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Viscosity and surface tension in the biological microparticle filtration process

ANDRZEJ GÓRKA

Military University of Technology, Institute of Optoelectronics,
2 Gen. W. Urbanowicza Str., 00-908 Warsaw, Poland, gorka@upcpoczta.pl

Abstract. The paper uses a simple model to present the effects of viscosity and surface tension on the basic parameters of the biological microparticle filtration process. A single filtration membrane with a uniform transmission structure in 2D space was used for the purposes of this analysis. The filtration system relations are described for the threshold filtration force, maximum and minimum height of the filtered liquid to guarantee stable filtration in a gravity field, and the duration of the process and forces affecting the microparticles during filtration. Based on these relations, the summary shows sample calculations for the base model of negative filtration of biological particles.
Keywords: microparticle filtration, filtration screens, biological particle filtration, filtration membranes, sedimentation, kinematic viscosity, dynamic viscosity, liquid surface tension
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1. Background

The movement of liquid in flood tanks during the filtration process is not widely discussed in the literature. This was confirmed by a literature search performed for the purposes of this paper, as well as papers concerning filtration processes utilising membrane filters with uniform transmission structures in 2D space [1]. Currently, on account of broader research related to blood, conducted for the purposes of oncology, the issue of biological microparticle filtration, including negative blood filtration, has gained substantial significance in recent years [2-8]. The number of papers, patents and studies concerning this subject has increased rapidly [9-15]. Various methods are described, mainly those developed for the vitro enrichment of tumour cells in blood samples. In general, blood cells are highly

sensitive to mechanical damage and ambient conditions, and frequently aggregate during the filtration process. Consequently, the fundamental principle and purpose of these methods is cell separation while maintaining their biological and biophysical functions. This is because cells isolated in this manner serve as a basis for further, broader biomedical testing. This paper presents a preliminary method for advanced MES procedures, and a mathematical description of the process of negative filtration of biological microparticles.

2. Introduction

The model of a single membrane filter with a uniform transmission structure in 2D space, as shown in paper [1], refers to a filtration process where neither the viscosity nor the surface tension force of the filtered liquid is taken into account. The introduction of these properties into the model enables a more accurate estimation of the basic filtration conditions and parameters, such as:

- varying pressure level in the flood tank in relation to the filtration membrane level during the entire filtration process cycle,
- maximum height $\boldsymbol{H}$ and minimum height $\boldsymbol{h}$ of the filtered fluid in the flood tank, ensuring stable conditions for the process throughout the entire filtration cycle,
- estimated duration of a stable filtration process,
- hydrostatic forces exerted by the filtered biological fluid on the biological particles deposited on the filter,
- volumetric flow rate of the filtered fluid,
- filter transmission level in relation to a uniform filtration membrane standard with a comparable viscosity and density of the filtered base fluid.

In standard biological testing, the phenomenon of gravimetric sedimentation is most commonly used for separating biological particles [16]. This is achieved using medical centrifuges. As a method it provides excellent separation of various microparticles that differ significantly in mass from the carrier fluid. The method is not, however, sensitive to the size of the particles segregated. Due to the change in density in the test liquid along the centrifugal force axis during the centrifugal sedimentation process, separation precision is unimpressive and, additionally, the process promotes the aggregation of biological microparticles. In general, the issue of microparticle motion in filtered fluids throughout the entire filtration process, as well as the forces propelling this motion, is currently not represented broadly in the literature. In most cases, the subjects addressed in the technical literature concern primarily the hydrodynamics of macro-scale flows [17]. In the medical literature, the issues related to biological microparticle filtration from heterogeneous bodily

fluids has only recently come under scrutiny, and then primarily concerning dynamic blood filtering [18-20]. This mainly concerns the dynamic enrichment of specific cell fractions from blood samples. Similarly to centrifugal filtration, this enrichment is dominated by gravimetric separation of microparticles. This study focuses on a simplified mathematical description of size-based separation of microparticles. To this end, a model of size-based filtration of biological microparticles developed previously has been used [1]. It concerns a filtration process that occurs during laminar and slow flows of particles in a gravity field without any Coriolis effect [21]. Additionally, this paper assumes that:

