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The use of the vibration method in testing steel load-bearing cables for the purpose of developing a method of measuring their state of tension

Abstract

In technical diagnostics of load-bearing tendons of bridge and building structures, vibration methods are popular due to simplicity, speed of results and low costs of their application. The aim of the research was to develop a method for measuring the load-bearing cables stress. The article presents the results of tests on the cable testing stand, which enable identification of the parameters of the cable dynamic model. In the research, a technique was applied consisting in impulse stimulation to vibrations of the tense steel cable and analysis of signals, which are a response of the cable to this excitation. Using the modal analysis method, the dependence of the frequency of various forms of the cable natural vibrations on the cable tensioning force was determined. Three types of cables were tested. The bending rigidity of the cable was determined. Using the modal analysis method as a reference method, the usefulness of a simpler method of evaluating cable natural frequency, was examined.

Keywords: technical diagnostics, vibration, cables stress, modal analysis.

1. Introduction

Steel cables become important load-bearing elements for bridge structures and building structures used as load-bearing tendons. They often work under difficult conditions of changing loads in a strongly corrosive environment, etc. For this reason they are subjected to wear and tear much faster than other elements. Their tension state has a decisive influence on the continuous failure-free operation and safety of their operation. Therefore, the availability of precise methods for testing and evaluating the tension condition of steel cables and, consequently, the safety of their operation is of great importance.

The most accurate method of measuring cable tension is a direct measurement with dynamometer. However, it is technically difficult and costly, as it involves interference in the cable structure, requiring a variety of highly qualified contractors [1]. Therefore, indirect methods of determining the tension force of the cables, which allow to avoid interference in the structure, are of great importance. Several methods are used in practice:

- method of measuring the tension forces on the cable bending principle [2],
- surveying methods,
- photogrammetric methods,
- vibration methods.

Each of them, in certain circumstances, has an advantage over the others. However, they also show a number of inconveniences and disadvantages, which makes it reasonable to undertake work on developing new methods aimed at improving and increasing the accuracy of stress measurement in linear load-bearing structures.

The paper is the first stage of research, whose aim is to develop a method for measuring the state of tension in the load-bearing tendons of bridge and construction structures, for the technical diagnostics of both monoline and multi-cable and cable systems. The scope of work presented in the article includes, important for further work, the stage of laboratory tests at a line inspection post.

2. Theoretical bases of vibration method and dynamic cable model for impulse stimulation

The growing interest in research into vibration methods and the practical application of their results is due to the fact that these methods are simple to use and inexpensive, enabling rapid results of measurements (non-destructive).In these methods, relationship between the values of modal frequencies and the axial force applied to the cable is used.

Vibration methods are indirect methods in which the estimation of cable axial force is based on measured values of cable natural vibration frequency. The full description of the dynamic properties of the oscillating cable also requires knowledge of the material and geometric parameters of the cables, such as cable flexural rigidity, axial stiffness, mass, length, deflection and slope of the cable (Fig. 1).

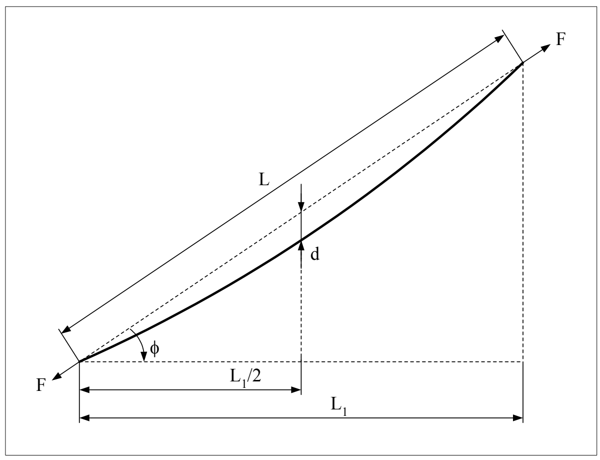

Fig. 1. Geometric parameters of the cable

Depending on the extent to which these parameters are taken into account in calculations, several models of dynamic cables can be distinguished, which form the theoretical basis of the method.

