

# An analytical evaluation method of the temperature dependence of resistive losses in electric motors

Zafer Dogan<sup>1\*</sup>✉, Tural Mehmetoglu<sup>2</sup>

<sup>1</sup>Faculty of Natural Sciences and Engineering, Department of Electrical Engineering, Gaziosmanpaşa University, Tokat, Turkey

✉ zafer.dogan@gop.edu.tr

<sup>2</sup>Amasya University, Taşova Vocational School, Amasya, Turkey

✉ turalmehmetoglu@yahoo.co.uk

## Abstract

In this work, the temperature dependence of motor resistive losses has been analytically evaluated by using a quantum mechanical approximation. It is well known that the Bloch-Gruneisen approximation is a fundamental quantum theory to the study of the resistivity of the conductor with the change of temperature by using analytical formulation. By using the proposed method, the motor resistive losses can be controlled with respect to the temperature changes. The accuracy of the proposed algorithm has been tested by comparison with different theoretical approaches. It is demonstrated that the new analytical method for the motor resistive losses controlled with the change of temperature will be used for improving motor power and mechanical systems.

**Keywords:** Electric motor, Power losses, copper resistive losses, Bloch-Gruneisen method

## 1. Introduction

It is well known that the accurate study of the physical origin of power losses in electric motors is important. The power and mechanical transmission performance of the rotating electrical motor can be strongly influenced by the change of winding temperature. There are many efficient experimental and theoretical methods [1-12] available on this subject, including fundamental books [17]. The different theoretical approaches are useful for the accurate assessment of power losses in the motors; therefore, we can control the power losses. It seems that the proposed methods are not successful in calculating the power losses of motors because they are all based on classical physics approaches. Since the temperature range of the materials used in motor windings is smaller than the Debye temperature, the power losses should be investigated using the assumptions of modern physics. Therefore, this study introduces a new perspective on the

accurate investigation of power losses, efficiency, and other characteristics, depending on the temperature changes of the power machines.

There are various types of losses in the motor such as mechanical losses, iron losses, and copper losses. In this study, the examination of copper losses to the variation of temperature was carried out by the Bloch-Gruneisen method, which is the approach of modern physics. The method is based on the idea of modern physics assumptions and analytical description of the temperature dependence of resistive losses by interpretation formulas. Note that the Bloch-Gruneisen approximation is one of the efficient methods to evaluate the temperature dependence of metals' resistivity. This is particularly important because the effectiveness of a motor technology strategy depends on the possibility to control the power losses. The proposed method for the evaluation of the temperature dependence of motor

resistive losses is simpler than existing theoretical and experimental approaches.

### 2 Basic formulae and definition

The resistive losses in a motor winding are defined as following form [13]:

$$P_{Cu} = mI^2 R_{AC} \quad (1)$$

where  $m$  is the winding phases,  $I$  is the current. Here  $R_{AC}$  is the resistance winding and defined by

$$R_{AC} = k_R \frac{Nl_{av}}{\sigma S_c} \quad (2)$$

where  $k_R$  is the resistance factor,  $N$  is the number of turns,  $\sigma$  is the specific conductivity of the winding material,  $S_c$  is the cross-sectional area of the winding material and  $l_{av}$  is the average length of turn [13]. The mass of the conductors is defined as:

$$m_{Cu} = \rho_d Nl_{av} S_c \quad (3)$$

where  $\rho_d$  is the density of the conductor. Inserting Eqs. (2) and (3) in Eq. (1), we have formulae for the resistive losses:

$$P_{Cu} = mI^2 R_{AC} = \frac{k_R}{\rho_d \sigma} J^2 m_{Cu} \quad (4)$$

where  $J = \frac{I}{S_c}$  is the current density in the winding material. Now, substituting the formula  $\sigma(T) = \frac{1}{\rho(T)}$  into Eq. (4), we can obtain

the resistive losses in the following form:

$$P_{Cu}(T) = \frac{mk_R \rho(T)}{\rho_d} J^2 m_{Cu} \quad (5)$$

As seen from Eq. (5) the temperature dependence of resistive losses can be evaluated directly for arbitrary temperature ranges. The

temperature dependences of electrical resistivity  $\rho(T)$  are explained by the Bloch-Gruneisen theory or classical physics formula and defined as, respectively [14-16]:

$$\rho(T) = \rho_0 + (m-1)\rho'\theta_D \left(\frac{T}{\theta_D}\right)^m J_m \left(\frac{T}{\theta_D}\right) \quad (6)$$

$$\rho(T) = \rho_0(1 + \alpha(T - T_0)) \quad (7)$$

where  $\rho'$  is the temperature coefficient of resistivity,  $\rho_0$  is residual resistivity,  $T$  is the temperature and  $\theta_D$  is the Debye temperature.

