BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 70(6), 2022, Article number: e143646
DOI: 10.24425/bpasts.2022.143646

CONTROL AND INFORMATICS

Cascade tanks system identification for robust predictive control

Wiktor JAKOWLUK [1]* and **Sławomir JASZCZAK** [2]

[1] Bialystok University of Technology, Faculty of Computer Science, Wiejska 45A, 15-351 Białystok, Poland
[2] West Pomeranian University of Technology in Szczecin, Faculty of Computer Science and Information Technology, Żołnierska 49, 71-210 Szczecin, Poland

Abstract. The main goal of estimating models for industrial applications is to guarantee the cheapest system identification. The requirements for the identification experiment should not be allowed to affect product quality under normal operating conditions. This paper deals with ensuring the required liquid levels of the cascade system tanks using the model predictive control (MPC) method. The MPC strategy was extended with the Kalman filter (KF) to predict the system's succeeding states subject to a reference trajectory in the presence of both process and measurement noise covariances. The main contribution is to use the application-oriented input design to update the parameters of the model during system degradation. This framework delivers the least-costly identification experiment and guarantees high performance of the system with the updated model. The methods presented are evaluated both in the experiments on a real process and in the computer simulations. The results of the robust MPC application for cascade system water levels control are discussed.

Key words: application-oriented input design; Kalman filter; model predictive control; system identification.

1. INTRODUCTION

System identification for the purposes of control received extensive interest over recent decades [1–3]. Many industrial processes show slow response, which makes identification experiments long and costly. It was shown that about three fourths of the total costs associated with real-life control system design are absorbed by modelling [4]. The costs of the identification experiment can be defined as the experiment duration as well as power and length of the excitation signal. Other more complicated costs are expressed by the acceptable control performance during model-based control. The often used framework for model-based control development is called application-oriented input design [5].

A key issue in system identification is to perturb the system under diagnosis using an informative input signal and to estimate the model parameters with maximum accuracy [6]. Early announcement of optimal input design for system identification described minimization of the error of the parameters to be estimated regarding the process constraints. The fundamental rule of optimal input design is to maximize the sensitivity of state variables to unknown parameters [1]. Most recently, an idea of the acceptable application performance degradation instead of the estimated parameters variance minimization was presented in [7]. Robust control considers the nominal model identification regarding constraints on the model uncertainty set.

In the worst-case scenario for the identification experiment, model parameters are estimated with the guaranteed error bounds expressed in the form of the noise disturbing the model of the system [8]. For optimal average performance input design for control, where the obtained uncertainty set suits H_∞, robust control can be found in [9]. However, the methods described above do not provide satisfactory control of the system. It is enough that the conditions of the identification experiment guarantee that the uncertainty of the model is smaller than an assumed threshold. This leads to the concept of the least-costly identification experiment design for control. The goal is to design the least-costly identification strategy where model uncertainty is relatively small and ensures satisfactory control performance. Instead of minimizing the performance degradation defined as experiment cost (i.e. input energy, experiment duration), the experiment cost includes the input design problem through the objective function [10]. Another method used to experiment with cost minimization is called plant-friendly system identification [11, 12]. The plant-friendly input signal design is related to the application-oriented system identification method. The common goal is to find a trade-off between minimal deviation from the operating point and precision of the model parameters identified [13]. In [14], the idea of a robust plant-friendly input design with constraints on input move size and output power was presented. The application-oriented input design minimizes the cost of the identification experiment and guarantees satisfactory application performance. The input signal design is performed in terms of the excitation signal spectrum selection [15]. There are some trends associated with the use of the application-oriented input design. One of them is to simultaneously improve the performance of the system by ameliorating its model. This problem can be solved by the model predictive control method where the input signal yielded by the MPC is designed to provide satisfactory control performance in

*e-mail: w.jakowluk@pb.edu.pl

Manuscript submitted 2022-06-25, revised 2022-08-29, initially accepted for publication 2022-10-17, published in December 2022.

a sequential manner [16, 17]. Robust MPC deals with designing controllers that satisfy stability and performance demands for diagnosed models.

The main goal of this research is to ensure the required liquid levels of the cascade tanks system using the MPC method. The MPC strategy augmented by the Kalman filter (KF) was used to predict the system's succeeding states subject to the designed reference input signal. For this purpose, the state-space model of the cascade tanks system was linearized around a working point and then discretized. The cascaded tanks system pump control signal was designed using the single weighted cost function method to estimate model parameters more precisely. The parameters of the diagnosed model can be re-estimated during normal operation using the application-oriented input design framework. This method uses a finite impulse response (FIR) filter to obtain an optimal input signal spectrum, subject to the trajectory power constraints. The A- and D-optimal criteria were verified as measures of optimality [14, 18].

The novelty of the problem considered in this paper is related to the input design method for system identification when the obtained model is used in the MPC. The advantage of this approach is a convex optimization problem formulation in the frequency domain.

2. CASCADE TANKS SYSTEM

2.1. System overview

The physical plant of the water tanks process consists of three tanks placed above each other and another tank responsible for water storage, as shown in Fig. 1.

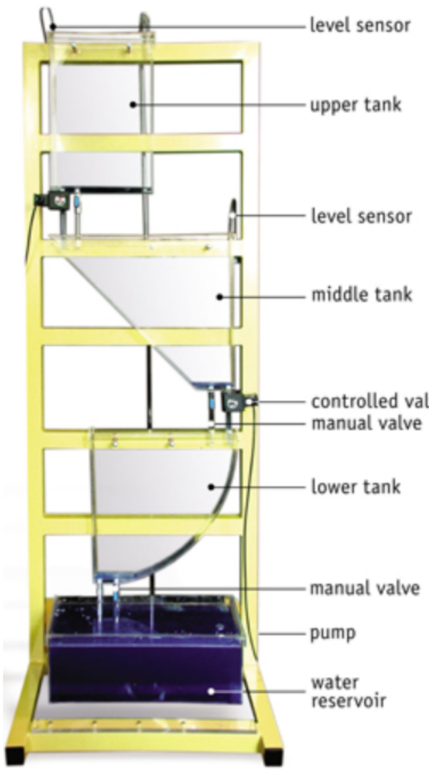

Fig. 1. Multitank system [19]

The upper tank has a constant cross-section, while the others are spherical or prismatic and have variable cross-sections. Water is poured into the upper tank from the reservoir tank by the pump driven by a DC motor. The water outflows the tanks by the force of gravity but output orifices act as flow resistors. The water levels are measured using pressure transducers and the frequency signals are transmitted to the digital inputs of the I/O board. The goal of the tanks system control is to reach and stabilize desired water levels via pump control.

