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# Compromise of Two-criteria Final Payoff of the Game Ship Control in Collision Situations

J. Lisowski
*Gdynia Maritime University, Gdynia, Poland*

ABSTRACT: The essence of the article is the use of multi-criteria static optimization of object motion, based on a set of optimal Pareto points in the space of possible variants of solutions for a new approach to the problem as a game control. Using the example of the two-criteria optimization of the final payoff of the object game control during the safe evasion of the encountered objects, six methods of multi-criteria static optimization are presented—Bentham's utilitarian rule, Rawls's principle of justice, Salukvadze's benchmark, Benson's weighted sums, Haimes's constraints, and goal-oriented programming. In the end, the results obtained by the two-criteria optimization are compared with regard to the values of the components of the final game payoff—the risk of collision and the deviation of the object from the safe route of the set trajectory of movement. The directions for the development of multi-criteria optimization methods, both static and dynamic, and the game are indicated.

## 1 INTRODUCTION

The latest reports from Ding et al. [10], Ehrgott et al. [11, 14], and Odu et al. [40] on the use of multi-criteria optimization show that they occur in various areas of application: in medicine, agriculture, product and production design, financing, and the design of moving objects (vessels, cars, aircraft, and drones), wherever you need to make optimal decisions in the face of compromises between two or more conflicting goals.

Thus, Balraj [2] presents the multi-criteria optimization of a rotary electrical discharge machining process. Cotton et al. [7] describe multi-criteria optimization for mapping programs to multi-processors. The use of multi-criteria optimization methods in radiation therapy planning is proposed by Craft [8]. Glavac et al. propose [16] the multi-criteria optimization of a car structure using a finite-element method. Another interesting application of multi-criteria optimization in humanitarian aid is proposed by Gutjahr et al. [17]. Hirsch et al. [19] present a multi-criteria optimization approach to the design and operation of a district heating supply system over its life cycle. The application of multi-criteria analysis methods for the determination of priorities in the implementation of irrigation plans are described by Karleusa et al. [21]. Maniowski [34] proposes the multi-criteria optimization of chassis parameters of Nissan 200 SX for drifting competitions. Multi-criteria optimization and its application to earthwork processes is presented by Paulovicova [41]. Sheikus et al. [48] describe the static optimization of rectification processes using mobile control actions. A multi-criteria optimization technique for SSSC-based power oscillation dumping controller design is proposed by Swain et al. [53]. Tahvili [55] presents the multi-criteria optimization of system integration testing. Roy [47] describes a multi-criteria supporting decision.

Analyses included in the literature [1, 8, 13, 38, 42, 43, 47, 52] show that multi-criteria optimization plays an important practical role, e.g., maximizing profit while minimizing production costs, maximizing efficiency while reducing the fuel consumption of a moving object, or reducing the weight of a device while increasing the strength of its individual components.

At the end of the eighteenth century, the English philosopher and economist Jeremy Bentham [3] formulated the following utilitarian principle: "The greatest utility for the largest number of criteria". He promoted the principle of utility as a standard for proper action on the part of governments and individuals. Actions are acceptable when they are aimed at promoting happiness or pleasure, and rejected when they tend to cause unhappiness or pain. By combining these criteria, we are actively trying to promote overall happiness.

The literature [9, 13, 23] shows that when implementing these requirements, very often there are contradictions, i.e., in a given space of decision variables, individual criteria cannot simultaneously achieve their extreme values, and their participation is measured by a weight factor.

Among all technical objects, moving objects constitute a significant amount, for which the method of controlling their movement significantly affects both the operating costs and the accuracy and safety of the transport tasks. This applies to land, sea, and air objects in terms of manned and unmanned facilities. Remote sensing devices, such as radar, lidar, and other highly specialized measurement solutions are used to identify detection processes and control moving objects. When planning and implementing the motion control of objects, there are many possible acceptable solutions, from which the best or optimal solution should be selected.

Different static and dynamic optimization methods can be used to find the optimal solution. Lazarowska [27] presents a multi-criteria trajectory base path planning algorithm for a moving object in a dynamic environment as an intelligent control system.

Summarizing the literature review, in the scientific studies conducted so far, the encountered moving objects have been treated as a limitation of the process state, and not as control objects [22, 25, 26, 28, 32, 33, 39, 51, 54, 57]. There is the possibility that active control of the encountered objects leads to cooperative or non-cooperative game control. In the work of [31], a game control method comparison was made when avoiding collisions with multiple objects using radar remote sensing, ensuring the lowest final payoff of the positional or matrix game, only in the form of the amount of deviation of the safe route of the cruise from the set trajectory.

The purpose of this article is to extend the scientific analysis of the single-criteria optimization of the final payoff to the two-criteria optimization of the final payoff, consisting of both the final deviation from the set trajectory and the final risk of collision. In order to determine a compromise between these two criteria, a comparative analysis of the six most frequently used methods of multi-criteria static optimization was performed.

The article presents an original scientific topic, previously unpublished, assigned to the scientific discipline - automation, electronics and electrical engineering, concerning the synthesis of systems for safe and optimal ship traffic control with the use of artificial intelligence methods and game theory.

## 2 MULTI-CRITERIA STATIC OPTIMIZATION TASK

Multi-criteria optimization is the most natural method of inference, consisting of determining the optimal solution and its acceptability from the point of view of the adopted criteria. When implementing these criteria, there are most often contradictions that, in a given space of decision variables, individual criteria cannot reach their extreme values at the same time. Then, there is a need to find a compromise solution.