In Eq. (6) the quantity  $J_m \left(\frac{T}{\theta_D}\right)$ , is Bloch-Gruneisen function and given by [15]

$$J_n \left(\frac{T}{\theta_D}\right) = \int_0^{\theta_D/T} \frac{x^n}{(e^x - 1)(1 - e^{-x})} dx \quad (8)$$

where the index  $n$  takes noninteger and integer values. By applying Bloch-Gruneisen theory, in the study [17] an efficient method for the evaluation of electric resistivity of metals is described as:

$$\rho(T) = \rho_1 + \rho_{ee} T^2 + \frac{\rho_{sd} T^3}{7.212} J_3 \left(\frac{T}{\theta_D}\right) + \frac{\rho_{ss} T^5}{124.14} J_5 \left(\frac{T}{\theta_D}\right) \quad (9)$$

where the parameters  $\rho_1$ ,  $\rho_{ee}$ ,  $\rho_{sd}$  and  $\rho_{ss}$  are determined for cooper as:  
 $\rho_1 = 7.6 \cdot 10^{-3} \mu\Omega \cdot cm$ ,  $\rho_{ee} = \rho_{sd} = 0$ ,  
 $\rho_{ss} = 2.38 \cdot 10^{-10} \frac{\mu\Omega \cdot cm}{K^5}$ .

We see from Eqs. (3), (5) and (6), it is necessary to have an accurate evaluation of  $n$ -dimensional Bloch-Gruneisen functions for determination of the electrical resistivity of solids. In a study [18], the authors obtained a new formula for the correct assessment of the Bloch-Gruneisen functions is given by:

$$J_m(T/\theta_D) = \lim_{N' \rightarrow \infty} \sum_{i=0}^{N'} (-1)^i F_i(-2) \frac{\gamma(m+1, (i+1)\theta_D/T)}{(i+1)^{m+1}} \quad (10)$$

where  $F_i(-\beta)$  are the binomial expansion coefficients:

$$F_m(n) = \begin{cases} \frac{n(n-1)\dots(n-m+1)}{m!} & \text{for integer } n \\ \frac{(-1)^m \Gamma(m-n)}{m! \Gamma(-n)} & \text{for noninteger } n \end{cases} \quad (11)$$

And  $\gamma(n+1, (i+\beta)x)$  are incomplete Gamma functions [19]:

$$\gamma(\alpha, y) = \int_0^y t^{\alpha-1} e^{-t} dt \quad (12)$$

In Eq. (10), the quantity  $N'$  is the summation limit.

### Numerical results and Discussions

We proposed a new approach for the analytical calculation of motor resistive losses using Bloch-Gruneisen and classical physic approximations. The developed method can be used to compute motor resistive losses with improved accuracy. To verify the accuracy for various cases we obtained the exact analytical formulae and compared them with the results of a direct classical physic approach. It is clear from Table 1, the comparison results between Eq. (7) and the proposed Bloch-Gruneisen method (Eq. (9)) are in agreement with all sets of parameters. But, as can be seen from the experimental results [20], the Bloch-Gruneisen method is very convenient in practical applications. It is well known that at low temperatures ( $T < \theta_D$ ), classical physics theories cannot explain the heat capacities of solids.

Table 1: The evaluation results of resistive losses of ASM motor by considering Bloch-Gruneisen and classical methods ( $\theta_D = 340$ ,  $k_R = 1.3$ ,  $m = 3$ ,  $J = 6 \times 10^6 \text{ A/m}^2$ ,  $m_{Cu} = 9.02 \text{ kg}$ ,  $\rho_d = 8960 \text{ kg/m}^3$ ) (kWatt(kW))

$T$ °K	$t$ °C	Eq. (7)	Eq. (9)	Experiment [20]
293	20	2.4367	2.4244	2.4106
298	25	2.5225	2.4842	-
300	27	2.5032	2.5417	2.5664
303	30	2.5317	2.5706	-
308	35	2.5792	2.6186	-
313	40	2.6267	2.6665	-
318	45	2.6743	2.7143	-
325	52	2.7408	2.7811	2.7784
328	55	2.7693	2.8097	-
333	60	2.8168	2.8574	-
338	65	2.8643	2.9049	-
343	70	2.9119	2.9524	-
350	77	2.9784	3.0188	2.9945
353	80	3.0069	3.0473	-
358	85	3.0544	3.0946	-
363	90	3.1019	3.1419	-
368	95	3.1494	3.1891	-
375	102	3.2159	3.2552	3.2713

This phenomenon was observed firstly by Einstein based on quantum theory and he formulated the heat capacity of a solid which had been a problem until the beginning of the twentieth century. Subsequently Debye, on the basis of Einstein's approach, proposed the general theory, for the exact definition of the heat capacity of solids in all temperature ranges. Therefore, the recommended approach provides a desired rate of numerical stability over a broad range of parameters, especially in the range of  $T < \theta_D$ . In this study, all results are obtained with the summation upper limit value  $N = 200$ . Also, this work represents progress in the direction of developing good approach for an accurate estimate of motor losses. As a conclusion, on the basis of the proposed method, using this approach allows us to accurately estimate the efficiency performance of motor power and mechanical systems.