2.2. System modeling

The main objective of system identification is to deliver a model that provides acceptable control performance. The water tank system (Fig. 1) is defined by the volumetric inflow $q_{\text{in}}(t)$ into the tank to the water outflow $q_{\text{out}}(t)$ through the output orifice. The equation of the water flow in the tank can be formulated as:

$$A_i \frac{\mathrm{d}h_i(t)}{\mathrm{d}t} = q_{\text{in}}(t) - q_{\text{out}}(t), \tag{1}$$

where: A_i is the cross-sectional area of the tank, $h_i(t)$ is the liquid level in the tank, and $i = 1,2,3$. The laminar water outflow rate (i.e. for $\alpha_i = 1/2$) from the tank is defined by Torricelli's law described as:

$$q_{\text{out}}(t) = S_i \cdot \sqrt{2gh_i(t)}, \tag{2}$$

where: S_i is the cross-sectional surface of the orifice i and g is the gravitational acceleration. For the real configuration of tanks and valves, if turbulence and acceleration of the water in the tube cannot be omitted, more general coefficients α should be used. Substituting equation (2) to (1) and considering that $q_{\text{in}}(t) = u(t)$, $x_1(t) = h_1(t)$, $x_2(t) = h_2(t)$, $x_3(t) = h_3(t)$, it is possible to formulate the following nonlinear state-space equations:

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = -\dfrac{C_1}{A_1} \cdot x_1^{\alpha_1}(t) + \dfrac{k_v}{A_1} \cdot u(t), \quad x_1(0) = h_{10}, \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = \dfrac{C_1}{A_2} \cdot x_1^{\alpha_1}(t) - \dfrac{C_2}{A_2} \cdot x_2^{\alpha_2}(t), \quad x_2(0) = h_{20}, \\ \dfrac{\mathrm{d}x_3(t)}{\mathrm{d}t} = \dfrac{C_2}{A_3} \cdot x_2^{\alpha_2}(t) - \dfrac{C_3}{A_3} \cdot x_3^{\alpha_3}(t), \quad x_3(0) = h_{30}, \end{cases} \tag{3}$$
$$y_1(t) = x_1(t) + v_1(t),$$
$$y_2(t) = x_2(t) + v_2(t),$$
$$y_3(t) = x_3(t) + v_3(t),$$

where: C_i is the solenoid valve output resistance of tank i, α_i is the outflow coefficient for the i tank, k_v is the maximum volumetric flow rate of the pump, and v_i is the stationary, zero-mean, white process of the $i-$th output. The water levels in the tanks $h_i(t)$ have real limitations:

$$h_{i,\max} \geq x_i(t) \geq 0, \quad i = 1,2,3. \tag{4}$$

The physical limitations and the system parameters of the cascade tanks process are listed in Table 1.

Table 1
Numerical parameters of the system

Parameter	Value	Unit	Description
$h_{i,\max}$	2.5*e*–1	[m]	Max. water level of tanks 1, 2, 3
$h_{i,\min}$	0.0	[m]	Min. water level of tanks 1, 2, 3
h_{i0}	2.0*e*–2	[m]	Initial condition of tanks 1, 2, 3
A_1	8.7*e*–3	[m^2]	Cross-sectional area of tank 1
A_2	1.1*e*–2	[m^2]	Cross-sectional area of tank 2
A_3	1.2*e*–2	[m^2]	Cross-sectional area of tank 3
k_v	1.5*e*–4	[m^3/s]	Max. rate flow of pump
C_i	2.5*e*–5	[–]	Valve resistance of tank 1, 2, 3
α_i	5.0*e*–1	[–]	Outflow factor for tank 1, 2, 3

The identification experiment was performed for fixed settings of the solenoid valve output resistance C_i and the outflow factor α_i for all tanks of the system. The coefficient values were then tuned based on the comparison of the flow results of the physical plant and the Simulink model. For the model of the system identification purposes, the pump control signal should be carefully selected.

3. OPTIMAL PUMP CONTROL SIGNAL DESIGN

System identification relies on an accurate mathematical model built using experimental data and a priori knowledge of the plant [20]. The accuracy of the model to be estimated is deeply related to an excitation signal [1, 12]. To design an optimal input signal, an adequate scalar norm of the Fisher information matrix (FIM) should be used. The FIM is defined as:

$$M = \frac{1}{N}\sum_{t=1}^{N} \mathsf{E}\left\{\left(\frac{\mathrm{d}}{\mathrm{d}\theta}y(t,\theta)\right)\Lambda^{-1}\left(\frac{\mathrm{d}}{\mathrm{d}\theta}y(t,\theta)\right)^{\mathrm{T}}\right\}, \tag{5}$$

where: y is the observable random variable and θ is the unknown parameter vector. The different criteria for optimal design performance could be found in [21]:

- A-optimality: $\operatorname{tr}(M^{-1})$, minimizes the variance of the parameters to be estimated,
- E-optimality: $\lambda_{\max}(M^{-1})$, minimizes the maximum eigenvalue of M^{-1},
- D-optimality: $\det(M)$ maximizes the determinant of the FIM and minimizes the volume of the ellipsoidal confidence region of the estimated parameters.