Model 1 - model based on the transverse vibrations of a tensioned string equation [3, 4, 5, 6]

$$F = 4mL^2 \left(\frac{f_n}{n}\right)^2, \quad (1)$$

where: F – tension force, N,
m – weight per unit length, kg/m,
L – length of cable, m,
f_n – nth natural frequency, Hz,
n – mode number.

This model does not takes into account the bending rigidity and deflection effect of the cable, which can lead to an estimation of the tension force with unacceptable high error. The usefulness of this model is limited to long cables with small cross-sections. The results obtained shall be treated as the first approximation of the estimated value of the tension force.

Model 2 - model based on the transverse vibrations of a tensioned beam equation [3, 4, 5, 6]

$$F = 4mL^2\left(\frac{f_n}{n}\right)^2 - \frac{EI}{L^2}(n\pi)^2, \quad (2)$$

where: EI – bending rigidity, N·m^2.

This model takes into account the bending rigidity of the cable, but still does not take into account the deflection effect of the cable, which may result in significant errors in estimating a cable tension under certain conditions. Despite this inconvenience, this method is often used in practice because of its simplicity and speed of obtaining results.

Model 3 - a set of practical formulas developed by Zui et al [7]

The method takes into account the influence of cable rigidity on bending and the effect of cable deflection on the determined value of cable tension force. It consists of a set of seven equations which refer to different geometric configurations of the cable, described by two coefficients defined by formulas:

$$\xi = \sqrt{\frac{F}{EI}} \cdot L, \quad (3)$$

$$\Gamma = \sqrt{\frac{mgL}{128EAe^3cos^5\phi}} \cdot \frac{0{,}31\xi+0{,}5}{0{,}31\xi-0{,}5}, \quad (4)$$

where: e – factor defined by the formula

$$e = \frac{d}{L_1} \text{(sag-to-span ratio)}, \quad (5)$$

ϕ– inclination angle,
g – gravitational acceleration.

Depending on the value of these coefficients, three cases are considered, allowing the selection of the right equation for the calculation.

Case 1 – $\Gamma \geq 3$, for small sag,

$$F = 4mL^2f_1^2\left[1 - 2{,}20\frac{C}{f_1^2} - 0{,}55\left(\frac{C}{f_1^2}\right)^2\right], \text{ for } 17 \leq \xi \quad (6)$$

$$F = 4mL^2f_1^2\left[0{,}865 - 11{,}6\left(\frac{C}{f_1^2}\right)^2\right], \text{ for } 6 \leq \xi \leq 17 \quad (7)$$

$$F = 4mL^2f_1^2\left[0{,}828 - 10{,}5\left(\frac{C}{f_1^2}\right)^2\right], \text{ for } 0 \leq \xi \leq 6 \quad (8)$$

Case 2 – $\Gamma \leq 3$, for large sag

$$F = 4mL^2f_2^2\left[1 - 4{,}40\frac{C}{f_2^2} - 1{,}10\left(\frac{C}{f_2^2}\right)^2\right], \text{ for } 60 \leq \xi \quad (9)$$

$$F = 4mL^2f_2^2\left[1 - 2{,}20\frac{C}{f_2^2} - 0{,}55\left(\frac{C}{f_2^2}\right)^2\right], \text{ for } 17 \leq \xi \leq 60 \quad (10)$$

$$F = 4mL^2f_2^2\left[1 - 2{,}20\frac{C}{f_2^2} - 0{,}55\left(\frac{C}{f_2^2}\right)^2\right], \text{ for } 0 \leq \xi \leq 17 \quad (11)$$

