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Propagation properties of anomalous hollow vortex beam in uniaxial crystal orthogonal to the optical axis

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The analytical equation of an anomalous hollow vortex beam propagating in a uniaxial crystal orthogonal to the optical axis is derived, and the intensity and phase properties of this beam propagating in the uniaxial crystal are illustrated using numerical examples. The influences of topological charge M and the ratio of refractive index n_e/n_o of a uniaxial crystal on the normalized intensity distribution and phase distribution for anomalous hollow vortex beam are discussed and analyzed in detail.

Keywords: uniaxial crystal, anomalous hollow vortex beam, laser propagation.

1. Introduction

Recently, the propagation properties of various laser beams in a uniaxial crystal have been widely studied because the uniaxial crystal has the application in the design of wave plates and optical devices [1]. Until now, based on the propagation theory of laser beams in a uniaxial crystal [2–4], the evolution properties of various laser beams propagating in a uniaxial crystal have been widely illustrated, such as those of Laguerre–Gauss and Bessel–Gauss beams [5], dark hollow beam [6], anomalous hollow beam [7], Hermite -cosine-Gaussian beam [8], partially polarized partially coherent beam [9], partially coherent flat-topped beam [10], Lorentz and Lorentz–Gauss beams [11], partially coherent Lorentz beam [12], twisted Gaussian Schell-model beam [13], Airy beam [14], partially coherent anomalous hollow beam [15], elliptical Gaussian vortex beam [16], four-petal Gaussian vortex beam [17], Laguerre–Gaussian correlated Schell-model beam [18], partially coherent four-petal Gaussian vortex beams [19], flat-topped vortex hollow beam [20], partially coherent flat-topped vortex hollow beam [21], multi-Gaussian Schell model beam [22] and rectangular multi-Gaussian Schell model beam [23].

On the other hand, new laser beams have been introduced to describe the optical field of a laser output. Among the laser beams, a new dark hollow beam called an anomalous hollow beam has been produced [24]. And the properties of the anomalous hollow beam propagating in the uniaxial crystal, turbulent atmosphere, optical system and free space have been widely studied [7, 25–27]. Based on the model of the anomalous hollow beam, the anomalous hollow beam with an orbital angular momentum has also been introduced and studied [28]. Until now, there have been no reports about the anomalous hollow vortex beam propagating in the uniaxial crystal orthogonal to the optical axis. In this paper, our aim is to study the intensity and phase properties of the anomalous hollow vortex beam propagating in the uniaxial crystal orthogonal to the optical axis.

2. Propagation theory

In the Cartesian coordinate system, the z-axis is set to be the propagation axis, and the electric field of an anomalous hollow vortex beam at the source plane $z = 0$ can be expressed as [28]

$$E(\mathbf{r}_0, 0) = \left(-2 + \frac{8x_0^2}{w_{0x}^2} + \frac{8y_0^2}{w_{0y}^2}\right)\exp\left(-\frac{x_0^2}{w_{0x}^2} - \frac{y_0^2}{w_{0y}^2}\right)(x_0 + iy_0)^M \tag{1}$$

where $\mathbf{r}_0 = (x_0, y_0)$ denotes the position vector at the source plane $z = 0$; M is the topological charge; w_{0x} and w_{0y} are the beam waist widths of an astigmatic Gaussian beam in x and y directions, respectively. When $w_{0x} = w_{0y}$, Eq. (1) reduces to a circular anomalous hollow vortex beam.