Sensitivity of the measurements to the parameters is guaranteed by the Cramér–Rao inequality described as:

$$\operatorname{cov}\left(\hat{\theta}\right) = \mathsf{E}\left[\left(\hat{\theta}-\theta\right)\left(\hat{\theta}-\theta\right)^{\mathrm{T}}\right] \geq M^{-1}. \tag{6}$$

The optimal pump control signal is designed to ensure reasonably accurate parameter estimates during the identification of the cascaded system. To solve the input design problem, the RIOTS_95 toolbox [22] dedicated to the problems of optimal control is used. This issue can be solved considering the upper tank equation formulated as:

$$\begin{aligned}&\dot{x}_1(t) = -\frac{a}{A_1}\cdot x_1^{\alpha_1}(t) + \frac{b}{A_1}u(t), \quad x_1(0) = h_{10},\\ &y_1(t) = x_1(t) + v_1(t).\end{aligned} \tag{7}$$

According to Table 1, $a = C_i$ and $b = k_v$, respectively. The FIM for the upper tank of the state-space model (3) can be expressed as:

$$M(T) = \int_0^T X_\theta^{\mathrm{T}}\Lambda^{-1}X_\theta\,\mathrm{d}t = \frac{1}{\sigma_n^2}\int_0^T \begin{bmatrix}x_a\\ x_b\end{bmatrix}\begin{bmatrix}x_a & x_b\end{bmatrix}\mathrm{d}t, \tag{8}$$

where: $x_a = \partial x_1/\partial a$, $x_b = \partial x_1/\partial b$ are the sensitivities of the measurements to the parameters and Λ is the variance matrix given by:

$$\Lambda^{-1} = \frac{1}{\sigma_n^2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}. \tag{9}$$

It is assumed that $\sigma_n = 1$ to obtain the optimal pump control signal for model parameters estimation. Proceeding to the optimal pump control design, it was assumed that $\alpha_1 = 0.5$. Then problem (7) is modified by defining the state vector augmented by the sensitivity equations and the FIM ingredients:

$$\begin{aligned}
&x_1 = x_1, && \dot{x}_1 = -\frac{a}{A_1}\sqrt{x_1} + \frac{b}{A_1}u, && x_1(0) = h_{10},\\
&x_2 = x_a, && \dot{x}_2 = -\frac{a}{2A_1}\frac{x_2}{\sqrt{x_1}} - \frac{\sqrt{x_1}}{A_1}, && x_2(0) = 0,\\
&x_3 = x_b, && \dot{x}_3 = -\frac{a}{2A_1}\frac{x_3}{\sqrt{x_1}} + \frac{u}{A_1}, && x_3(0) = 0,\\
&x_4 = m_{11}, && \dot{x}_4 = x_2^2, && x_4(0) = 0,\\
&x_5 = m_{12} = m_{21}, && \dot{x}_5 = x_2x_3, && x_5(0) = 0,\\
&x_6 = m_{22}, && \dot{x}_6 = x_3^2, && x_6(0) = 0,
\end{aligned} \tag{10}$$

where: m_{ij} are ingredients of the FIM. Finally, the optimal input design problem considering Mayer's canonical formulation, which maximizes the FIM determinant with respect to input and state trajectory constraints, is:

$$\begin{aligned}&J = \max_u \left[x_4\left(T_f\right)x_6\left(T_f\right) - x_5^2\left(T_f\right)\right],\\ &\text{subject to}\\ &0 \leq u(t) \leq 1, \quad t \in \left[0, T_f\right],\\ &x_1(t) \leq 0.25, \quad t \in \left[0, T_f\right],\end{aligned} \tag{11}$$

where: $T_f = 20$ [s] is the fixed termination time chosen arbitrarily. The pump control signal obtained by solving optimization problem (11) is shown in the Fig. 2.

It can be noticed that for two seconds the pump is off and after that it pumps the water at full power for about seven seconds. During operation time, the pumped water can fill about half the volume of the upper tank. It is because valve resistance, and not the valve cross-section of the tank, was assumed for calculation.

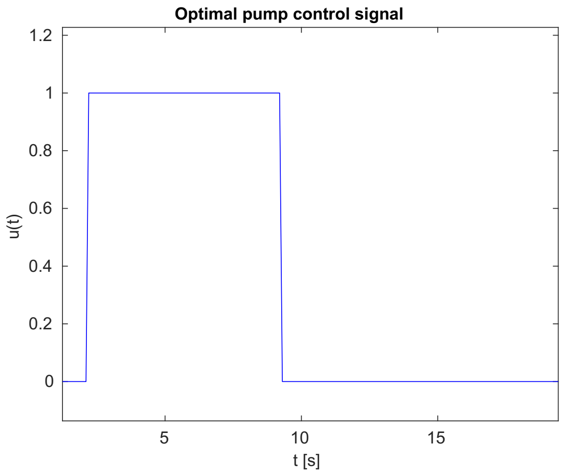

Fig. 2. Pump control signal

4. IDENTIFICATION OF THE LINEARIZED MODEL

To provide the acceptable water levels in tanks of the cascade system (Fig. 1), the model predictive control strategy is used. To apply both MPC and the Kalman filter methods, the controlled system should be linearized and discretized. In general, the discrete-time model can be expressed as:

$$\begin{aligned} x(t+1) &= A_d x(t) + B_d u(t) + v(t), \\ y(t) &= C_d x(t) + w(t), \end{aligned} \tag{12}$$

where: $x(t)$ is a state vector, $u(t)$ means an input signal, $y(t)$ denotes a measured output, whereas $v(t)$ and $w(t)$ represent the zero-mean process and measurement noise, respectively.