Case 3 – for very long cables

$$F = 4mL^2f_n^2\left[0{,}828 - 10{,}5\left(\frac{C}{f_n^2}\right)^2\right], \text{ for } 200 \leq \xi \quad (12)$$

for $n \geq 2$,

$$\text{where: } C = \sqrt{\frac{EI}{mL^4}}, \quad (13)$$

In the first case, referring to a low cable deflection, the value of the first eigenfrequency of the cable is used for the calculation. In the second and third cases, which refer to high cable deflection, the value of the force determined shows a high sensitivity to changes in the value of the cable's first eigenfrequency. As a result minor errors in the measurement of the cable's first eigenfrequency may cause significant errors in the cable tension value. It is therefore appropriate in these cases to perform calculations with the use of values of the second (or higher) modal frequencies of the cable [1, 7].

It follows from the above remark that using this method it is only possible to determine the values of the first two modal frequencies of the cable. This is important from a practical point of view, as these frequencies are easy to determine with acceptable uncertainty.

The above mathematical cable models have been selected for use in further studies, obviously taking into account the range of their applicability. Their choice was dictated by the fact that they may be easily implemented into the measurement procedure and the fact that these models were verified by numerous authors, as demonstrated by the publications with the results of the research [3, 5, 8].

3. Implementation of laboratory tests at a cable testing stand

The first stage of work was carried out on a skidding machine installed in the GIG Cable Testing Department. The aim of this step was to determine the dependency between different forms of eigenvibration of the cable and the tension force, and to develop a method of carrying out cable vibration measurements and signal processing in order to make it possible to use a portable meter of the cable tension in the project. Figure 2 shows the view of the test stand.

Fig. 2. Stand for testing cables

In this work, a technique of testing mechanical objects was applied, consisting in impulse stimulation to mechanical vibrations of the tensioned cable and then in measurement and analysis of vibration signals which are the response of the cable to this stimulation [9].

The cable was stimulated to vibration by an impulse method, using a large modal hammer type 086 D50 from PCB Piezotronics. The force pulse was applied perpendicular to the line axis. The hammer's point of impact was determined arbitrarily about 1.5 m from the fixed end of the cable. Cable vibrations in response to impulse stimulation were measured with a laser scanning vibrometer type PSV-400 from Polytec and with a piezoelectric vibration transducer type 4396 from Brüel&Kjær. Using a laser vibrometer, the cable vibration velocity was measured at several points evenly distributed along the cable length. A vibration transducer was used to measure the cable vibration acceleration at a point at a distance of approx. d = 80 cm ÷ 100 cm (depending on the length of the tested cable sample) from the fixed end of the cable. The transmitter was fixed to the cable by means of a metal fitting and a steel clamp (Fig. 3). Both the time history of the force and the time history of vibration signals of the line response to impulse excitation were recorded for further analysis.

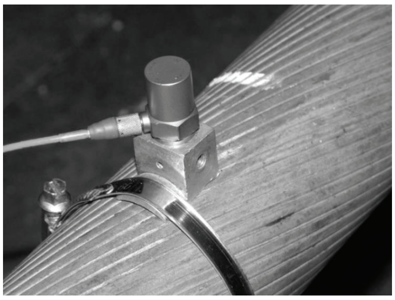

Fig. 3. Mounting of the vibration transducer to the tested cable

The exemplary time history of the force pulse and its frequency spectrum for cable 1 is shown in Figures 4 and 5.

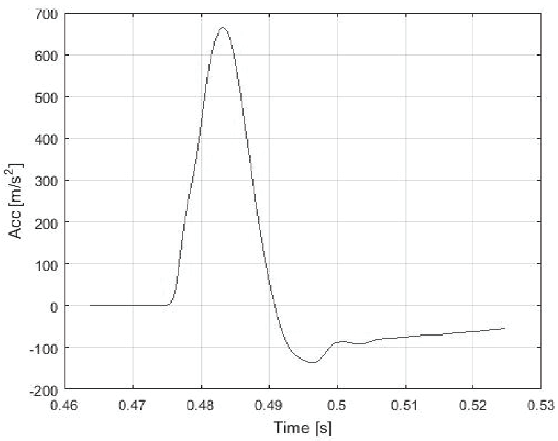

Fig. 4. Time history of force pulse

Pulse duration specifies the maximum frequency of excited vibrations. The duration time τ = 14 ms was obtained for the tip used. The pulse spectrum and consequently frequency analysis of the object vibration excited in this way is limited to a frequency of about f_{max} = 80 ÷ 90 Hz.