### 2.1 *Control Quality Index*

The synthesis of the optimal control of moving objects is most often carried out as convex optimization, a special class of mathematical optimization problems that covers least squares and linear programming problems, and can be solved numerically very efficiently. According to Boyd and Vandenberghe [5], a mathematical optimization problem has the following form:

$$\text{minimize } F(x) \tag{1}$$

$$\text{subject to } g_s(x) \le 0,\ s = 1, 2, \ldots, S \tag{2}$$

$$h_w(x) = 0;\ w = 1, 2, \ldots, W \tag{3}$$

to describe the problem of finding an $x$ that minimizes $F(x)$ among all $x$ that satisfy conditions (2) and (3).

We call $x \in R^n$ the optimization variable, and function $F: R^n \rightarrow R$ is the objective function or cost function. The inequalities $g_s(x) \le 0$ are called inequality constraints, and the corresponding functions $g_s: R^n \rightarrow R$ are called the inequality constraint functions. The equations $h_w(x)=0$ are called the equality constraints, and the functions $h_w$: $R^n \rightarrow R$ are the equality constraint functions. The set of points for which the objective and all of the constraint functions are defined is as follows:

$$D = \bigcap_{s=1}^{S} dom\, g_s \cap \bigcap_{w=1}^{W} dom\, h_w \tag{4}$$

which is called the domain of the optimization problem (1). A point $x \in D$ is feasible if it satisfies the constraints $g_s(x) \le 0$, $s = 1, \cdots, S$ and $h_w(x)=0$, w=1,…, $W$. Problem (1) is said to be feasible if at least one feasible point exists, and is infeasible otherwise. The set of all feasible points is called the feasible set or the constraint set.

The optimal value $F^*$ of problem (1) is defined as follows:

$$F^* = \inf\left\{F(x) \mid g_s(x) \le 0,\ s = 1, \ldots, S,\ \ h_w(x) = 0,\ w = 1, \ldots, W\right\} \tag{5}$$

The task of multi-criteria optimization is to find such a vector of decision variables, $x$, as shown in the following equation:

$$x = [x_1, x_2, ..., x_i, ..., x_N]; \quad i = 1, 2, ..., N \tag{6}$$

which optimizes the vector of the decision objective function $F$ as a control quality index:

$$F(x) = \sum_{c=1}^{C} k_c F_c(x), \; c = 1, 2, ..., C \tag{7}$$

where $k_c$ is the weight factor for the $F_c$ component of the control objective function, taking into account both its percentage share and the physical units of the components themselves.

## 2.2 *Pareto Optimal Front*

The well-known 80/20 rule states that 80% of the results come from only 20% of the causes, in other words, more modest means can be achieved with less effort. The development of this principle was made in 1897 by Italian economist Vilfred Pareto.

The definition of a set of optimal Pareto points in the space of variants can be expressed as follows, "A given variant is Pareto optimal if none of its grades can be corrected without worsening at least one of the others".

According to the considerations of Messac et al. [37], the set of non-dominated solutions from the entire permissible search space is called the optimal set in the Pareto sense, and these solutions form the so-called Pareto front. The solutions from this set are not dominated by any others, so in this sense, they are optimal solutions for the problem of multi-criteria optimization.

Eshenauer et al. [12] show that as the non-Pareto optimal variants can be improved for all criteria, the introduction of the optimal Pareto concept reduced the problem of finding a solution to a task with multiple criteria for selecting a point from this set.

# 3 GAME CONTROL OF SHIP

As an example of a multi-criteria static optimization task, one can consider the process of game control of the ship in situations of passing many encountered objects, which is illustrated in Figure 1.

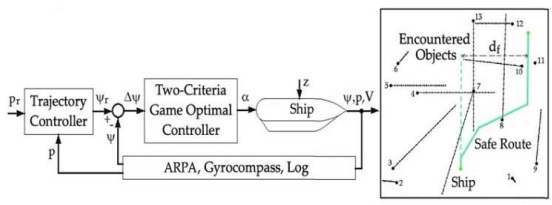


Figure 1. Block diagram of the system for the game control of the ship in collision situations: $p_r$—reference trajectory; $p$—real position of ship; $\psi_r$—reference course; $\alpha$—rudder deflection; $z$—disturbances (wave, wind, and sea current); $\psi$—ship course; $V$—velocity of ship

Ship controlling their movement by means of course and speed changes are characterized by the mutual distance and bearing from the ARPA radar remote sensing system, allowing for determining the risk of collision. The task of two-criteria static optimization of the ship safe trajectory is to look for the minimum final payoff value of the control objective function:

$$F = \sum_{c=1}^{2} F_c = k_1 F_1 + k_2 F_2 = k_1 r_f + k_2 d_f = F_{\min} \tag{8}$$

where $r_f$ (%) is the final value of risk collision; $d_f$ (m) is the final deviation of ship safe determined route from the set trajectory, shown in Figure 1; and $k_1$ (%$^{-1}$) and $k_2$ (m$^{-1}$) are sought compromise values of solutions to the static optimization problem.

According to the author of [31], the ship collision risk is defined as a reference to two assessments of the navigation situation, shown in Figure 2. The first assessment contains the parameters $D_{jmin}$ and $T_{jmin}$ of the real situation of the proximity of the objects. The second assessment concerns the same situation, but the safety is determined by parameters $D_s$, $T_s$, and $D_j$. Thus, this reference has three relative elements, namely, $D_{jmin}/D_s$, $T_{jmin}/T_s$, and $D_j/D_s$, all of which are proposed to express the risk of collision, $r_j$, as the following mean square form of these three relative elements:

$$r_j = \left[ \varepsilon_1 \left( \frac{D_{j\min}}{D_s} \right)^2 + \varepsilon_2 \left( \frac{T_{j\min}}{T_s} \right)^2 + \varepsilon_3 \left( \frac{D_j}{D_s} \right)^2 \right]^{-0.5} \tag{9}$$

where $\varepsilon_1$, $\varepsilon_2$, and $\varepsilon_3$ are the weighting factors, depending on the visibility at sea and the intensity of marine traffic.