Recalling the following formula [29]

$$(x + iy)^M = \sum_{l=0}^{M} \frac{M!\,i^l}{l!(M-l)!} x^{M-l} y^l \tag{2}$$

the electric field of an anomalous hollow vortex beam can be written as

$$E(\mathbf{r}_0, 0) = \sum_{l=0}^{M} \frac{M!\,i^l}{l!(M-l)!} x_0^{M-l} y_0^l \left(-2 + \frac{8x_0^2}{w_{0x}^2} + \frac{8y_0^2}{w_{0y}^2}\right)\exp\left(-\frac{x_0^2}{w_{0x}^2} - \frac{y_0^2}{w_{0y}^2}\right) \tag{3}$$

Under paraxial approximation, we assume that the optical axis of the uniaxial crystal coincides with the x-axis of the Cartesian coordinate system, and the beam propagates along the z-axis, thus the electric field of the beam propagating in the uniaxial crystal orthogonal to the optical axis can be expressed as:

$$E_x(\mathbf{r}, z) = \exp(ikn_\mathrm{e}z)\frac{kn_\mathrm{o}}{2\pi iz} \times \int \mathrm{d}x_0\mathrm{d}y_0 \exp\left\{-\frac{k}{2izn_\mathrm{e}}\left[n_\mathrm{o}^2(x-x_0)^2 + n_\mathrm{e}^2(y-y_0)^2\right]\right\}E_x(\mathbf{r}_0, 0) \tag{4a}$$

$$E_y(\mathbf{r}, z) = \exp(ikn_\mathrm{o}z)\frac{kn_\mathrm{o}}{2\pi iz} \times \int \mathrm{d}x_0\mathrm{d}y_0 \exp\left\{-\frac{kn_\mathrm{o}}{2iz}\left[(x-x_0)^2 + (y-y_0)^2\right]\right\}E_y(\mathbf{r}_0, 0) \tag{4b}$$

where $k = 2\pi/\lambda$ is the wavenumber with λ denoting the wavelength; $\mathbf{r} = (x, y)$ is the position vectors at the observation plane. From Eq. (4a), it can be seen that the only x component of the beam propagating in uniaxial crystals undergoes diffraction spreading asymmetry in the x-y plane due to the influences of the uniaxial crystal. Therefore, only the x-polarized anomalous hollow vortex beam propagating in uniaxial crystals orthogonal to the optical axis is analyzed in this work.

Substituting Eq. (3) into Eq. (4a), and recalling the following formulae [29]:

$$\int_{-\infty}^{+\infty} x^n \exp(-px^2 + 2qx)\mathrm{d}x = n!\exp\left(\frac{q^2}{p}\right)\left(\frac{q}{p}\right)^n\sqrt{\frac{\pi}{p}}\sum_{k=0}^{[n/2]}\frac{1}{k!(n-2k)!}\left(\frac{p}{4q^2}\right)^k \tag{5}$$

$$H_n(x) = \sum_{k=0}^{[n/2]}\frac{(-1)^k n!}{k!(n-2k)!}(2x)^{n-2k} \tag{6}$$

The analytical propagation equation of the anomalous hollow vortex beam propagating in the uniaxial crystal orthogonal to the optical axis can be derived as

$$E(x, y, z) = \frac{kn_\mathrm{o}}{2\pi iz}\exp(ikn_\mathrm{e}z)\exp\left(-\frac{kn_\mathrm{o}^2}{2izn_\mathrm{e}}x^2 - \frac{kn_\mathrm{e}}{2iz}y^2\right) \times \sum_{l=0}^{M}\frac{M!i^l}{l!(M-l)!}\left(-2I_1 + \frac{8}{w_{0x}^2}I_2 + \frac{8}{w_{0y}^2}I_3\right) \tag{7}$$

where

$$I_1 = 2^{-M}i^M\sqrt{\frac{\pi}{a_x}}\sqrt{\frac{\pi}{a_y}}\left(\frac{1}{\sqrt{a_x}}\right)^{M-l}\left(\frac{1}{\sqrt{a_y}}\right)^l\exp\left(\frac{c_x^2}{a_x}\right)\exp\left(\frac{c_y^2}{a_y}\right) \times H_{M-l}\left(-\frac{ic_x}{\sqrt{a_x}}\right)H_l\left(-\frac{ic_y}{\sqrt{a_y}}\right) \tag{8}$$