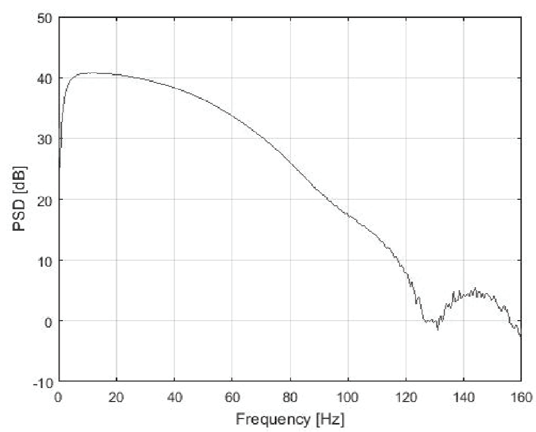

Fig. 5. Spectrum of force pulse

The tests were performed for prepared samples of three types of steel cables used in supporting structures. The basic technical data of the cables are shown in Table 1.

Tab. 1. Technical specifications of the tested cable samples

Cable number		1	2	3
Cable type		Lack of data	WS6x36-NFC	34x7-WSC
Diameter, *D*	mm	70	50	29
Length, *L*	m	5.05	4.65	4.10
Weight per unit length, m	kg/m	28.3	9.27	5.10
Bending rigidity, *EI*	N·m2	Lack of data	Lack of data	Lack of data

Each of the cable samples was subjected to different static loads in the form of tensile force. For each load, the frequency spectrum of the response of the cable to impulse stimulation was determined. Cable vibration damping is small (Fig. 6), so the spectrum has clearly marked maxima corresponding to the modal frequencies of the cable. Spectrum fragments corresponding to the first and second type of vibrations have a very good signal-to-noise ratio, which makes it possible to precisely determine the values of frequencies of these vibrations. For higher types of vibrations, the influence of noise is visible and the determination of frequency values is subjected to greater uncertainty (Fig. 7). This is caused by the limitation of the frequency analysis band, resulting from the duration of the pulse stimulation force referred to above.

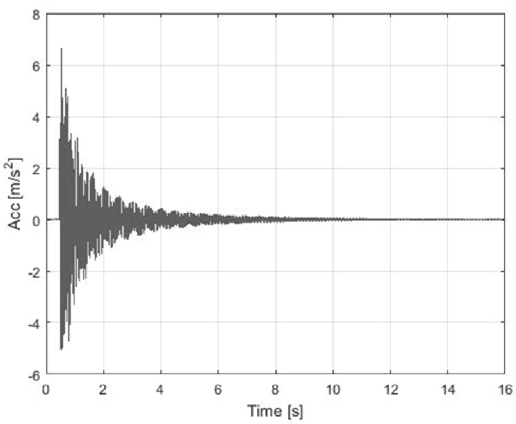

Fig. 6. Time history of cable response to impulse stimulation (cable no 2, F = 200 kN)

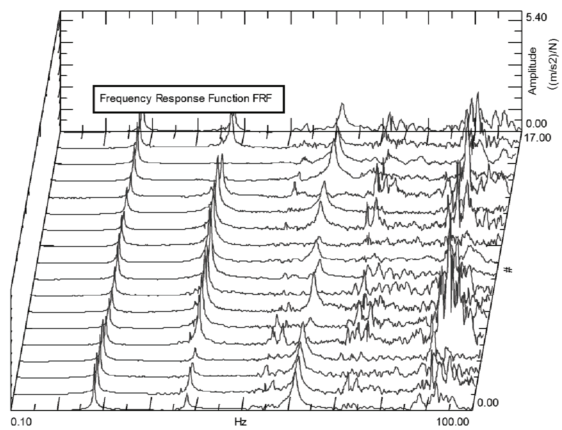

Fig. 7. Spectrum of cable response to impulse stimulation (cable no 2, F = 200 kN)

