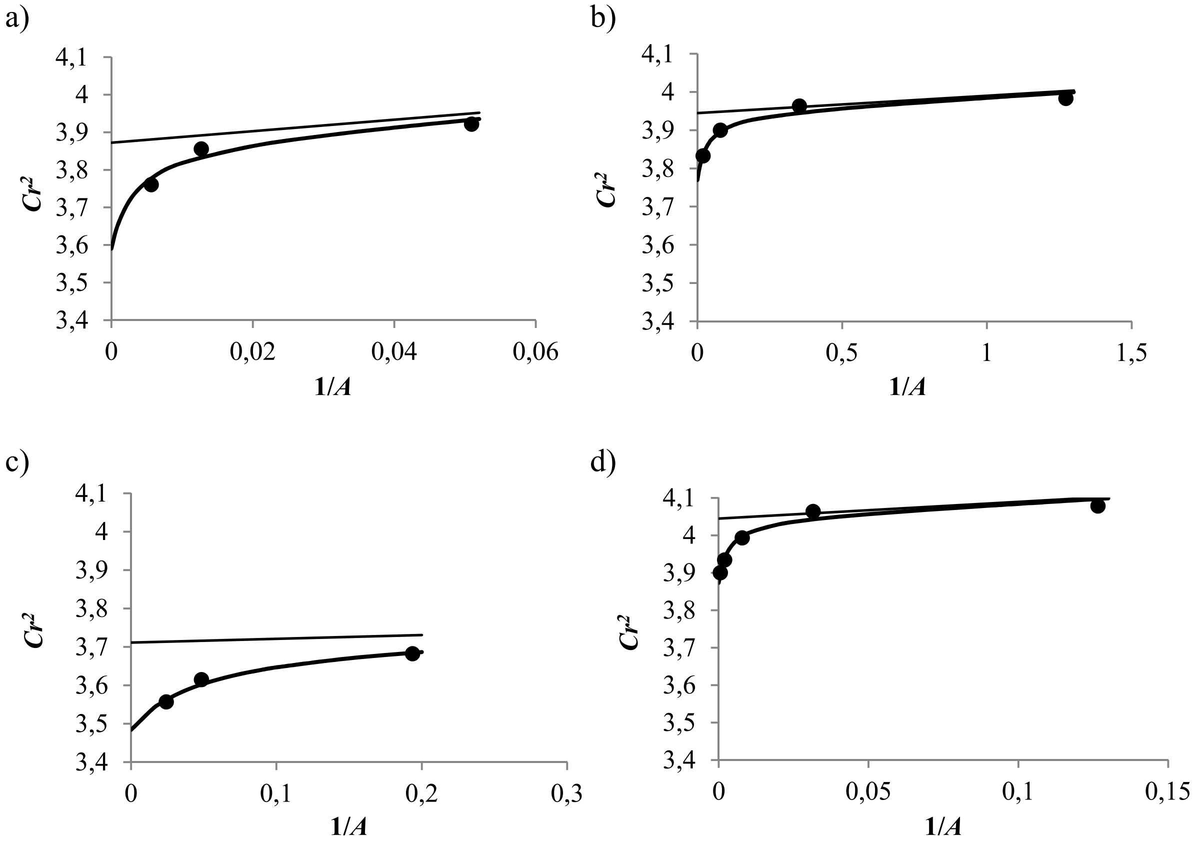

Fig. 6. Dependence of parameter C_r^2 to 1/A for the aluminium alloy analysed: a) D21-T6, b) EN AW-7075 (AlZnMg), c) 24 S-T, d) 75 S-T6

Tab. 1. Parameters of the MFSL model for the aluminium alloy analysed

Aluminum alloy	*a*	*b*	*c*	*f*
D21-T6	-0,282	1,535	3,872	305,7
EN AW-7075 (AlZnMg)	-0,175	0,045	3,945	29,7
24 S-T	-0,227	0,097	3,712	20,5
75 S-T6	-0,171	0,449	4,045	302,6

Fig. 7 shows plotting of the points from the results of experiments and curves received from the model of dependence (3). From dependences (4), (5) there were determined the coordinates of point *T* (compliant with Fig. 5) given in Table 2. From the plots one can see that parameter C_r decreases with an increase in cross-sectional area *A*. The size of the slope/decrease is comparable for all the aluminium alloy analysed.

Tab. 2. Characteristic values received for model MFSL for the aluminium alloy s analysed

Aluminum alloy	A_T	$C_{r\infty}$
D21-T6	0,427	1,89
EN AW-7075 (AlZnMg)	0,012	1,94
24 S-T	0,028	1,87
75 S-T6	0,116	1,96

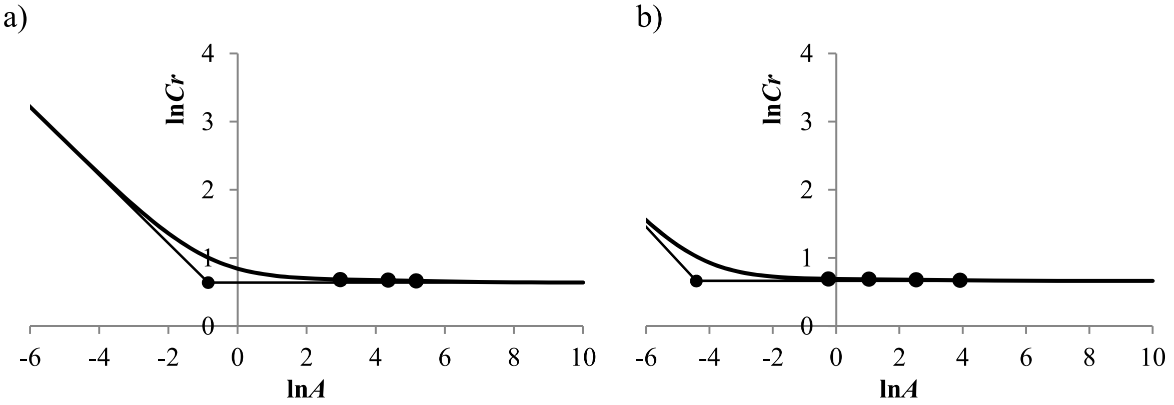

Fig. 7. MFSL model curve for characteristic T and experimental points for the select aluminium alloys analysed: a) D21-T6, b) EN AW-7075 (AlZnMg)

5. Conclusion

A decrease in the strength properties of the construction material as a result of a change in the cross-sectional area can be described with fractal approaches. The decrease/slope gradually disappears together with an increase in the cross-sectional area and so in the case of large objects it is indispensable to apply the nonlinear approach. Knowing the fractal approach model parameters for a given group of construction materials, it is possible to evaluate the fatigue life for the objects with a different geometry than the specimens tested in the lab.

The paper analyses the multifractal scaling law for selected aluminium alloys. The approach was applied in the scope of high-cycle fatigue for fatigue characteristics σ_a-N determined experimentally. For each of the materials analysed there were determined mean characteristic quantities of the MFSL model: structure transition value $A_{Tśr} = 0.15$, infinite fatigue strength $C_{r\infty\ \acute{s}r} = 1.92$. Convergent values A_T and $C_{r\infty}$ for the materials analysed show a similar susceptibility to a change in the cross-section and a similar structure of the materials analysed.

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