The nonlinear state-space model (3) is linearized around the steady-state point [23]. The equilibrium state can be calculated from:

$$\begin{aligned} u_{\text{std}} &= C_1 x_{1\text{std}}^{\alpha_1} = C_2 x_{2\text{std}}^{\alpha_2} = C_3 x_{3\text{std}}^{\alpha_3}, \\ x_{\text{std}} &= \begin{bmatrix} x_{1\text{std}} \\ x_{2\text{std}} \\ x_{3\text{std}} \end{bmatrix} = \begin{bmatrix} \left(\dfrac{u_{\text{std}}}{C_1}\right)^{1/\alpha_1} \\ \left(\dfrac{u_{\text{std}}}{C_2}\right)^{1/\alpha_2} \\ \left(\dfrac{u_{\text{std}}}{C_3}\right)^{1/\alpha_3} \end{bmatrix}. \end{aligned} \tag{13}$$

Assuming that α_i, C_i are constant values for $i = 1,2,3$, (see Table 1) and $x_{i\text{std}} = 2.0e-2$, the linearized state-space matrices obtained using the first-order Taylor expansion are as follows:

$$\begin{aligned} A_l &= \begin{bmatrix} -\tau_1 & 0 & 0 \\ \tau_2 & -\tau_2 & 0 \\ 0 & \tau_3 & -\tau_3 \end{bmatrix}, \quad B_l = \begin{bmatrix} \dfrac{k_v}{A_1} \\ 0 \\ 0 \end{bmatrix}, \\ C_l &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, \quad \tau_i = \frac{C_i \alpha_i}{A_i x_{\text{std}}^{1-\alpha_i}}, i = 1,2,3. \end{aligned} \tag{14}$$

The results of linearization are given by:

$$\begin{aligned} A_l &= \begin{bmatrix} -0.0133 & 0 & 0 \\ 0.0096 & -0.0096 & 0 \\ 0 & 0.0294 & -0.0294 \end{bmatrix}, \\ B_l &= \begin{bmatrix} 0.12 \\ 0 \\ 0 \end{bmatrix}, \quad C_l = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}. \end{aligned} \tag{15}$$

The zero-order hold discretization at the sampling rate of 10 Hz according to (11) gives:

$$A_d = e^{A_l}, \quad B_d = \int_0^1 e^{A_l(1-t)} B_l \,\mathrm{d}t, \quad C_d = C_l. \tag{16}$$

The identification experiment was performed by energizing the pump using the control signal shown in Figure 2. The initial water level conditions and valve resistances of cascade system tanks are given in Table 1. The results of the identification experiment, performed using the real system, as well as the linearized model (3) are shown in Figs. 3 and 4 below.

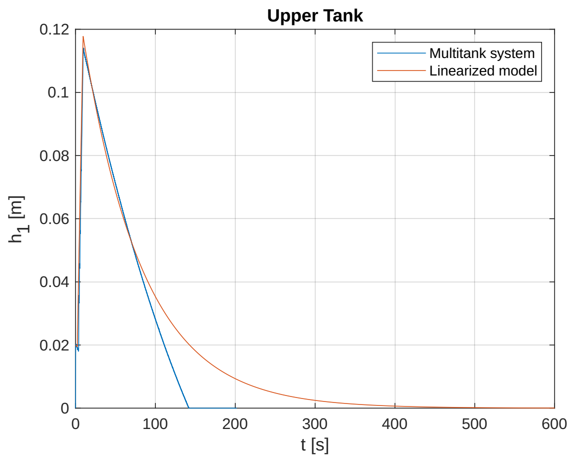

Fig. 3. Upper tank experiment results

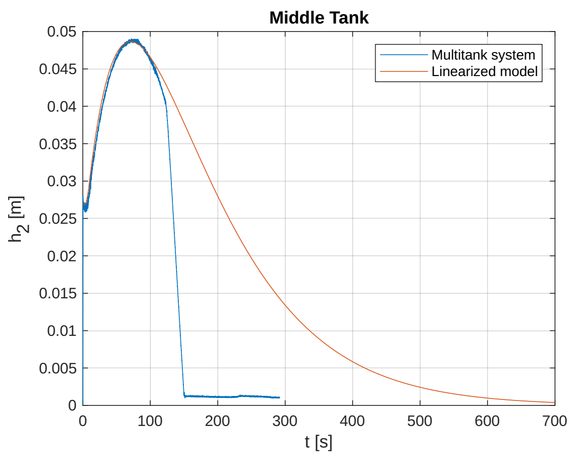

Fig. 4. Middle tank experiment results

The lower tank experiment results are very similar to that obtained for the middle one. It can be noted from Figs. 3 and 4 that the fitting of the flows in the linearized model compared to the real plant flows is satisfactory around the equilibrium point.

5. PLANT MODEL PARAMETER ADAPTATION

The objective of the application-oriented input design is to obtain the linear model of the system that provides acceptable application performance. This type of experiment can be used for updating the plant model parameters during the model predictive control operation.

The construction of the excitation signal relies on the input signal spectrum design. The spectrum of the input signal affects the model parameters to be estimated during the identification experiment [5]. The spectral density of the input signal can be written as:

$$\Phi_u(\omega) = \sum_{k=-(m-1)}^{m-1} c_k \mathfrak{R}_k\left(e^{j\omega}\right), \tag{17}$$

where the scalar basis functions $\left\{\mathfrak{R}_k\left(e^{j\omega}\right)\right\}_{k=0}^{\infty}$ are proper, stable rational so that $\mathfrak{R}_{-k}\left(e^{j\omega}\right) = \mathfrak{R}_k\left(e^{-j\omega}\right)$, and the real coefficients are $c_{-k} = c_k^{\mathrm{T}}$. The basis function used for model identification is $\mathfrak{R}_k\left(e^{j\omega}\right) = e^{-j\omega k}$. The finite-dimensional parametrization approach for finding coefficient c_k is applied. This can be achieved by using the positive real lemma which arises from the Kalman-Yakubovich-Popov lemma [24]. The partial expansion defined by (17) ensures that only first m factors are used to compute the input spectrum.

In this section the discrete-time linear dynamic model of the cascade system $G(q,\theta)$, defined by (27), and the transfer function $H(q,\theta)$ describing the noise dynamic are considered during identification. The open-loop system diagram used for model parameter estimation is presented in Fig. 5. Here, $u(t)$ is the input signal, $y(t)$ is the output signal and $e(t)$ is the noise signal. The transfer functions $G(q,\theta)$ and $H(q,\theta)$ are parameterized by the parameter vector θ, and q is the time-shift operator.