- the direction of particle migration during the emptying of a symmetric flood chamber is decided by the distribution of forces, caused by the tightly packed holes in the migration membrane and the specific gravity of the base fluid of both the test liquid and the filtered biological particles,
- the specific gravity of the biological particles being separated does not differ significantly from the averaged specific gravity of the filtered base fluid,
- the viscosity and surface tension forces at the interfaces do not affect the movement of particles during flood chamber emptying,
- the number and size of the filtered particles does not affect the filtration flow rate,
- filtration flow rate depends mainly on the height of the filtered liquid column in the flood tank, on the size and density of the hole distribution in the filtration membrane, and on the pressure in the flood tank at the filtration membrane.

3. Effects of surface tension on the distribution of forces in a single filtration channel

Surface tension is a result of short-range interactions between liquid microparticles. For particles located within the liquid, the interactions are mutually balanced. However, for particles located on the liquid's surface or at the edge of the interaction, they are no longer completely balanced. This results in a cohesive force directed towards the inside of the liquid.

Under liquid equilibrium, this leads to the formation of a permanent interface with the lowest possible separation surface area and minimal potential energy. Each change in the separation surface area involves the performance of work. This relation is defined by surface tension coefficient $\boldsymbol{\delta}$, as a ratio of this work to the change in surface area, or as a ratio of force tangent to the surface subject to the change as a result of the change in edge length of this surface. If the liquid is in contact with a solid (e.g. the filtration membrane), an additional interaction between liquid particles and the filter surface occurs. This interaction is defined

by an adhesion force which disrupts the cohesive forces. Consequently, in the case of microparticle filtration in a gravity field, the cohesive and adhesive forces determine the free flow of liquid from the membrane hole to the space at the bottom tank that collects the filtration products. The small size of the membrane filter holes, within the (5 ÷ 20) μm range, promotes droplet formation at the outlet of each hole (Fig. 1).

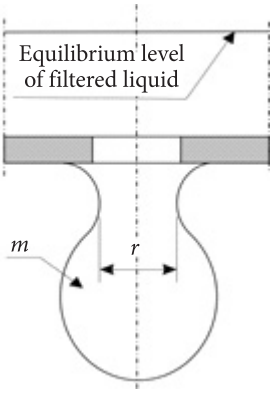

Fig. 1. Outflow droplet from a membrane filter

This effect occurs only for gravitational filtration, where the Coriolis effect is absent [21] and the pressure force is weak. A droplet formed in this way, of mass ***m*** and constriction radius ***r***, in a sense balances the distribution of forces inside the outflow channel [1], and detaches from the membrane only when its weight exceeds the force which surface tension exerts on the droplet (stalagmometric method of measuring the surface tension coefficient of a Newtonian fluid) [22, 23]. Based on this, it can be stated that surface tension "de facto" counteracts the force of gravity of the liquid flowing out of the outflow channel. This leads to a reduction in the hydraulic lever effect from the AGEA subarea [1] (Fig. 2). When these forces are in equilibrium, the filtration process is stopped, and the liquid column height corresponding to this equilibrium determines the total filtration duration. According to the stalagmometric method of surface tension measurement, the equation describing this tension $\boldsymbol{\delta}$ for the test liquid can be written as follows [22]:

$$\delta = \frac{m \cdot g}{2 \cdot \pi \cdot r} \tag{1}$$

where: m — test liquid droplet mass;
g — acceleration due to gravity;
r — constriction radius of the test liquid droplet formed (Fig. 1).

In practical tests, when the values of the test liquid and reference liquid surface tension and constriction radii are similar, relation (2) can be used:

$$\frac{\delta}{\delta_w} = \frac{m}{m_w} \tag{2}$$

where: δ — test liquid surface tension value;
δ_w — reference liquid surface tension value;
m — test liquid droplet mass;
m_w — reference (standard) liquid droplet mass.

Given the above, based on equations (1) and (2), the threshold filtration force value in a single outflow channel subject to gravity pressure can be described with equation (3):

$$F_f = 2 \cdot \pi \cdot r_w \cdot m \cdot \frac{\delta_w}{m_w} \tag{3}$$

where: r_w — reference liquid constriction radius.