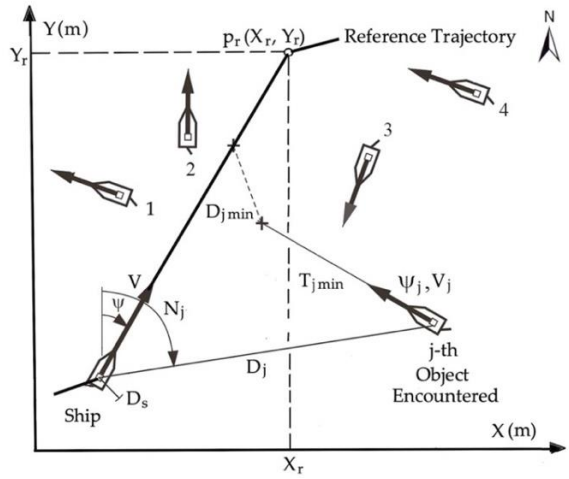


Figure 2. Displaying the situation of passing ship with encountered objects, in particular with the $j$-th object; $\psi$—ship course; $V$—velocity of ship; $\psi_j$—$j$-th object course; $V_j$—velocity of $j$-th object; $X_r$, $Y_r$—ship reference coordinates; $p_r$—reference position of ship; $D_j$ —distance to $j$-th object; $N_j$—bearing to $j$-th object; $D_{jmin}$—distance of the closest point of approach; $T_{jmin}$—time to the closest point of approach; $D_s$—a safe distance of approach

Figure 3 illustrates an example of set of acceptable solutions of the task of safe control of the ship in a

situation of passing a larger number of encountered objects as a task of two-criteria optimization of control due to the risk of collision and deviation from the set motion trajectory. The Pareto-optimal front shape is shown as a set of non-dominated solutions to the game control task due to the final payoff value of the control objective function, which consists of the final value of risk collision and the final deviation of the ship safe determined route from the reference trajectory.

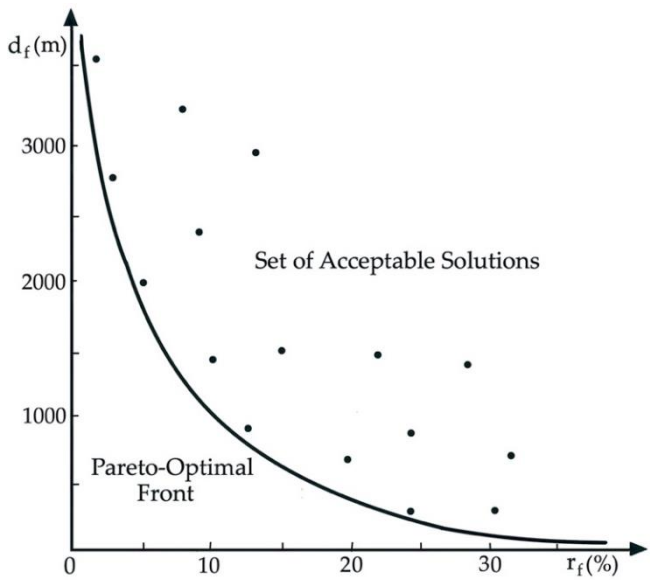


Figure 3. Front Pareto multi-criteria optimization of the ship game controlling while safe passing the encountered objects: $F_1 = r_f$ (%)—value of the final collision risk; $F_2 = d_f$ (m)—value of the final deviation trajectory

## 4 MULTI-CRITERIA STATIC OPTIMIZATION METHODS

Many practitioners [4, 15, 20, 24, 29, 30, 50] have worked for many years to answer the question of whether the optimal Pareto point is the best optimization method.

The methods of solving this task can be distinguished by the following: Bentham's utilitarianism rules (UR), Rawls' principle of justice principle (JP), Salukvadze reference point (RP), Benson weighted sum (WS), Haimes ε-restrictions (εR) and Goal programming (GP).

### 4.1 *Bentham's Utilitarianism Rule UR Method*

The use of the J. Bentham [3] principle allows for accepting the criterion of the sum of the partial $F_c$ criteria (Figure 4).

First, lines with a constant value of the sum of the components of the objective function were drawn. Then these lines were shifted in parallel in the direction of decreasing the value of this sum of components. The last line, tangent to the Pareto-optimal front, marks the point of the UR of the two-criteria optimality of the safe passing of the ship while passing the encountered objects.

The coordinates of this point determine the values of the weighting coefficients $k_1$ and $k_2$ of the components of the final payoff value of the control objective function. The location of the UR point in the $k_1$ and $k_2$ coordinates system allowed to assess the compromise of the quality of safe control of the ship between the risk of collision and deviation from the set cruise route, convertible into the costs of transport operating and the time obligations of the shipowner.

