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Fuzzy decision making under transition from silence to confusion*

by

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Abstract: We often fall into silence. After that, we make a decision through confusion, in most cases. In silence, cool mind runs parallel with warm heart. Traversing over these two, the phenomenon arises, which can be interpreted as a fuzzy event, which can be called waver. In other words, two states of nature develop into conflict, and are covered by a fuzzy event. In confusion, we consider that the states of nature, which had been moving in conflict, not only undergo an inversion, but also a transformation takes place from warm heart into dry mind. It is therefore possible to derive a fuzzy function, resulting from the fuzzification of the transition matrix from silence to confusion, absorbing noise, and taking expectation to link the membership function with the multi-attribute utility function. This short note shows that we can calculate the expected utility by using both the probability of a fuzzy event and the subjective importance of the two states of nature for the decision maker. Further, we can obtain an optimum action, based on the theory of maximum expected utility.

Keywords: fuzzy event, expected utility, decision making, silence, confusion

1. Introduction

In terms of a constructive approach to decision making in a fuzzy environment, first of all, Tanaka, Okuda and Asai (1976) applied the notion of probability of a fuzzy event to the statistical decision making, based on the utility function theory. Uemura (1991) and Uemura and Sakawa (1993) considered a fuzzy utility function in a way as if a utility function were mapped onto a membership function in a fuzzy event, and constructed a decision making procedure, based on fuzzy utility function. Further, Uemura (1995) applied a normal possibility theory, which had been constructed by Tanaka and Iwabuchi (1992), to decision making in a fuzzy event, and developed a normal possibility rule. In these

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here mentioned theories, related to decision making, with emphasis on those constructed by Uemura, though, we do not dispose of good examples for the single-dimensional membership functions. By expanding the respective functions into two dimensions, though, we can derive the decision making model in silence, if we regard a fuzzy event as representing a waver, associated with the traversing over two conflicting states of nature (Uemura, 2001). In the present writing, we rather consider the process, in which silence turns into confusion. In other words, we observe the transition from "cool head and warm heart" to "dry mind and cool head". This kind of transition is described through a probability model with transition probabilities between the respective states. Considering the transition from "warm heart" to "dry mind", the transition matrix should be adequately fuzzyfied. In addition, we can also take into account the fact that the transition occurs with some noise. In this context, we assume that this noise is absorbed by the fuzzyfied transition matrix. Thus, altogether, we analyse the decision making model as shown highly schematically in Fig.1.

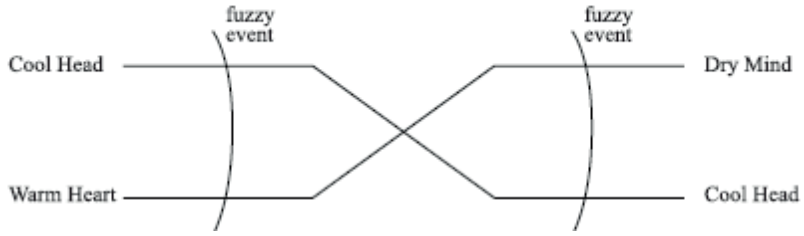

Figure 1. Transition from silence to confusion, that is: from "cool head and warm heart" to "dry mind and cool head"

2. Transition from silence to confusion

We treat the here considered states of nature, (s_1, s_2), in the respective two dimensions. Concerning the transition in time, we speak of silence, with the existence of conflict, and the inversion, leading to confusion. The transition is expressed through:

$$(S_{2t}, S_{1t}) = \begin{bmatrix} M_{11}M_{12} \\ M_{21}M_{22} \end{bmatrix} \begin{pmatrix} S_{1\,t-1} \\ S_{2\,t-1} \end{pmatrix}$$

Since what we focus here on is the transformation from warm heart into dry mind, the noise of the transition can be absorbed by applying fuzzification of the transition matrix, appearing in the above formula. The fuzzyfied transition matrix is illustrated in Fig. 2.