When the weight of the liquid in a single outflow channel reaches the value of the threshold filtration force, the process of liquid filtration through the given membrane filter stops. The liquid column height ***h*** corresponding to this determines the threshold filtration height for a single outflow channel (Fig. 2).

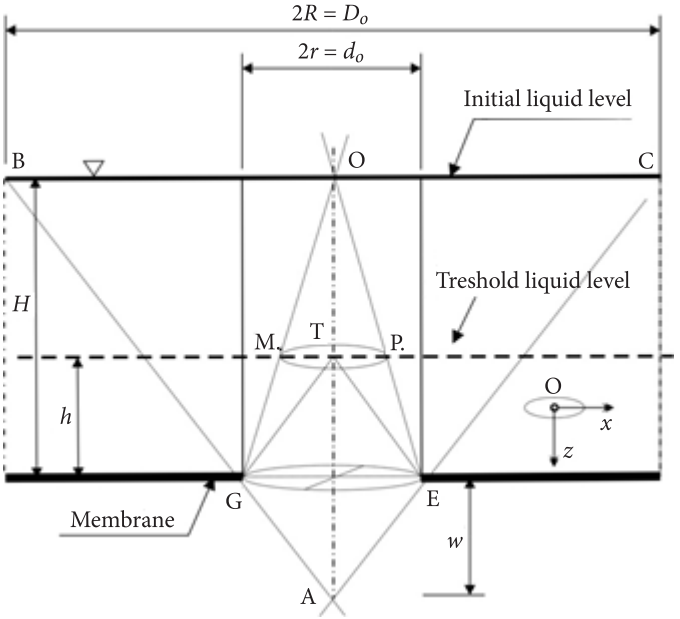

Fig. 2. Threshold liquid outflow level ***h*** in the single outflow channel of a membrane filter [1]

Based on relation (3), it can be stated than the higher the viscosity of the liquid being filtered, and therefore the mass of the outflow droplet being formed, the higher the threshold liquid height in a single outflow channel of the given membrane filter. In membrane filtration model [1], liquid droplets flowing out of a single filtration channel are modelled as AGEA outflow cones (Fig. 2). Based on this model, and assuming that the approximate equilibrium between gravity forces from this space and the weight of the droplets formed by the surface tension phenomenon, the threshold filtration height can be determined. The above approximate equilibrium of these forces is described by the following equilibrium relation (4):

$$F_{lh} \sim F_f \tag{4}$$

where: F_f — gravity force of a liquid droplet formed by the surface tension phenomenon in a single outflow channel of the membrane filter;
F_{lh} — gravity force of liquid of the AGEA outflow cone (Fig. 2).

Based on equilibrium relation (4) and taking into account relation (5)

$$w = \frac{r \cdot h}{(R - r)} \tag{5}$$

threshold filtration height $\boldsymbol{h}$ can be determined using equation (6):

$$h = \frac{6 \cdot (R - r) \cdot m \cdot \delta_w}{\gamma \cdot r^2 \cdot m_w} \tag{6}$$

where: R, r — as in Fig. 2;
m_k, m_w, δ_w — as in equation (2);
γ — specific gravity of the liquid filtered.

Assuming that the constriction radius from Fig. 1 and the hole radius in a single outflow channel from Fig. 2 are equal to each other, and when $\boldsymbol{R = 2r}$ and $\boldsymbol{m \sim m_w}$, then the threshold height for single filtration channel $\boldsymbol{h}$, based on equation (4), is described by simplified equation (7):

$$h = \frac{6 \cdot \delta_w}{\gamma \cdot r} \tag{7}$$

In the gravity filtration process, initial liquid height $\boldsymbol{H}$ should be higher than threshold filtration height $\boldsymbol{h}$ (Fig. 2). This height is determined by the viscosity of the filtered liquid, and the hydrostatic interaction forces between the liquid and biological microparticles on the filtration membrane level. For the negative filtration of blood, the survivability of the cells held on the filter is generally decided by the liquid column hydrostatic pressure, assuming that the filtration membrane is smooth and made of a biologically neutral material. At the same time, it is assumed

that due to the laminar nature of the filtration process in a gravity field, the falling speed of microparticles during the filtration process does not introduce significant differences to this process [1].