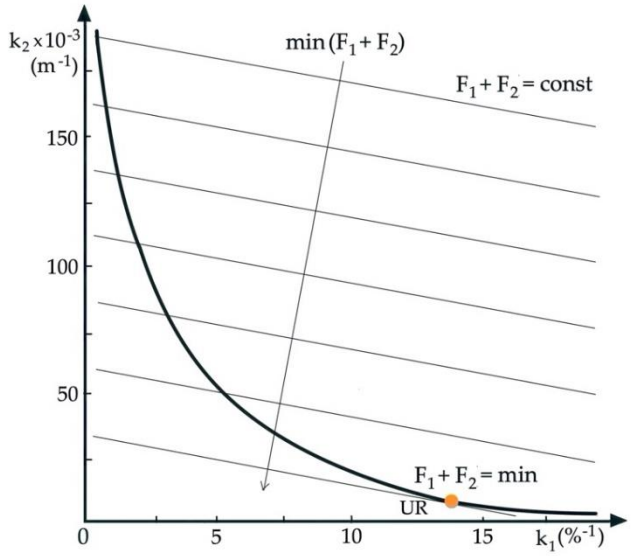


Figure 4. The sum of partial criteria according to J. Bentham, and the optimal solution Bentham's utilitarianism rule (UR) on the Pareto front; weighting values of optimal game final payoff: $k_1$=13.9 %$^{-1}$, $k_2$=0.007 m$^{-1}$

The sum of partial criteria according to J. Bentham, and optimal solution Bentham's utilitarianism rule (UR) on Pareto front; weighting values of optimal game final payoff: $k_1$=13.9 %$^{-1}$, $k_2$=0.007 m$^{-1}$.

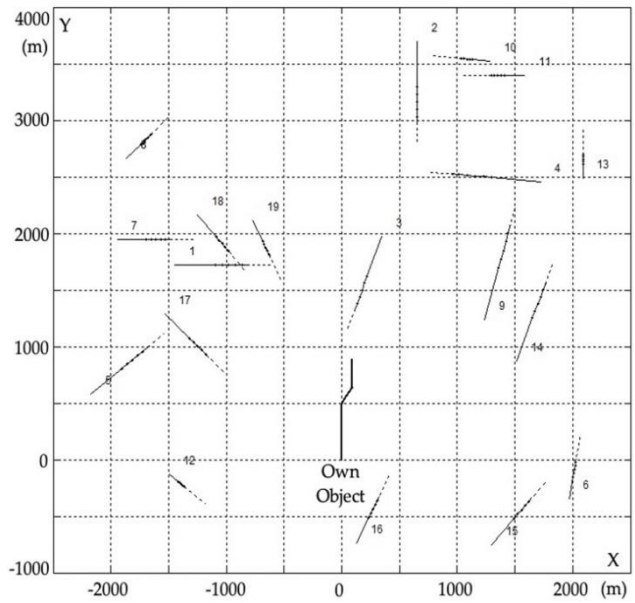


Figure 5. Optimal ship trajectory while safely passing 19 encountered objects for the Bentham's utilitarianism rule UR method multi-criteria optimization of the final payoff value of the control objective function: $r_f$ = 24 %; $d_f$ = 150 m; $F_{\min}^{UR} = 334.65$ —minimum value of objective function (8)

### 4.2 *Rawls' Principle of Justice JP Method*

In 1971, American philosopher John Rawls formulated [44] the following principle of justice, "Least usability, as big as it can". In his theory, Rawls refers to the principle of "Justice as Fairness"—the distribution of goods is justice (just) if it is impartial (fair), i.e., if it offers everyone the same opportunities. Figure 6 presents the principle of max-minimization of J. Rawls.

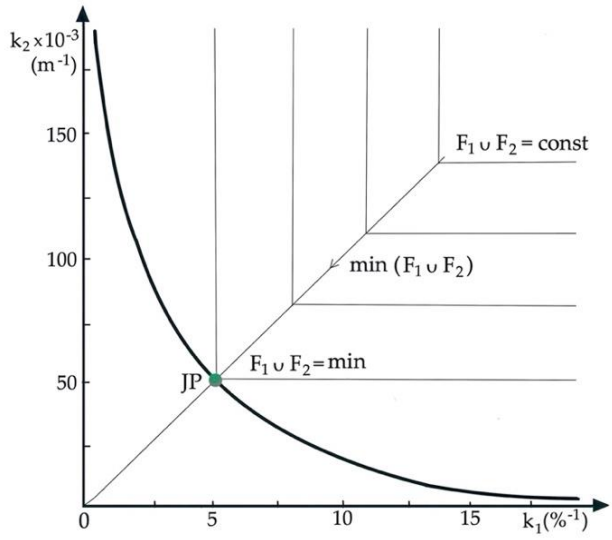


Figure 6. The max-minimization of partial criteria according to J. Rawls, and the optimal solution on the Pareto front; weighting values of optimal game final payoff: $k_1$=5.0 $\%^{-1}$, $k^2$=0.052 $m^{-1}$

The position of the optimal point on the Pareto-optimal front results from the condition of the minimum value of the logical sum of the components of the control objective functions.

Figure 7 shows the results of a computer simulation of the optimal controlling own object while passing 19 encountered objects, using the Rawls' principle of justice JP method on the Pareto front.