$$I_2 = 2^{-M-2} i^{M+2} \sqrt{\frac{\pi}{a_x}} \sqrt{\frac{\pi}{a_y}} \left(\frac{1}{\sqrt{a_x}}\right)^{M-l+2} \left(\frac{1}{\sqrt{a_y}}\right)^{l} \exp\left(\frac{c_x^2}{a_x}\right) \exp\left(\frac{c_y^2}{a_y}\right)$$
$$\times H_{M-l+2}\left(-\frac{ic_x}{\sqrt{a_x}}\right) H_l\left(-\frac{ic_y}{\sqrt{a_y}}\right) \tag{9}$$

$$I_3 = 2^{-M-2} i^{M+2} \sqrt{\frac{\pi}{a_x}} \sqrt{\frac{\pi}{a_y}} \left(\frac{1}{\sqrt{a_x}}\right)^{M-l} \left(\frac{1}{\sqrt{a_y}}\right)^{l+2} \exp\left(\frac{c_x^2}{a_x}\right) \exp\left(\frac{c_y^2}{a_y}\right)$$
$$\times H_{M-l}\left(-\frac{ic_x}{\sqrt{a_x}}\right) H_{l+2}\left(-\frac{ic_y}{\sqrt{a_y}}\right) \tag{10}$$

with

$$a_x = \frac{1}{w_{0x}^2} + \frac{kn_\mathrm{o}^2}{2izn_\mathrm{e}} \tag{11a}$$

$$c_x = \frac{kn_\mathrm{o}^2}{2izn_\mathrm{e}} x \tag{11b}$$

$$a_y = \frac{1}{w_{0y}^2} + \frac{kn_\mathrm{e}}{2iz} \tag{12a}$$

$$c_y = \frac{kn_\mathrm{e}}{2iz} y \tag{12b}$$

The intensity and phase distributions of the anomalous hollow vortex beam propagating in the uniaxial crystal orthogonal to the optical axis can be illustrated and analyzed using the derived Eqs. (7)–(12).

3. Numerical examples and analysis

In this section, the intensity and phase properties of the anomalous hollow vortex beam propagating in the uniaxial crystal orthogonal to the optical axis are illustrated and analyzed using numerical examples. Unless specified otherwise in captions of figures, the parameters of the uniaxial crystal and the anomalous hollow vortex beam are chosen as $n_\mathrm{o} = 2.616$, $\lambda = 1.064$ μm, $w_{0x} = 10$ μm, $M = 2$ and Rayleigh length $z_\mathrm{R} = \pi w_{0x}^2 n_\mathrm{o}/\lambda$.

The contour graphs of the normalized intensity of the anomalous hollow vortex beam propagating in the uniaxial crystal orthogonal to the optical axis for different $n_\mathrm{e}/n_\mathrm{o}$ are illustrated in Figs. 1 and 2. It can be found that the anomalous hollow vortex beam keeps its initial central dark centre as the propagation distance increases, and the width