4. Effects of dynamic viscosity on the basic parameters of the filtration process

Viscosity is a property related to inter-particle interactions, which refer to the internal friction in the liquid. This friction is defined as laminar movement resistance of one part of the medium against another, neighbouring part of the medium. According to Newton's law [24], static force $\boldsymbol{F_L}$ necessary to impart a layer of thickness $\boldsymbol{s}$ and active surface $\boldsymbol{A}$ in a medium of viscosity $\boldsymbol{\eta}$, with velocity gradient $\boldsymbol{dv/ds}$ in a plane parallel to the laminar flow direction is described by the following equation:

$$F_L = \eta \cdot A \cdot \frac{dv}{ds} \tag{8}$$

In the case of a liquid flow in a narrow outflow channel of a membrane filter, the description of the above phenomenon is based on the Hagen-Poiseuille equation [25]. According to this equation, volume $\boldsymbol{V}$ of a liquid flowing during time $\boldsymbol{t}$ through a channel of radius $\boldsymbol{r}$ and length $\boldsymbol{\Delta h}$ under pressure difference $\boldsymbol{\Delta p}$ [26] is:

$$V = \frac{\pi \cdot r^4 \cdot \Delta p \cdot t}{8 \cdot \eta \cdot \Delta h} \tag{9}$$

where: r — radius of a single outflow channel in a membrane filter;
$\Delta p = \rho \cdot g \cdot \Delta h$ — pressure difference resulting from the difference in liquid levels in the outflow channel;
$\Delta h = H - h$ — difference in liquid levels in the outflow channel;
g — acceleration due to gravity;
ρ — test liquid density;
η — test liquid dynamic viscosity;
t — filtration time of test liquid volume.

In analogy to the Ostwald viscometer [27], which measures flow time for equal volumes of test liquid $\boldsymbol{t_k}$ and standard liquid $\boldsymbol{t_w}$, and assuming that pressure difference $\boldsymbol{\Delta p = \rho \cdot g \cdot \Delta h}$ is proportional to liquid density, the single outflow channel calibration relation for a microparticle membrane filter can be expressed as equation (10):

$$\frac{\eta}{\eta_w} = \frac{\rho \cdot t_k}{\rho_w \cdot t_w} \tag{10}$$

where: η_k — test liquid dynamic viscosity;

η_w — standard liquid dynamic viscosity;
ρ — test liquid density;
ρ_w — standard liquid density.

Based on equation (10), test liquid dynamic viscosity can be determined, see equation (11):

$$\eta = k \cdot \rho \cdot t_k \qquad k = \frac{\eta_w}{\rho_w \cdot t_w} \tag{11}$$

where: k — membrane filter calibration constant.

Due to the specific nature of the biological microparticle filtration process, one of the main limitations of the negative filtration process is the maximum force which the liquid can exert on the membrane bottom. This partly stems from the threshold rupture strength of very thin membranes, and from the acceptable force of the liquid interaction on the filtered biological cells. Because membranes are made of fairly strong materials, an important threshold limitation for the filtration process is the breaking strength of cells. Force $\boldsymbol{F_g}$, when hydrostatic pressure causes the destruction of a cell deposited on a filter membrane, determines the acceptable height of the filtered liquid. Assuming that force $\boldsymbol{F_g}$ on a filtration membrane in a single outflow channel is exerted by hydrostatic pressure $\boldsymbol{\Delta p = F_g/(\pi \cdot r^2)}$, then the acceptable difference in liquid column levels $\boldsymbol{\Delta h}$ is described by equation (12):

$$\Delta h = \frac{F_g}{\rho \cdot g \cdot \pi \cdot r^2} \tag{12}$$

Based on the above relations, force $\boldsymbol{F_g}$ in extreme cases determines the acceptable amount of liquid that can be applied to a filter without destroying the biological particles deposited on the filter. For the above condition, the volume of liquid in a single outflow channel of the proposed filtration model, based on equation (9), is described by equation (13):

$$V = \frac{A \cdot F_g \cdot t}{8 \cdot \pi \cdot \eta \cdot \Delta h} \tag{13}$$

where: $\boldsymbol{A = \pi \cdot r^2}$ — surface area of a single outflow channel of the filter.