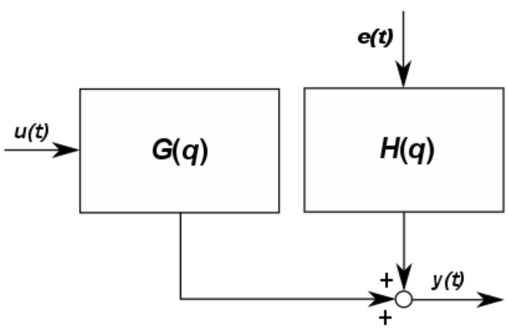

Fig. 5. System set-up

The model (Fig. 5) response is expressed as:

$$y(t) = G(q,\theta)u(t) + H(q,\theta)e(t). \tag{18}$$

The model's unknown parameters are estimated using the prediction error method (PEM). The one-step-ahead predicted output of the model is:

$$\hat{y}(t|\theta) = H^{-1}(q,\theta)\,G(q,\theta)\,u(t) + \left[I - H^{-1}(q,\theta)\right]y(t). \tag{19}$$

Consequently, the one-step-ahead prediction error becomes:

$$\begin{aligned}\varepsilon(t,\theta) &= y(t) - \hat{y}(t|\theta)\\ &= H^{-1}(q,\theta)\left[y(t) - G(q,\theta)u(t)\right].\end{aligned} \tag{20}$$

In this paper the criterion function is defined as the quadratic criterion required by the Moose2 toolbox [25]:

$$V_N(\theta) = \frac{1}{2N}\sum_{t=1}^{N} \varepsilon(t,\theta)\Lambda^{-1}\varepsilon(t,\theta)^{\mathrm{T}}, \tag{21}$$

where Λ is a zero-mean, white noise variance matrix. The values of the parameters are estimated from:

$$\hat{\theta}_N = \arg\min_{\theta} V_N(\theta). \tag{22}$$

The obtained estimates $\hat{\theta}_N$, with probability α, are located inside the system identification ellipsoid centered around real parameters:

$$\varepsilon_{SI} = \left\{\theta \,|\, \left(\hat{\theta}_N - \theta^0\right) M \left(\hat{\theta}_N - \theta^0\right)^{\mathrm{T}} \leq \frac{\chi_\alpha^2(n)}{N}\right\}, \tag{23}$$

where: $\chi_\alpha^2(n)$ is the α-percentile of the χ^2 distribution with $n-$degrees of freedom, symbol M denotes the FIM defined in (5) and N signifies the number of observations.

The different constraints of the identification experiment are related to the application and quality of the model. The degradation in performance between the model and the system is defined by the application cost function. With the increasing value of the application cost, the system performance degrades. Acceptable degradation is formulated as:

$$V_{\mathrm{app}}(\theta) \leq \frac{1}{\gamma}, \tag{24}$$

where: γ is the pre-specified value that guarantees that the performance degradation is less than γ^{-1} when using the identified model [26]. The parameters that fulfill inequality (24) are contained in the application set Θ_{app}:

$$\Theta_{\mathrm{app}}(\gamma) = \left\{\theta | V_{\mathrm{app}}(\theta) \leq \frac{1}{\gamma}\right\}. \tag{25}$$

Implementing equation (25) in application-oriented input design, we guarantee, with high probability, that estimated parameters are accurate. This requirement is equivalent to:

$$\varepsilon_{SI}(\alpha) \subseteq \Theta_{\mathrm{app}}(\gamma), \tag{26}$$

for appropriately selected values of α and γ.

The application-oriented input design is performed using the Matlab-based toolbox Moose2 [27]. This package handles only discrete-time polynomial models with identifiable parameters. The model of the *idpoly* type is defined as:

$$A(q)y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t), \tag{27}$$

where: A, B, C, D, and F are polynomials, and q is the time-shift operator. The optimal input is constructed using an FIR parameterization of the spectrum with 20 lags, that is 20 coefficients in the spectral density function (17). The cascade tanks system upper reservoir implementation is: $A = 1.0$; $B = [0.0 \;\; \theta_1]$; $F = [1.0 \;\; \theta_2]$; $C = 1.0$; $D = 1.0$; $\theta^0 = [1.20e-1 \;\; 1.33e-2]$ – initial parameters values; $\lambda = 1.0$ – variance of measurement noise; $T_s = 0.1$ – sampling duration [s]; $N = 500$ – number of samples; $\gamma = 100$ – allowed application degradation; $\alpha = 0.95$ – confidence level of degradation. The goal of input design is to solve the optimization problem as:

$$\begin{aligned} &\underset{\Phi_u(\omega)}{\text{minimize}} \; \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_u(\omega)\mathrm{d}(\omega) \\ &\text{subject to} \;\; \varepsilon_{\mathrm{SI}}(0.95) \subseteq \Theta_{\mathrm{app}}(100) \\ &\qquad\qquad\;\; \Phi_u(\omega) \geq 0, \; \forall\omega. \end{aligned} \tag{28}$$

The optimal input signal spectra (17) obtained for D- and A-optimal criteria are displayed in Fig. 6.

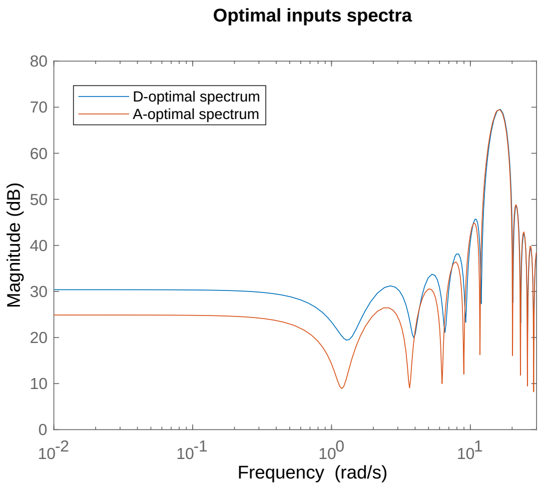

Fig. 6. Designed optimal input signals spectra

One hundred independent Monte Carlo runs have been made to estimate polynomial model (27) parameters while the output of the model is disturbed by normally distributed random numbers. The ellipsoidal confidence regions of the estimated model parameters for chosen criteria are shown in Figs. 7 and 8.