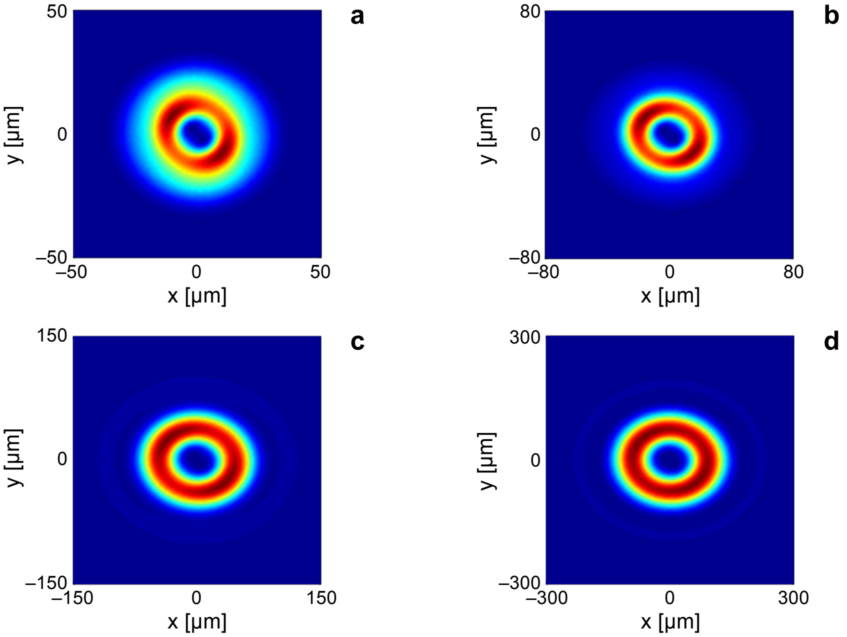

Fig. 1. The contour graphs of the normalized intensity of anomalous hollow vortex beam in uniaxial crystal with $n_e/n_o = 1.1$, $w_{0y} = 10$ µm, and $z = z_R$ (**a**), $z = 2z_R$ (**b**), $z = 5z_R$ (**c**), $z = 10z_R$ (**d**).

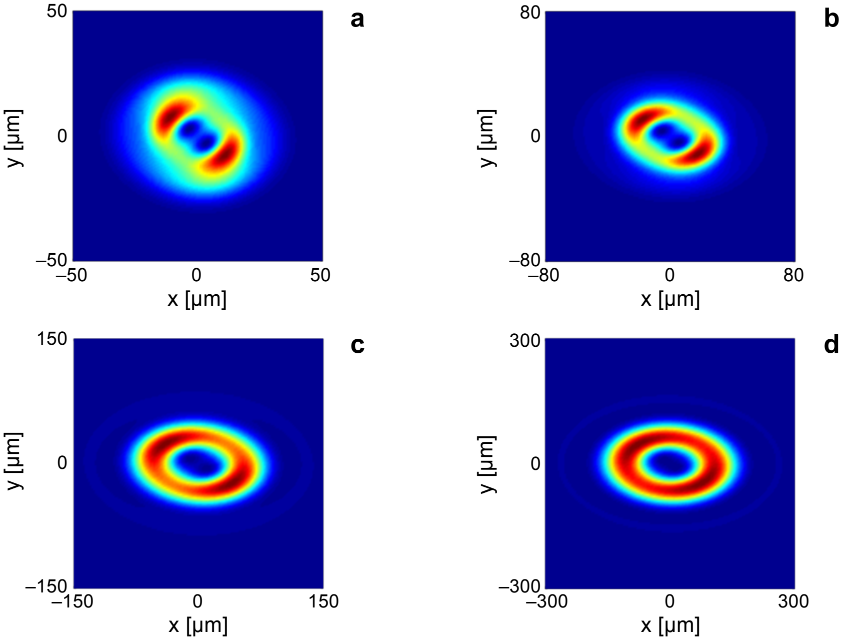

Fig. 2. The contour graphs of the normalized intensity of anomalous hollow vortex beam in uniaxial crystal with $n_e/n_o = 1.3$, $w_{0y} = 10$ µm, and $z = z_R$ (**a**), $z = 2z_R$ (**b**), $z = 5z_R$ (**c**), $z = 10z_R$ (**d**).

of beam will spread rapidly as the propagation distance increases. The beam propagating in the uniaxial crystal with smaller n_e/n_o (Fig. 1) can almost keep its circular symmetry, and the beam propagating in the uniaxial crystal with larger n_e/n_o (Fig. 2) will evolve into an elliptical symmetry beam with a central dark centre. In Fig. 3, the propagation of the anomalous hollow vortex beam with elliptical symmetry in the uni-