At the same time, based on equation (9) and assuming that $\boldsymbol{V = A \cdot \Delta h}$, the difference in liquid levels in a single outflow channel of a membrane filter can be determined using equation (14):

$$\Delta h = \sqrt{\frac{A \cdot \Delta p \cdot t}{8 \cdot \pi \cdot \eta}} \tag{14}$$

Because the difference in pressures, resulting from the difference in liquid levels in the flood tank of $\Delta p = \rho \cdot g \cdot \Delta h$, liquid volume V in a single channel of the proposed filtration model, taking into account equations (12) and (13), can be determined using equation (15):

$$V = \frac{A^2 \cdot \rho \cdot g \cdot t}{8 \cdot \pi \cdot \eta} \tag{15}$$

Comparing equations (12) and (14), and assuming that $\Delta p = F_g/A$, the filtration time for a given value of force F_g is described by equation (16):

$$t = \frac{8 \cdot \pi \cdot F_g \cdot \eta}{\left(A \cdot \rho \cdot g\right)^2} \tag{16}$$

Furthermore, based on equation (12), due to the value of biological particle destruction force F_z, the threshold acceptable height H_g of the liquid filtered in a flood tank can be determined from the following relation:

$$h \ll H_g \leq \left(h + \frac{F_z}{A \cdot \rho \cdot g} \right) \tag{17}$$

where: H_g — maximum acceptable height of filtered liquid column in the flood tank.

Equations (12-17) are used as the basis for designing any microparticle gravity filtration process using membrane filters. They are also used to determine the acceptable volume, depending on the strength of the filtered biological particles, of the filtered liquid to estimate the maximum filtration time, and, due to the size of the filtration holes, to calculate the total filtration surface area, volume of the liquid filtered, and acceptable liquid column height in a single outflow channel.

5. Kinematic viscosity in assessing the physical properties of biological particles

Biological particle filtration models assume that the filtered bodily fluids are uniform, with averaged physicochemical properties, in spite of them being morphologically varied and exhibiting different biological activities [28]. This assumption is fully justified, because the physical properties of these liquids do not differ substantially [29-31]. Furthermore, due to the quantitative and weight proportions of the microparticles in a given volume of a bodily fluid [31], and due to the laminar nature of the separation of these particles, the following analysis

also assumes that bodily fluids are uniform, with averaged physical properties. At the same time, it is assumed that the bodily fluids are binary, and that they comprise a base fluid and microcells of constant dimensions, distributed evenly throughout the volume of the bodily fluid. Consequently, for known averaged rheological parameters of binary and uniform bodily fluids, there are significant amounts of microparticles of small dimensions, and due to the comparable specific gravities of these microparticles and of the base liquid of the bodily fluid, the weight of the separated biological particles can be estimated using the Stokes' method employed for determining liquid viscosity coefficients [32, 33]. Due to the small sizes of these particles and the requirements to maintain their biological activity, determining their mass using direct measurement methods presents significant difficulty. The proposed method assumes that the microparticles are spherical in shape, and that their mass only depends on their size, i.e. radius $\boldsymbol{r_k}$, the cell surface is smooth, and the number of cells in a measurement sample is proportional to the number of holes blocked by these cells in the measurement membrane. According to the definition of dynamic viscosity, which is characterised by inter-particle forces in the studied uniform liquid, a free-falling cell draws adjacent layers with it, which only move in relation to subsequent layers of the studied liquid with a certain resistance. The resistance to the movement of these layers at a specific speed under specific thermal conditions characterises the viscosity of the studied liquid. This resistance, which the free-falling cell meets in the standard base matrix, depends on: cell size, mass, fall velocity and, primarily, the viscosity of the medium where the movement occurs. According to Stokes' law [32] for a sphere falling in a liquid, the force of resistance is described by equation (18):

$$T = 6 \cdot \pi \cdot \eta \cdot r_k \cdot \vartheta \tag{18}$$

where: T — force of resistance of a free-falling cell in a liquid;
r_k — cell radius;
η — dynamic viscosity coefficient of the medium;
υ — constant fall velocity of a cell in the studied medium.