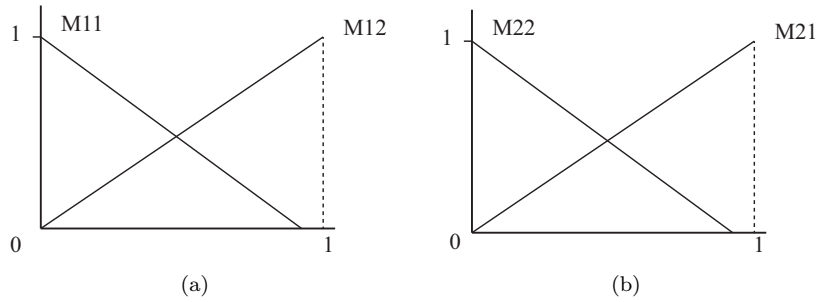

Figure 2. An illustration for the fuzzyfied transition matrix of the process

3. Decision making in confusion

In order to introduce the model that we consider, let us specify the following notations, for the magnitudes, which are specified by the decision maker (see Fig. 3 for the illustration):

- value of the multi-attribute utility function in the previous time instant: $U_{Dk}(S_{1\ t-1}, S_{2\ t-2})$
- membership function in a fuzzy event: $\mu_F(S_{1\ t-1}, S_{2\ t-1})$
- a priori probability: $\pi(S_{1\ t-1}, S_{2\ t-1})$.

At this point, a utility function for the present instant, is given by taking the respective mean, as shown in the formula below:

$$U_{Dkijt} = \int_0^1 \int_0^1 \mu_{M_{ij}}(S_{1\,t-1}, S_{2\,t-1}) U_{Dk}(S_{1\,t-1}, S_{2\,t-1}) dS_{1t-1} dS_{2\,t-1}.$$

Then, the definition of probability of a fuzzy event is given through the following formula:

$$P_{iFt} = \int_0^1 \mu M_{ij}(S_{1t-1}, S_{2t-1}) \int_0^1 \mu_F(S_{1\,t-1}, S_{2\,t-1}) \pi(S_{1\,t-1}, S_{2\,t-1}) dS_{1\,t-1} dS_{2t-1}.$$

Then, from the following equation, we can calculate the expected utility with respect to each of the states of nature:

$$E_{jt}[Dk] = \sum_{i=1}^{2} Pi_{Ft} U_{Dkijt}$$

If we suppose that the decision maker assigned the importance equal a to one of the states of nature, and equal b to the other, then we can calculate the expected utility value, taking into account each of the actions possible at the instance of making of the decision. The optimum action is the one, which maximizes the expected utility value:

$$EE_t[D_k] = aE_{1t}[D_k] + bE_{2t}[D_k]$$

$$D_{t*} \triangleq max_k EE_t[D_k].$$

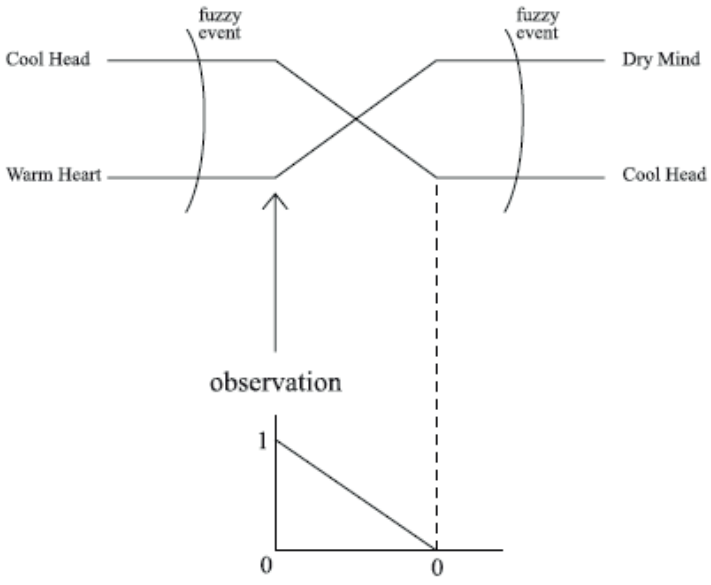

Figure 3. An illustration for the change of assessments in the transition process

4. Conclusion

In this short note, we have constructed a model for decision making under transition from silence to confusion. The process in question can be interpreted as the transition from "cool head and warm heart" to "dry mind and cool head". Given the nature of this process, we can expect a certain level of noise to be involved in it. This noise can be absorbed by the appropriate fuzzyfication of the transition matrix. The model allows for calculation of the expected utility function value at present time by taking the expectation between the membership function of the transition and the multi-attribute utility function. Finally, we are capable of deriving the expected utility from the probability of the fuzzy event and the value of the utility function. On the top of this, if the decision maker assigns importance values to the states of nature, we can also calculate the respective value of the utility function. The resulting decision rule maximizes the expected value of the utility, based on the degrees of importance. An extension to this study might be oriented at decision making under transition from silence to confusion given that we dispose of some observations.

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