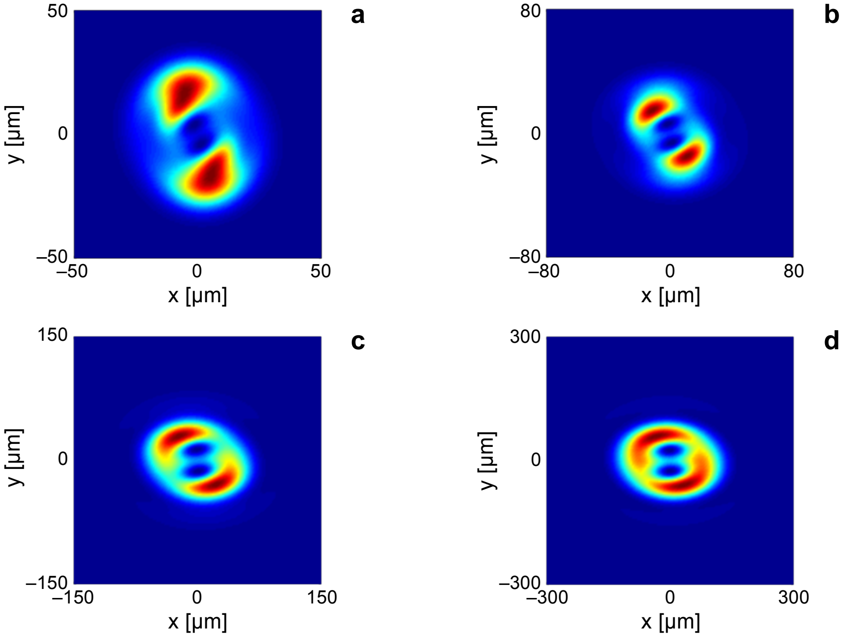

Fig. 3. The contour graphs of the normalized intensity of anomalous hollow vortex beam in uniaxial crystal with $n_e/n_o = 1.1$, $w_{0y} = 13$ µm, and $z = z_R$ (**a**), $z = 2z_R$ (**b**), $z = 5z_R$ (**c**), $z = 10z_R$ (**d**).

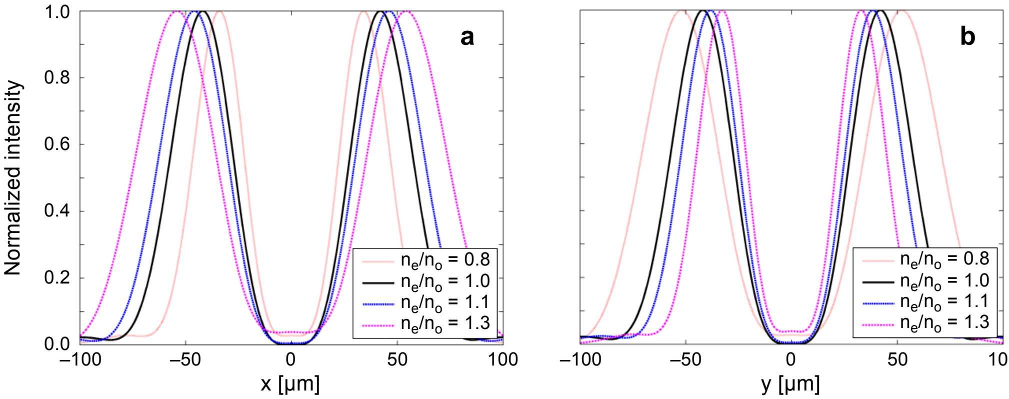

Fig. 4. Cross-section of anomalous hollow vortex beam in uniaxial crystal for different n_e/n_o at the propagation distance $z = 5z_R$ along x axis (**a**), and along y axis (**b**).