A cell of mass $\boldsymbol{m_k}$ and radius $\boldsymbol{r_k}$, moving at a constant speed in a standard liquid of density $\boldsymbol{\rho_c}$ and known dynamic viscosity, is affected by three principal forces:

— internal friction force $\boldsymbol{T}$ according to Stokes' law, equation (18),
— weight of the falling cell, equation (19),

$$F = m_k \cdot g = \frac{4}{3} \cdot \pi \cdot r_k^3 \cdot \rho_k \cdot g \tag{19}$$

where: ρ_k — mass density of the cell;
g — acceleration due to gravity;

— buoyancy force, equation (20).

$$F_w = \frac{4}{3} \cdot \pi \cdot r_k^3 \cdot \rho_c \cdot g \tag{20}$$

where: ρ_c — mass density of the medium (e.g. a uniform bodily fluid);
g — acceleration due to gravity.

In the case of steady cell movement in a gravity field, the equilibrium equation describing this movement can be expressed as follows:

$$T = F - F_w \tag{21}$$

Assuming that:

$$v = \Delta h / t$$

where: v — steady speed of cell fall in a single outflow channel of a molecular filter;
Δh — difference in test liquid levels in the flood tank;
t — filtration time for known volume $\boldsymbol{V}$ of the test liquid.

For cases where $\boldsymbol{\rho_k > \rho_c}$, the calibration viscosity of the test liquid can be expressed as follows [33]:

$$\eta_w = \frac{2 \cdot r_k^2 \cdot g}{9 \cdot \Delta h} \cdot \left(\rho_k - \rho_c\right) \cdot t \tag{22}$$

At the same time, based on equations (12) and (14), the following can be stated:

$$\frac{t}{\Delta h} = \frac{8 \cdot \pi \cdot \eta}{A \cdot \rho \cdot g} \tag{23}$$

where: $A = \pi \cdot r_k^2$

Based on equations (22) and (23), when $\boldsymbol{r \sim r_k}$ and $\boldsymbol{\eta \sim \eta_w}$, the averaged substitute mass density of test liquid (defined as a uniform suspension of microparticles and base liquid) $\boldsymbol{\rho}$ in a single outflow channel containing a single cell of diameter $\boldsymbol{r_k}$ can be calculated using equation (24):

$$\rho = \frac{16}{9} \cdot \left(\rho_k - \rho_c\right) \tag{24}$$

As indicated by equation (16), force $\boldsymbol{F}$ exerted by the liquid on the filtered cell in a single outflow channel is described by equation (25):

$$F = \frac{(A \cdot \rho \cdot g)^2 \cdot t}{8 \cdot \pi \cdot \eta} \tag{25}$$

If $\boldsymbol{r \sim r_k}$, $\boldsymbol{\eta_w \sim \eta}$, $\boldsymbol{V = A \cdot \Delta h}$ and taking into account equation (22), force $\boldsymbol{F}$ exerted by the liquid on the filtered cell in a single outflow channel at the filtration membrane height is described by equation (26):

$$F = \frac{9 \cdot V \cdot g \cdot \rho^2}{16 \cdot (\rho_k - \rho_c)} \tag{26}$$

While based on equation (24), the density of the material of a single cell can be determined from relation (27):

$$\rho_k = \frac{9}{16} \cdot \rho + \rho_c \tag{27}$$

Based on the definition of material density, described as the ratio of mass $\boldsymbol{m_k}$ to cell volume $\boldsymbol{V_k}$, cell mass $\boldsymbol{m_k}$ — after taking into account equation (27) — can be calculated using equation (28):

$$m_k = V_k \cdot \rho_k = \frac{4}{3} \cdot \pi \cdot r_k^3 \cdot (\frac{9}{16} \cdot \rho + \rho_c) \tag{28}$$

where: $\rho_k = \frac{m_k}{V_k}$; $V_k = \frac{4}{3} \cdot \pi \cdot r_k^3$

m_k — cell mass;

V_k — cell volume.