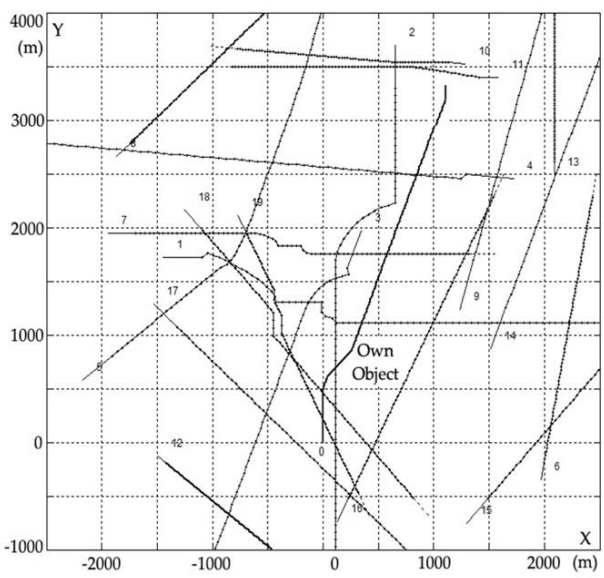


Figure 7. Optimal ship trajectory while safely passing 19 encountered objects for the Bentham's utilitarianism rule JP method multi-criteria optimization of the final payoff value of the control objective function: $r_f$ = 9 %; $d^f$ = 1037 m; $F_{\min}^{JP} = 98.91$ —minimum value of two-criteria objective function (8)

The appointment of a fair solution, according to J. Rawls, is a more difficult issue than finding a solution when using the aggregate criterion for choosing a weighted sum.

### 4.3 *Salukvadze Reference Point RP Method*

In 1971, the Georgian automatist Mindia E. Salukvadze [48] proposed an approach based on the concept of a reference point, namely, "In the Pareto collection, the nearest point in relation to the reference point is sought", presented by Stadler [49]. The Pareto collection looks for the nearest point in relation to the reference point.

Salukvadze proposed the intersection point tangent as a point of reference to the set of acceptable solutions (Figure 8).

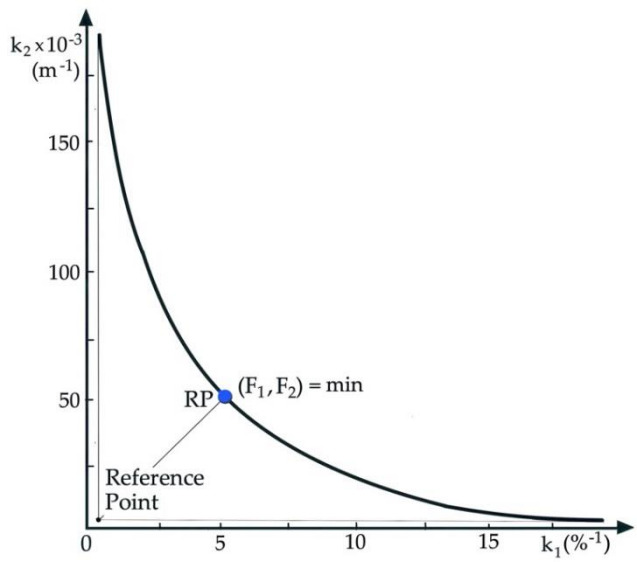


Figure 8. An approach using a reference point, according to M. E. Salukvadze, and an optimal solution on the Pareto front; weighting values of optimal game final payoff: $k_1$=5.1, $k_2$=0.05

The Salukvadze reference point method was developed by Wierzbicki [58, 59], who developed the mathematical foundations of reference point methods based on the conical separation of sets.

Figure 9 shows the results of a computer simulation of the optimal controlling the ship while passing 19 encountered objects, using the Salukvadze reference point RP method on the Pareto front.

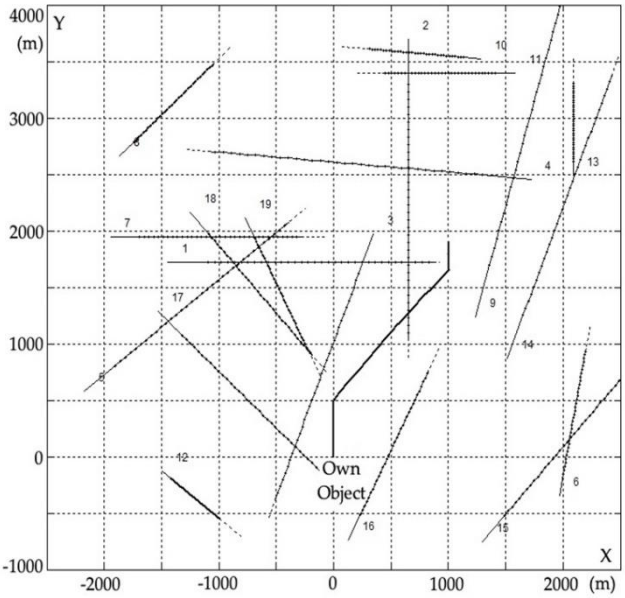


Figure 9. Optimal ship trajectory while safely passing 19 encountered objects for the Salukvadze reference point RP method multi-criteria optimization of the final payoff value of the control objective function: $r_f$ = 12 %; $d_f$ = 1000 m; $F_{\min}^{RP} = 111.21$ —minimum value of the two-criteria objective function (8)

### 4.4 *Benson Weighted Sum WS Method*

In the method of the American computer scientist Harrold Phillip Benson, described in 1988, the metric is used to measure the distance of the tested solution from an ideal solution that meets all of the criteria. In the literature [35, 36, 46], it is shown that minimizing the distance between the ideal solution and the tested solution allows for finding to the best solution belonging to the set of acceptable solutions.

The initial solution is randomly selected from among the acceptable solutions to the problem.

The graphical interpretation of this method is the search for a tangent to the permissible set, inclined at an angle determined by the weight coefficients $w$ ($w_1$ and $w_2$). The vector $W$, formed from the w coefficients, is perpendicular to the tangent sought, and the solution is the common points of the set edge and tangent.