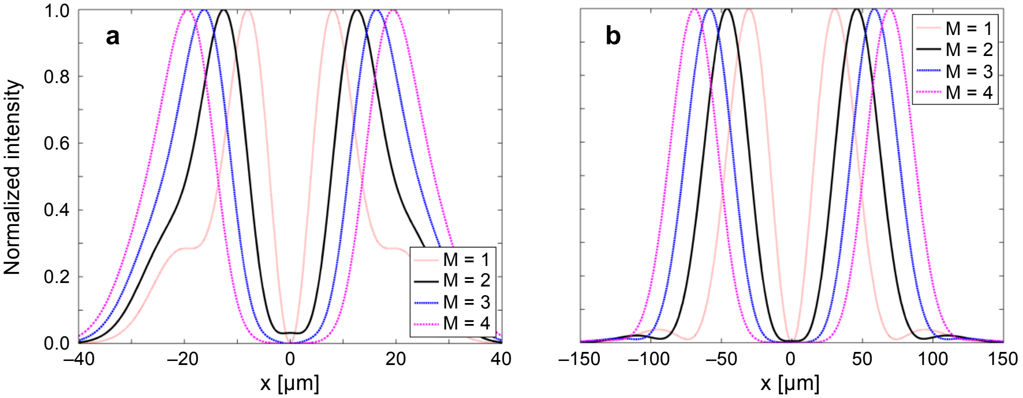

Fig. 5. Cross-section ($y = 0$) of anomalous hollow vortex beam in uniaxial crystal with $n_e/n_o = 1.1$ for different M; $z = z_R$ (**a**), and $z = 5z_R$ (**b**).

axial crystal with $n_e/n_o = 1.1$ is given. It can be found that the anomalous hollow vortex beam with elliptical symmetry propagating in the uniaxial crystal with smaller n_e/n_o will keep its initial spot pattern and spread as the propagation distance increases.

Figures 4 and 5 show the cross-section of the anomalous hollow vortex beam propagating in the uniaxial crystal for different n_e/n_o and M, respectively. As can be seen,

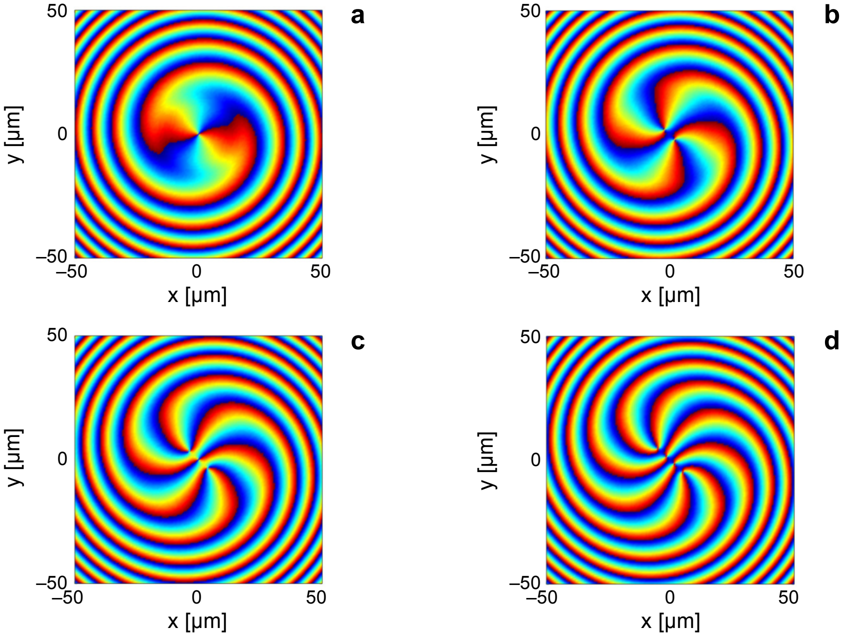

Fig. 6. The phase of anomalous hollow vortex beam propagating in uniaxial crystal with $n_e/n_o = 1.1$ at the propagation distance $z = z_R$ for $M = 1$ (**a**), $M = 2$ (**b**), $M = 3$ (**c**), and $M = 4$ (**d**).