On this basis, weight-separated cell $\boldsymbol{G_k}$ in a single outflow channel is calculated using equation (29):

$$G_k = V_k \cdot \gamma_k = V_k \cdot g \cdot \left(\frac{9}{16} \rho + \rho_c \right) \tag{29}$$

where: γ_k — specific gravity of the separated cell;

g — acceleration due to gravity;

V_k — volume of the separated cell;

ρ — averaged mass density of the test liquid, i.e. microparticle and base liquid suspension;

ρ_c — mass density of the base liquid (e.g. a uniform bodily fluid).

Based on equations (26-29), and knowing the viscosity of the studied binary liquid (defined as a suspension of microparticles and base liquid) and the viscosity of the uniform bodily fluid (defined as a uniform base liquid), it is possible to make a preliminary estimation of the physical properties of the separated blood cells using a calibration liquid and a membrane filter of uniform transmission structure in 2D space.

6. Conclusion

The paper presents the basic relations describing the effects of viscosity and surface tension on the parameters of negative filtration involving binary and uniform bodily fluids in a gravity field. A single filtration membrane with a uniform transmission structure in 2D space was used to analyse the negative filtration process.

Based on the relations, it was possible to estimate the following basic parameters of the filtration process for such bodily fluids:

- level of variable pressure in the flood tank,
- maximum $\boldsymbol{H_g}$ and minimum height $\boldsymbol{h}$ of the filtered fluid for known dimensions of the flood tank,
- estimated duration of a stable filtration process,
- hydrostatic forces exerted by the filtered biological fluid on the biological particles deposited on the filter,
- volumetric flow rate of the filtered fluid for a known filtration membrane structure,
- gradient of biological particle flow velocity in the flood tank,
- filter transmission level in relation to a uniform filter membrane standard with comparable viscosity and density of the filtered base fluid.

The above parameters of the filtration process form a basis for broader studies into microparticle separation from bodily fluids, in particular cancer cells from peripheral blood for the purposes of oncological tests. Sample estimate calculations utilising the relations described in this paper for commonly known physical characteristics of water and peripheral blood as a binary and uniform bodily fluid are presented below:

6a) Estimation of minimum water column filtration height *h* in a flood tank

Input data:

- water surface tension $\boldsymbol{\delta_w = 71.97\ [N \cdot 10^{(-3)}/m]}$,
- filtration hole radius in a single filtration channel $\boldsymbol{r = 5\ [\mu m]}$,
- specific gravity of water $\boldsymbol{\gamma = 0.981 \cdot 10^{(-5)}\ [N/m^3]}$.

Based on equation (7):

$$h = \frac{6 \cdot \delta_w}{\gamma \cdot r} = 8.8\ [\text{mm}] \tag{30}$$

6b) Estimation of pressure force F_g of a liquid column with height Δh = (10 ÷ 100) [mm] on a cell located on the surface of a filtration membrane in a single filtration channel

Input data:
— blood density ρ_{blood} = **1.055 g/cm^3**,
— plasma density ρ_o = **1.024 g/cm^3**,
— filtration hole radius in a single filtration channel r = **5 [µm]**.

From equation (27), the estimated averaged density of a single cell ρ_k in a filtration channel is:

$$\rho_k = \frac{9}{16} \cdot \rho_{blood} + \rho_o = 1.6174 \left[\frac{\mathrm{g}}{\mathrm{cm}^3} \right] \tag{31}$$

Based on equation (26), the force of pressure of a liquid column of a height between 10 and 100 [mm] on a cell in single filtration channel F_g is:

$$F_g = \frac{9 \cdot V \cdot g \cdot \rho_{blood}^2}{16 \cdot (\rho_k - \rho_c)} = (0.8125 \div 8.125) \left[\mu\mathrm{N} \right] \tag{32}$$

6c) Estimation of the mass of a droplet formed at the outlet of a single outflow channel with a hole of radius r = 5 [µm]

Input data:
— water surface tension δ_w = **71.97 [N·10$^{(-3)}$/m]**,
— filtration hole radius in a single filtration channel r = **5 [µm]**,
— test liquid droplet mass m,
— standard liquid droplet mass m_w,
— acceleration due to gravity g = **9.81 [m/s^2]**,
— constriction radius for standard liquid r_w.