This method, with a correctly drawn starting point, can find the optimal Pareto solutions, even for non-convex decision spaces. One of the disadvantages of this solution is the non-differentiable purpose function. In this case, you cannot use gradient-based methods to solve this problem (Figure 10).

The advantage of this method is the ability to find all Pareto optimal solutions when determining the abstract ideal (abstract) solution. The disadvantage of this method is the need to normalize the objective function, which is not always an easy task. In addition, an ideal solution should be determined, which requires the optimization of each criterion separately. Using this method as the a'priori method, the task of the decision-maker is to provide an ideal point, which, in many cases, can be determined intuitively, based on knowledge of the decision problem.

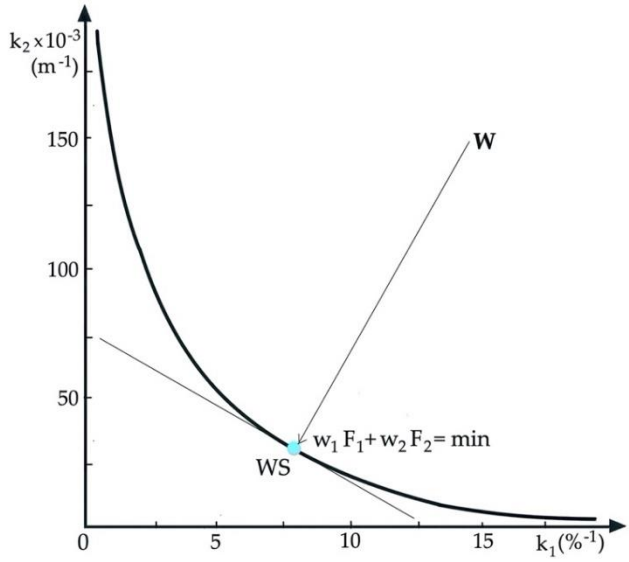


Figure 10. The Benson weighted sum method and the optimal solution on the Pareto front; weighting values of optimal game final payoff: $k_1$=7.9, $k_2$=0.03

Figure 11 shows the results of a computer simulation of the optimal controlling the ship while passing 19 encountered objects, using the Benson weighted sum WS method on the Pareto front.

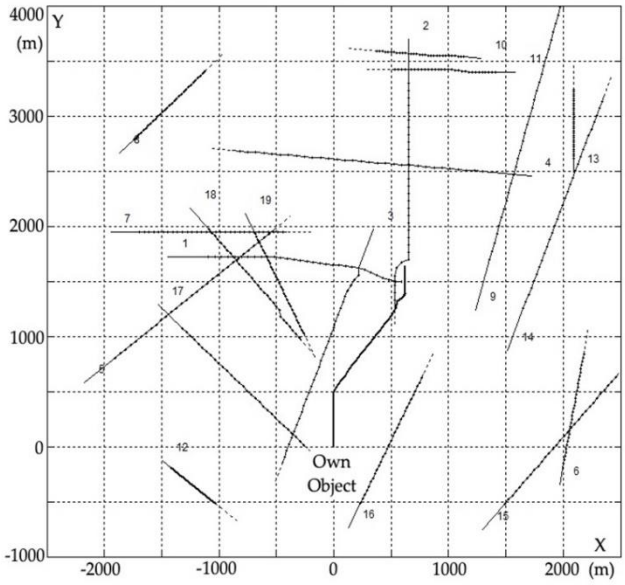


Figure 11. Optimal ship trajectory while safely passing 19 encountered objects for the Benson weighted sum WS method multi-criteria optimization of the final payoff value of the control objective function: $r_f$ = 17 %; $d_f$ = 593 m; $F_{\min}^{WS} = 149.69$ —minimum value of the two-criteria objective function (8)

### 4.5 *Haimes ε-Restrictions εR Method*

The method developed by Yacov Y. Haimes [18] in 1971 consists of selecting one of the criterion functions as a function of the goal and creating constraints from the other criteria functions. The most important criterion to be optimized is selected, assuming that the values of the other criteria meet the minimum assumed requirements (Figure 12).

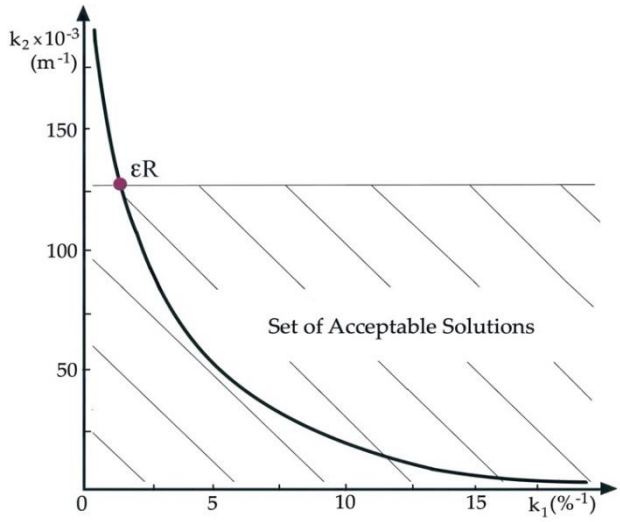


Figure 12. The Himes method of ε-restriction and optimal solution on the Pareto front; weighting values of optimal game final payoff: $k_1$=2.53, $k_2$=0.128

Figure 13 shows the results of a computer simulation of the optimal controlling the ship while passing 19 encountered objects, using the Haimes ε-restrictions εR method on the Pareto front.