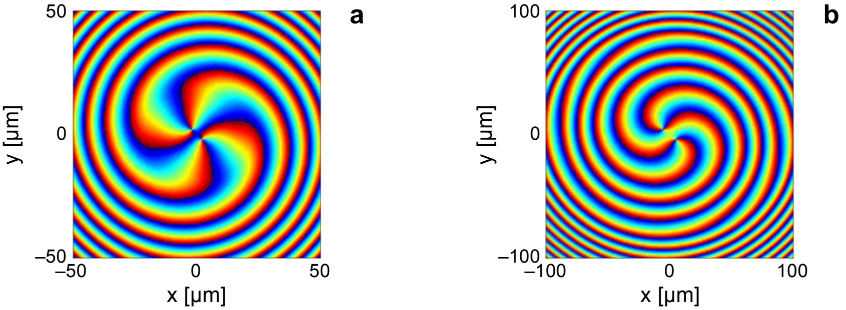

Fig. 7. The phase of anomalous hollow vortex beam propagating in uniaxial crystal with $n_e/n_o = 1.1$; $z = z_R$ (**a**), and $z = 5z_R$ (**b**).

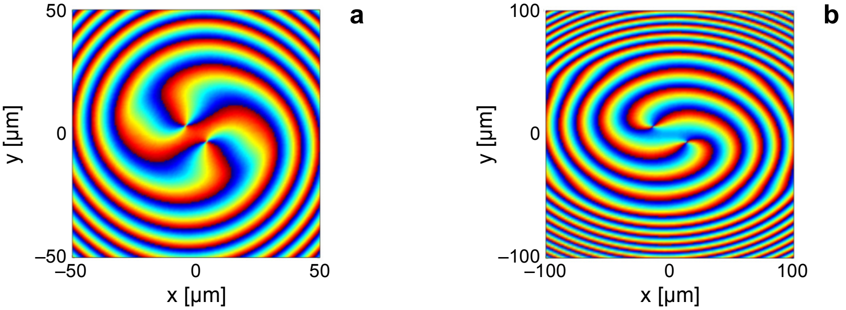

Fig. 8. The phase of anomalous hollow vortex beam propagating in uniaxial crystal with $n_e/n_o = 1.3$; $z = z_R$ (**a**), and $z = 5z_R$ (**b**).

the beam propagating in the uniaxial crystal will spread faster along the x axis as n_e/n_o increases, and spread slower along the y axis as n_e/n_o increases. The reason for this is that the beam propagation in the uniaxial crystal along the x axis and the y axis has different Rayleigh length, and the Rayleigh length along the x axis $\pi w_{0x}^2 n_o/\lambda$ is smaller than the Rayleigh length along the y axis $\pi w_{0x}^2 n_e/\lambda$ as n_e/n_o increases. And the beam with larger M will have a larger central dark centre.

The phases of the anomalous hollow vortex beam propagating in the uniaxial crystal for different M and n_e/n_o are given in Figs. 6–8. It is found that the beam with different topological charge M will keep its number of coherent vortex unchanged, while the partially coherent vortex beam will have the changed number of coherent vortices [21] as the propagation distance increases. When $M = 2$, the phase distributions of the anomalous hollow vortex beam propagating in the uniaxial crystal with smaller n_e/n_o have the double clock wise spiral distribution (Fig. 7); and as n_e/n_o of the uniaxial crystal increases, the phase distributions become irregular with the propagation distance z increasing, and the phase distributions become more irregular in the x direction than the y direction due to the $n_e/n_o > 1$ of the uniaxial crystal, which is caused by the different Rayleigh lengths along the x axis and y axis.

4. Conclusions

In this paper, the analytical equation of an anomalous hollow vortex beam propagating in the uniaxial crystal orthogonal to the optical axis is derived, and the propagation properties of the beam are analyzed using numerical examples. The results show that the beam propagation in the uniaxial crystal will keep its initial central dark centre and spread as the propagation distance increases. The beam propagating in the uniaxial crystal will spread faster along the x axis as n_e/n_o increases, and spread slower along the y axis as n_e/n_o increases. The phase distributions of the beam propagating in the uniaxial crystal with larger n_e/n_o are more irregular in the x direction than y direction due to the difference in Rayleigh lengths along the x direction and the y direction.

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