The comparison of the ellipsoidal confidence regions (Fig. 7, 8) of the first tank parameters estimation confirms that the spaces occupied by the estimated parameters are different. The confidence region obtained using the D-optimal criterion indicates performance degradation of less than 1%. But the values of the estimates achieved for the A-optimal input signal

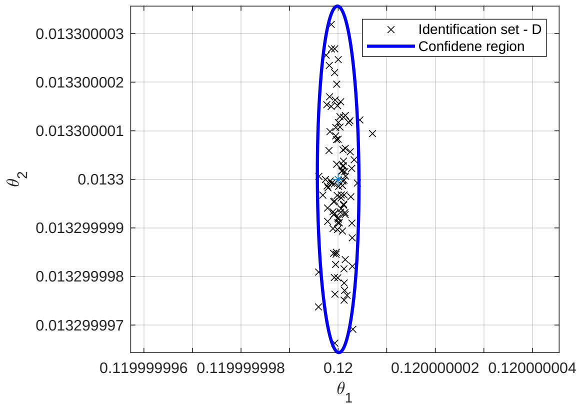

Fig. 7. D-optimal experiment application set

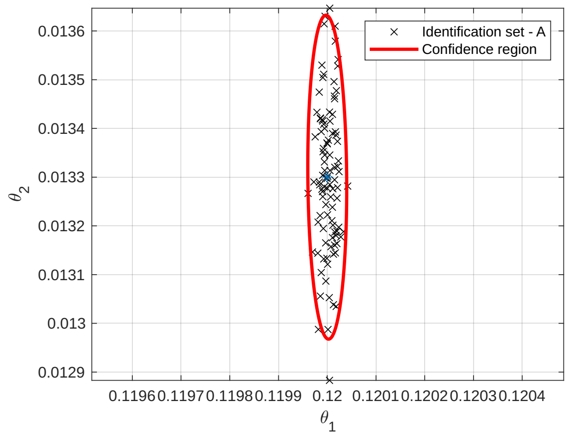

Fig. 8. A-optimal experiment application set

have greater dispersion (i.e. about 3%). In the given case, for different input signal amplitude constraints, the A-optimal identification experiment produces worse results.

For the model of the system parameter adaptation purposes, a D-optimal input design should be preferred. The problem considered in this section describes how to find a model set that guarantees the performance of the controller to be chosen.

6. MULTITANK SYSTEM CONTROL

To control the water levels of the linearized cascade system, the robust MPC method was used. The MPC diagram containing the Kalman filter (KF) is presented in Fig. 9.

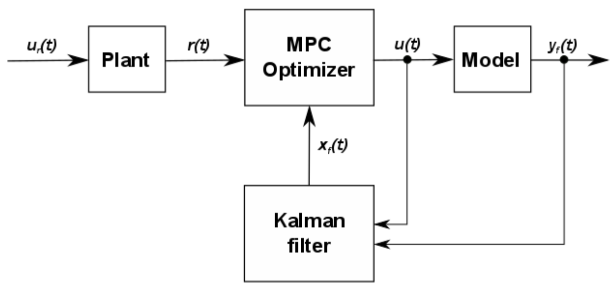

Fig. 9. Schematic overview of the robust MPC

The state-space model (3) linearized around its equilibrium point was discretized at the sampling rate of 10 Hz. The discrete-time matrices referring to the system (12) are:

$$A_d = \begin{bmatrix} 9.9e-1 & 0 & 0 \\ 9.6e-4 & 9.9e-1 & 0 \\ 1.4e-6 & 2.9e-3 & 9.9e-1 \end{bmatrix}, \\ B_d = \begin{bmatrix} 1.2e-2 \\ 5.8e-6 \\ 5.6e-9 \end{bmatrix}, \quad C_d = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}. \tag{29}$$

The robust MPC structure consists of the reference plant, the model under diagnosis, and the Kalman filter as the prediction model. The core of any recursive MPC is to use an accurate model of the process and incoming data to predict the system's output regarding future input [28]. To predict the future output of the system from time t, recognition of a system's state estimate $\hat{x}_f(t)$ is required:

$$\hat{y}_f(t+k|t) = C_d A_d^k \hat{x}_f(t) + \sum_{i=0}^{k-1} A_d^{k-i-1} B_d u(t+i), \tag{30}$$

where: $k \geq 0$. The state estimate $\hat{x}_f(t)$ is obtained by the Kalman filter using the output data acquired from the model. The input signal can be calculated using the following cost function:

$$J(t) = \sum_{i=0}^{N_y} \left\| \hat{y}_f(t+i|t) - r(t+i) \right\|_Q^2 + \sum_{i=0}^{N_u} \left\| \Delta\hat{u}(t+i) \right\|_R^2, \tag{31}$$

where: $\hat{y}(t+i|t)$ and $\Delta\hat{u}(t+i)$ are i – step-ahead predictions of the output and input signals. The $r(t)$ denotes the known reference trajectory over the prediction horizon. Q and R are experimentally chosen weighting matrices. The values of N_y and N_u define the prediction region and the control horizon, respectively. They determine the number of future time intervals to construct the input signal. The input signal sequence was calculated as a solution to the optimization problem:

$$\begin{aligned} &\underset{u(t)}{\text{minimize}}\ J(t) \\ &\text{subject to}\ \ \hat{y} \in Y \\ &\qquad\qquad \hat{u} \in U. \end{aligned} \tag{32}$$

where: Y and U are the constraint regions of the outputs and inputs, respectively. The objective of any MPC task is to provide control of the process while satisfying a set of constraints [29, 30]. Robust MPC deals with designing a controller that satisfies performance demands using the prediction model, e.g. (27). The recursive KF algorithm was implemented to estimate the internal-states x_f of the model under diagnosis using a series of noise measurements.