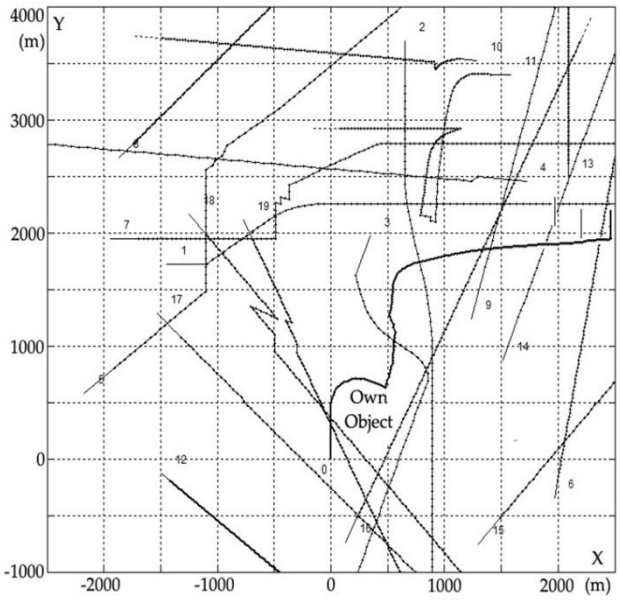


Figure 13. Optimal ship trajectory while safely passing 19 encountered objects for the Haimes ε-restrictions εR method multi-criteria optimization of the final payoff value of the control objective function: $r_f$ = 3 %; $d_f$ = 2481 m; $F_{\min}^{\varepsilon R} = 325.16$ — minimum value of the two-criteria objective function (8)

The advantages of this method are finding different optimal Pareto solutions using different values for the $\varepsilon$ parameter. The main advantage of this approach over the weighted sum method is the ability to find a solution belonging to the set of Pareto-optimal solutions when the problem space is both convex and concave. However, the disadvantage of such a solution is the significant dependence of the result on the selected parameter ε and the original optimization function. In some cases, the wrong choice of parameters for this method may not find any solution or may give the entire searched domain as a solution. However, the most important problem of this method is the fact that in reality, a simple one-criterion problem is solved on the basis of only one parameter, after the prior elimination of solutions that do not meet the $\varepsilon$ criterion.

### 4.6 *Goal Programming GP Method*

The goal programming method [6] consists of replacing a multi-criteria task with the following task:

$$\begin{aligned} &\min F = \min_{x} \alpha \\ &F_K(x) - w_K \alpha \le c_K \end{aligned} \tag{10}$$

where ($c_1$, $c_2$, ..., $c_K$) represent the coordinates of the $C$ point defining the purpose of the search, and ($w_1$, $w_2$, ..., $w_K$) are the coordinates of the vector $W$, defining the direction of the search.

Then, according to the authors of [45, 56], the task is reduced to searching for the $C$ point from the set of acceptable solutions, in which the values of the criteria are closest to some of the ideal values determined by the coordinates ($c_1$, $c_2$, ..., $c_K$).

The optimal search is carried out in the criterion space, starting from point $C$ in the direction determined by vector $W$. The solution is a rectangular point with sides parallel to the system axis, a lower left corner at point $C$, and a diagonal parallel to the vector $W$ (Figure 14).

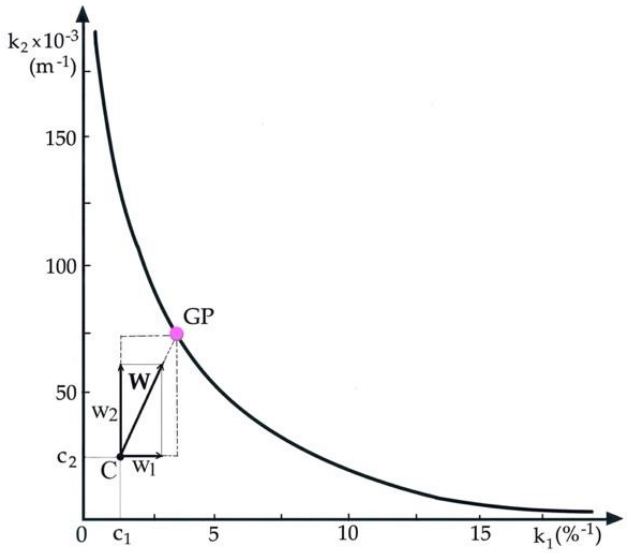


Figure 14. The method of goal programming and the optimal solution on the Pareto front; weighting values of optimal game final payoff: $k_1$=3.52, $k_2$=0.075

The coordinate values of point $C$ ($c_1$ and $c_2$) come from the person making the arbitrary decision. The choice of vector components $W$ ($w_1$ and $w_2$) determines the importance of the individual optimization criteria.

Figure 15 shows the results of a computer simulation of the optimal controlling the ship while passing 19 encountered objects, using the goal programming GP method on the Pareto front.