**Conclusions**

In this study, we have presented implementation of quantum theory to the calculation of motor resistive losses with changing temperature. As a conclusion, the proposed method and the approach used allows

us to accurately estimate the efficiency performance of motor power and mechanical systems.

**References**

- [1] Zhang L., Tian X., Boggs S. A.: Determination of total resistive loss in multiple circuits, three-phase cable system. *IEEE Transactions on Power Delivery*, 26(3), 2011, 1939-1945.
- [2] Dems M., Komez K., Lecointe J-Ph.: Variation of additional losses at no-load and full-load for a wide range of rated power induction motors. *Electric Power Systems Research*, 143( 2), 2017, 692-702.
- [3] Green T. C., Hernandez-Aramburo C. A., Smith A. C.: Losses in grid and inverter supplied induction machine drives. *IEEE Proceedings Electric Power Applications*, 150(6), 2003, 712-724.
- [4] Chiang H.K., Tseng C.H., Hsu W.L.: Implementation of a sliding mode controller for synchronous reluctance motor drive considering core losses. *Journal of the Chinese Institute Engineers*, 26(1), 2003, 81- 86.
- [5] Tootoonchian F., Nasiri-Gheidari Z., Lesani, H.: Design, analysis, and implementation of extra low air-gap single-phase axial-flux induction motors for low-cost applications. *International Transactions Electrical Energy Systems*, 26(12), 2016, 2516-2531.
- [6] Livshits P., Rysin A., Sofer S., Fefer, Y.: Increased resistive losses of copper interconnects in ULSI devices—a reliability issue. *IEEE Transactions on Device Materials Reliability*, 11(3), 2011, 484-489.
- [7] Gmyrek Z., Boglietti A., Cavagnino A., Estimation of Iron Losses in Induction Motors: Calculation Method, Results, and Analysis. *IEEE Transactions Industrial Electronics*, 57(1), 2010, 161- 171.
- [8] XU, Ke, Yang J., Chuansheng T., Zhao Q.: Finite Element Analysis and Experimental Study on the Thermal Resistance Characteristics of Motor Coolers. *Journal of Power Technologies*, 10(3), 2020, 279-289.
- [9] Domeki H., Ishihara Y., Kaido, C. Kawase Y., Kitamura S., Shimomura T., Takahashi N., Yamada T., Yamazaki K.: Investigation of benchmark model for estimating the iron loss in rotating machine. *IEEE Transactions on Magnetics*, 40(2), 2004, 794-797.
- [10] Gmyrek Z., Anuszczyk J.: The rotational power loss calculation in the square sample. *Computer Engineering in Applied Electromagnetism*, New York: Springer-Verlag, 2005.
- [11] Yang Liu, Liming Shi, Yaohua Li: Comparison Analysis of Loss Calculation Methods and Measurement Techniques in Power Electronics and Motor Systems. *International Conference on Electrical Machines and Systems*, Busan, Korea, 26-29 October (2013).
- [12] Mohammad A., Lokshin E., Averbukh M.: Energy Losses Modeling in Induction Motors Fed by Danfoss VF Micro Drive FC51. *28-th International Conference Electrical and Electronics Engineers*, Israel, 3-5 December (2014).
- [13] Pyrhonen J., Jokinen T., Hrabovcova V.: *Design of Rotating Electrical Machines*, Wiley, New Delhi, 2014.
- [14] Landau L. D., Lifshits E. M.: *Statistical Physics*, Pergamon Press, London, 1980.
- [15] Gruneisen E.: The temperature dependence of the electrical resistance of pure metals. *Annalen der Physik*, 16(3), 1933, 530-540.



- [16] Rego L. G. C., Kirczenow G.: Quantized Thermal Conductance of Dielectric Quantum Wires. *Physics Review Letters*, 81(6), 1998, 232-236.
- [17] Poker D. B., Klabunde C.E.: Temperature dependence of electrical resistivity of vanadium, platinum, and copper. *Physics Review B*, 26(12), 1982, 7012-7015.
- [18] Mamedov B. A., Askerov I. M.: A new algorithm for accurate evaluation of the generalized Bloch-Gruneisen function and its applications to MgB<sub>2</sub> superconductor. *Physics Letters A*, 362(3), 2007, 324-326.
- [19] Gradshteyn I. S., Ryzhik I. M.: *Tables of Integrals, Sums, Series and Products*, 4<sup>th</sup>ed., Academic Press, New York, 1980.
- [20] Matula R. A.: Electrical Resistivity of Copper, Gold, Palladium and Silver, *Journal of Physical and Chemical Reference Data*, 8(4), 1979, 1147-1298.