The cascade system water flows control problem was solved using the model predictive controller from the Matlab-Simulink environment. The Kalman filter state estimator (Fig. 9) was disturbed by zero-mean Gaussian noise. Process $v(t)$ and the observation $w(t)$ noise (12) have arbitrarily selected variance values of $1e-5$ and $1e-1$, which correspond to weighting matrices Q and R, respectively. The predictions of the system state variables were made using discrete-time parameters (29) with a time duration of 700 sec. The states of the cascaded system were predicted using the fixed-step, 4^{th}-order Runge–Kutta method with a grid interval of 0.1 sec, and the initial conditions are shown in Table 1.

The waveforms of the state variables obtained using the Simulink model of the plant are shown in Fig. 10 and represents the water flows in each tank of the linearized system (3). Figure 11 displays the input signal obtained for the discrete-time model (29) controlled by the robust MPC method. The blue (solid) line represents water flow in the upper tank, the red (dotted) curve illustrates water flow in the middle tank and the black (dash-dotted) plot depicts the lower tank water flow.

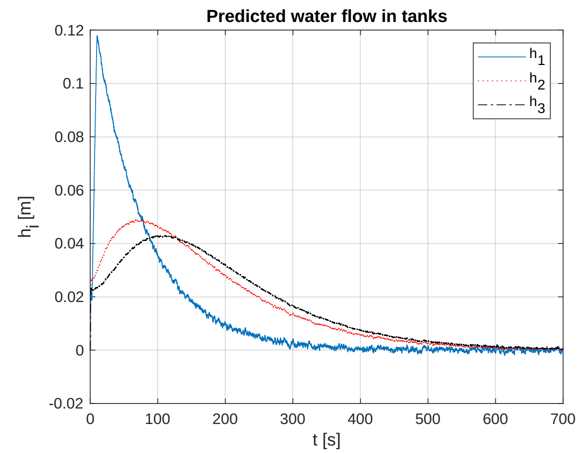

Fig. 10. States of the cascaded system controlled by MPC

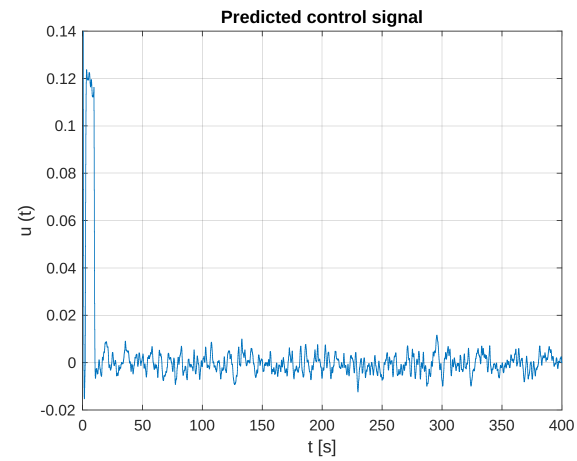

Fig. 11. Input signal of the prediction model controlled by MPC

The input signal $u(t)$ predicted by the MPC considering the plant model (29) is disturbed by observation noise $w(t) = 1e-1$ (Fig. 11). Increasing the measurement noise would certainly make it difficult to control the expected water levels in the tanks of the system. When comparing the trajectories of state variables received for the linearized model (Fig. 3 and 4) to those obtained from the predictive controller (Fig. 10), they were confirmed to be very similar. It should be stated that the proposed control strategy is robust for the model parameter changes of even up to thirty percent. However, significant changes in the operating point of the cascade system should be diagnosed using the application-oriented input design method.

7. CONCLUSIONS

In this paper, we proposed the robust MPC method for the water levels control in the laboratory cascade tanks system. This method allows for maintaining the assumed water levels in the tanks despite changes in system parameters. The presented framework consists of modelling the multitank system, the optimal pump control signal design, and the tanks water levels predictive control. Optimal water inflow into the first tank was designed using the well-known single weighted cost function method. The obtained model of the cascade system was linearized around its equilibrium point, discretized, and then used for robust MPC purposes. The system performance loss can be detected independently during normal operation. Then the above-described application-oriented technique can be applied to the parameter of model re-estimation. The robust predictive control design was verified on desk using the Simulink environment. The simulation results show the effectiveness of the proposed scheme.

ACKNOWLEDGEMENTS

The study presented herein was supported by grant WZ/WI-IIT/4/2020 from the Bialystok University of Technology and funded from the resources for research by the Ministry of Science and Higher Education.

REFERENCES

[1] R. Kalaba and K. Spingarn, *Control, identification, and input optimization*. softcover reprint of the original 1st ed., 1982, Springer US, 2012, pp. 225–299.

[2] R.S. Sánchez-Peña, J. Quevedo Casín, and V. Puig Cayuela, Eds., *Identification and Control. The Gap between Theory and Practice*, Springer-Verlag, London, 2007, pp. 203–242.

[3] E. Ikonen and K. Najim, *Advanced Process Identification and Control*, Taylor and Francis, Boca Raton, 2019, pp. 137–180.

[4] M. Gevers, "Identification for control: from the early achievements to the revival of experiment design," *Eur. J. Control*, vol. 11, pp. 335–352, 2005, doi: 10.3166/ejc.11.335-352.

[5] M. Annergren, C.A.A. Larsson, H. Hjalmarsson, X. Bombois, and B. Wahlberg, "Application-oriented input design in system identification: Optimal input design for control," *IEEE Control Syst. Mag.*, vol. 37, no. 2, pp. 31–56, 2017, doi: 10.1109/MCS.2016.2643243.

[6] R. Pintelon and J. Schoukens, *System Identification: A Frequency Domain Approach*. 2nd ed., John Wiley & Sons: New York, NY, USA, 2001, pp. 151–224.