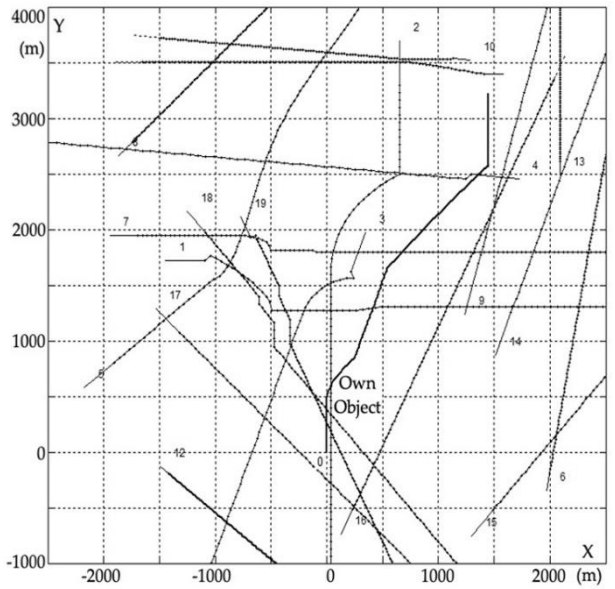


Figure 15. Optimal ship trajectory while safely passing 19 encountered objects for the goal programming GP method multi-criteria optimization of the final payoff value of the control objective function: $r_f$ = 7 %; $F_{\min}^{GP} = 132.94$ — minimum value of the two-criteria objective function (8)

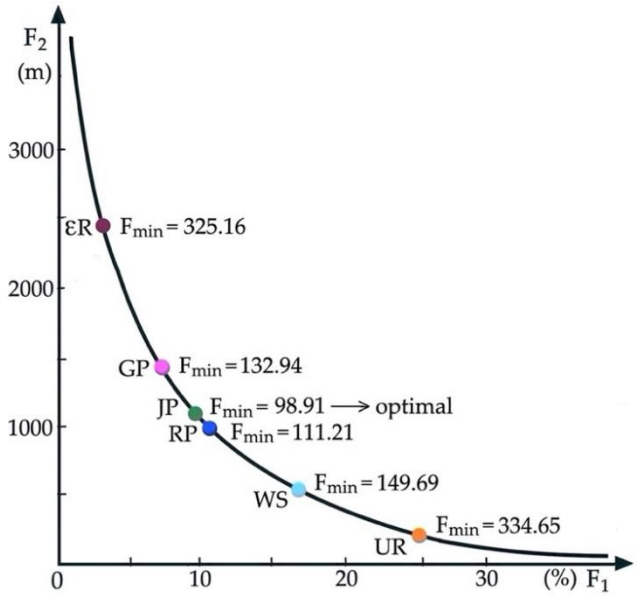


Figure 16. Comparison of multi-criteria optimization methods of the ship game control process in collision situations: εR—ε-restrictions; GP—goal programming; RP—reference point; JP—justice principle; WS—weighted sum; UR—utilitarianism rule; $F_1 = r_f$—the final collision risk; $F_2 = d_f$—the final deviation trajectory; $F_{min}$—minimum value of the two-criteria objective function (8)

## 5 COMPARISON OF METHODS

Figure 16 shows a comparison of the multi-criteria optimization methods of the object traffic control process at the reference trajectory.

In the case of safe control of an object in collision situations, the most optimal compromise solution for two-criteria optimization tasks is the Rawls principle of justice (JP) method, i.e., controlling the movement of the ship, ensuring both a small final deviation in the cruise route and a small final risk of collision, providing the smallest value of the control objective function. But in other tasks of multi-criteria optimal control, depending on the shape of the Pareto-optimal front, there may be another better method, among the six presented in the article.

The most extreme optimal solutions to this problem are provided by the Haimes ε-restrictions (εR) and Bentham utilitarianism rule (UR) methods.

The Haimes ε-restrictions (εR) and goal programming (GP) methods provide the lowest final risk of collision, but with a large final cruise route deviation. The weighted sum (WS) and utilitarianism (UR) methods provide the highest final collision risk within the allowable range, but with the smallest deviation from the prescribed voyage route.

The degree of cooperation in the anti-collision maneuvers between objects has a significant impact on the value of the final optimal solution.

## 6 CONCLUSIONS

The methods of static multi-criteria optimization presented here constitute the most described part of all of the multi-criterial optimization methods. The differences in the value of the determined optimum are the most dependent on the shape of the Pareto front of the specific optimization task.

Commonly accepted solutions for multi-criteria optimization tasks are sets bringing the front of Pareto optimal solutions closer. The basis for building approximate collections is the relation of domination in the sense of Pareto. It allows for introducing a partial order in the set of assessments of acceptable solutions and for selecting from the non-dominant solution assessments in order to build approximate sets. The size of the approximate set is not specified and, in practice, contains many assessments of equivalent solutions, without objective possibilities to indicate the best solution or the best solutions in the approximate set.

The relation of the dominance of solution assessments can be extended to approximate sets and can be used to determine the relation of the preferences of approximate sets, and thus to the preliminary assessment of the quality of approximate sets or the effectiveness of the optimizers that were used to obtain these sets.

In summary, traditional multi-criteria optimization methods are still very popular. This is because they give good results when finding potential solutions for a small number of solutions in the sense of Pareto, and also because knowledge about them and the availability of materials is currently widespread. Nevertheless, these methods are not without flaws. For a small amount of the optimal sets they do quite well, however, a larger set causes a significant

increase in the cost of calculations. This is because, usually, traditional methods need to be run several times to determine the optimal Pareto set. In addition, some techniques, such as the weighted criteria method, are sensitive to the shape of the Pareto-optimal front.

Despite the constant popularity presented in the article traditional multi-criteria optimization methods, in many problems, evolutionary algorithms are increasingly becoming their alternative. This is because they are better at dealing with a potentially large number of solutions in the Pareto sense.

This review of methods does not exhaust all of the issues related to multi-criteria optimization, especially regarding moving objects—land, air, and sea—in terms of manned and unmanned vehicles.

Future works could also consider methods of multi-criteria dynamic optimization. Particular attention should also be paid to the multi-criteria optimization of differential games problems related to the motion control of many moving objects.

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