From equation (3):

$$F_f = 2 \cdot \pi \cdot r_w \cdot m \cdot \frac{\delta_w}{m_w} \tag{33}$$

and assuming that $m \sim m_w$ and $r \sim r_w$, mass of the test liquid droplet formed can be calculated using the relation:

$$m = \frac{F_f}{g} \tag{34}$$

On this basis, the mass of a water droplet formed at the outlet of a single filtration channel of radius r = **5 [µm]** is m = **130.45 [µg]**.

6d) Estimation of approximate blood filtration time for known filtration process parameters

Input data:
— dynamic viscosity of blood $\boldsymbol{\eta = 5.0 \cdot 10^{(-3)}}$ **[N·s/m²]**,
— force of pressure of a liquid column with height $\boldsymbol{\Delta h = 100}$ **[mm]** on a cell in single filtration channel $\boldsymbol{F = 8.125 \cdot 10^{(-7)}}$ **[N]**,
— acceleration due to gravity $\boldsymbol{g = 9.81}$ **[m/s²]**,
— filtration hole radius in single filtration channel $\boldsymbol{r = 5}$ **[µm]**,
— surface area of the filtration hole of a single filtration channel:

$$\boldsymbol{A = 78.5 \cdot 10^{(-6)}}\ \textbf{[mm}^2\textbf{]}$$

— blood density $\boldsymbol{\rho_{blood} = 1.055}$ **g/cm³**.
Based on equation (16), the blood filtration time for the given parameters of the filtration process is:

$$t = \frac{8 \cdot \pi \cdot F_g \cdot \eta}{\left(A \cdot \rho_{blood} \cdot g\right)^2} = 125.6\ [\mathrm{s}] \tag{35}$$

6e) Estimation of the averaged mass of a cell filtered in a single outflow channel for a binary and uniform bodily fluid

Input data:
— radius of the filtered cell $\boldsymbol{r_k = 5}$ **[µm]**,
— blood density $\boldsymbol{\rho_{blood} = 1.055}$ **g/cm³**,
— plasma density $\boldsymbol{\rho_o = 1.024}$ **g/cm³**.
Based on equation (28), the averaged mass of a single filtered cell in an outflow channel is:

$$m_k = \frac{4}{3} \cdot \pi \cdot r_k^3 \cdot \left(\frac{9}{16} \cdot \rho_{blood} + \rho_o \right) = 0.169\ [\mathrm{pg}] \tag{36}$$

The above calculated estimates and conclusions from the analysis of the negative filtration process may provide a basis for studies into biological microparticle separation and enrichment from bodily fluids, conducted mainly for the purposes of tests in the field of oncology.

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A. GÓRKA

Lepkość i napięcie powierzchniowe w procesie filtracji mikrocząstek biologicznych

Streszczenie. W artykule przedstawiono wpływ lepkości i napięcia powierzchniowego na podstawowe parametry procesu filtracji mikrocząstek biologicznych. Do analizy wykorzystano pojedynczą membranę filtracyjną o jednorodnej strukturze transmisji w przestrzeni 2D. Dla powyższego układu filtracji opisano zależności określające graniczną siłę filtracji, maksymalną i minimalną wysokość filtrowanej cieczy gwarantującą stabilne warunki procesu filtracji w polu grawitacyjnym, a także zależności opisujące czas procesu i siły działające na mikrocząstki w procesie filtracji. W podsumowaniu publikacji na podstawie tych zależności przedstawiono przykładowe obliczenia dla bazowego modelu negatywowej filtracji cząstek biologicznych.

Słowa kluczowe: filtracja mikrocząstek, sita filtracyjne, filtracja cząstek biologicznych, membrany filtracyjne, sedymentacja, lepkość kinematyczna, lepkość dynamiczna, napięcie powierzchniowe cieczy
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