[7] X. Bombois, G. Scorletti, M. Gevers, P.M.J. Van den Hof, and R. Hildebrand, "Least costly identification experiment for control," *Automatica*, vol. 42, no. 10, pp. 1651–1662, 2006, doi: 0.1016/j.automatica.2006.05.016.

[8] G.G. Yin, S. Kan and L.Y. Wang, "Identification Error Bounds and Asymptotic Distributions for Systems with Structural Uncertainties," *Journal of Systems Science and Complexity*, vol. 19, no. 1, pp. 22–35, 2006, doi: 10.1007/s11424-006-0022-7.

[9] H. Hjalmarsson, "From experiment design to closed-loop control," *Automatica,* vol. 41, pp. 393–438, 2005, doi: 10.1016/j.automatica.2004.11.021.

[10] C.R. Rojas, J.C. Agüero, J.S. Welsh, and G.C. Goodwin, "On the equivalence of least costly and traditional experiment design for control," *Automatica*, vol. 44, no. 11, pp. 2706–2715, 2008, doi: 10.1016/j.automatica.2008.03.023.

[11] S. Narasimhan and R. Rengaswamy, "Plant friendly input design: Convex relaxation and quality," *IEEE Trans. Automat. Control*, vol. 56, pp. 1467–1472, 2011, doi: 10.1109/TAC.2011.2132290.

[12] W. Jakowluk, "Free final time input design problem for robust entropy-like system parameter estimation," *Entropy,* vol. 20, no. 7, p. 528, 2018, doi: 10.3390/e20070528.

[13] D.E. Rivera, H. Lee and M.W. Braun, "'Plant-friendly' system identification: A challenge for the process industries," in *Proc. IFAC Symp. System Identification*, Netherlands, 2003, pp. 917–922, doi: 10.1016/S1474-6670(17)34873-5.

[14] A. Kumar and S. Narasimhan, "Robust plant friendly optimal input design," in *Proc. 10th IFAC Symposium on Dynamics and Control of Process Systems*, India, 2013, pp. 553–558, doi: 10.3182/20131218-3-in-2045.00110.

[15] X. Bombois, G. Scorletti, P. Van den Hof and M. Gevers, "Least costly identification experiment for control. A solution based on a high-order model approximation," *Proc. of the 2004 American Control Conference*, 2004, vol.3, pp. 2818–2823, doi: 10.23919/ACC.2004.1383893.

[16] C.A. Larsson *et al.*, "Model predictive control with integrated experiment design for output error systems," in *Proc. European Control Conf.*, Switzerland, 2013, pp. 3790–3795, doi: 10.23919/ECC.2013.6669533.

[17] J.M. Maciejowski, *Predictive Control with Constraints*, Englewood Cliffs, NJ: Prentice-Hall, 2002, pp. 108–115.

[18] R. Isermann and M. Münchhof, *Identification of Continuous-Time Systems: Linear and Robust Parameter Estimation*. Springer, 2011, pp. 453–499.

[19] "Multi Tank," *Inteco.* [Online]. Available: www.inteco.com.pl/products/multi-tank [Accessed: 18. Oct. 2022].

[20] W. Jakowluk, "Optimal input signal design for fractional-order system identification," *Bull. Pol. Acad. Sci. Tech. Sci.*, vol. 68, no. 4, pp. 883–891, 2019, doi: 10.24425/bpas.2019.127336.

[21] A.C. Atkinson, A.N. Donev, and R.D. Tobias, *Optimum Experimental Designs, with SAS*. Oxford Univ. Press, Oxford, 2007, pp. 119–147.

[22] A. Schwartz, E. Polak, and Y. Chen, "Riots a Matlab toolbox for solving optimal control problems," Version 1.0 for Windows, May 1997. [Online]. Available: http://www.schwartz-home.com/RIOTS/ [Accessed: 18 Oct. 2022].

[23] S. Jaszczak and P. Nikończuk, " Identification of the plant dynamic using genetic algorithms," *21st International Conference on Methods and Models in Automation and Robotics* (MMAR), Poland, 2016, pp. 516–519, doi: 10.1109/MMAR.2016.7575189.

[24] T. Iwasaki and S. Hara, "Generalized KYP lemma: unified frequency domain inequalities with design applications," *IEEE Trans. Autom. Control*, vol. l50, no. 1, pp. 41–59, 2005, doi: 10.1109/TAC.2004.840475.

[25] M. Annergren and C. A. Larsson, "Moose2 - A toolbox for least-costly application-oriented input design," *SoftwareX*, vol. 5, pp. 96–100, 2016, doi: 10.1016/j.softx.2016.05.003.

[26] H. Hjalmarsson, "System identification of complex and structured systems," *Eur. J. Control*, vol. 15, no. 3, pp. 275–310, 2009, doi: 10.3166/ejc.15.275-310.

[27] M. Annergren and C.A. Larsson, "MOOSE2: Model based optimal input signal design toolbox." Version 2.0, 2015. [Online]. Available: www.kth.se/is/dcs/research/sysid/moose/moose2-toolbox-1.194509 [Accessed: 18 Oct. 2022].

[28] W. Jakowluk and S. Boersma, "Fractional-order nonlinear system identification using MPC technique," in *Proc. Computer Information Systems and Industrial Management*, Poland, 2021, vol. 12883, pp. 381–393, doi: 10.1007/978-3-030-84340-3_31.

[29] P. Kurtyna-Mazurek, T. Szolc, M. Henzel, and K. Falkowski, "Control system with a non-parametric predictive algorithm for a high-speed rotating machine with magnetic bearings," *Bull. Pol. Acad. Sci. Tech. Sci.*, vol. 69, no. 6, p. e138998, 2021, doi: 10.24425/bpasts.2021.138998.

[30] S.P. Diwan and S.S. Deshpande, "Computationally efficient nonlinear model predictive controller using parallel particle swarm optimization," *Bull. Pol. Acad. Sci. Tech. Sci.*, vol. 70, no. 4, p. e140696, 2022, doi: 10.24425/bpasts.2022.140696.