Using the method of modal analysis of signals measured with a laser scanning vibrometer, for each tensioning force the natural frequencies of the individual types of vibrations were determined. The results of the analysis are presented in Tables 2 and 3 as well as in Fig. 8. The presented results are the outcome of processing about 20 registrations, made in points evenly distributed along the tested cable.

Tab. 2. Frequencies of natural vibrations – cable no 1

F	f_1	f_2	f_3	f_4	f_5
kN	Hz	Hz	Hz	Hz	Hz
50	10.441	27.369			
200	13.067	32.135	39.113		
400	15.757	36.844	47.203	57.840	
600	17.872	40.611	53.462	63.272	76.172
800	19.727	43.872	59.523	68.033	83.510
1000	21.789	46.858	65.216	71.916	90.529

Tab. 3. Frequencies of natural vibrations of cables no 2 i 3

Cable no 2				Cable no 3			
F	f_1	f_2	f_3	*F*	f_1	f_2	f_3
kN	Hz	Hz	Hz	kN	Hz		Hz
100	13.50	29.29	48.90	150	26.24	53.36	82.57
200	17.93	38.18	61.34	200	29.59	59.94	92.21
300	20.70	43.29	69.03	250	32.82	66.33	101.71
400	23.51	49.03	77.10	300	35.57	71.78	109.81
500	26.32	53.69	83.89	350	38.16	76.90	117.33
600	28.53	57.96	90.19	400	40.38	81.26	123.78
700	30.67	62.22	93.18				
800	32.49	65.79	100.99				
1000	35.92	72.54	107.73				

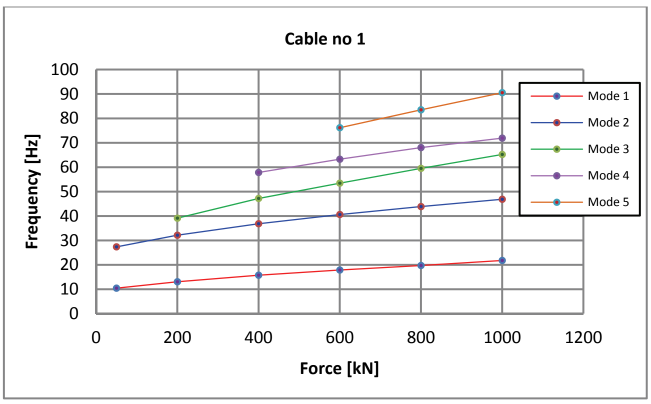

Fig. 8. Relationship between the natural frequencies and the axial load of the cable, evaluated by modal analysis method – cable no 1

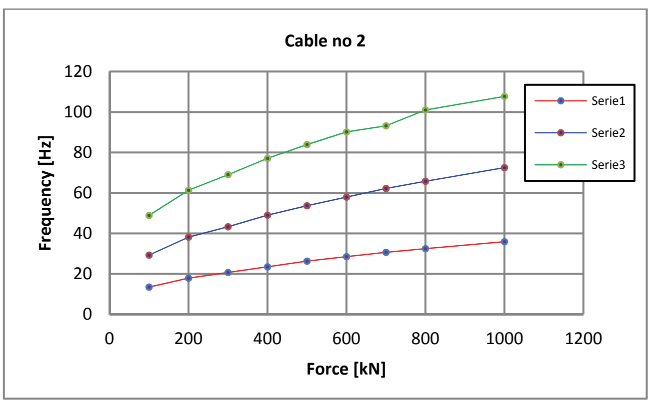

Fig. 9. Relationship between the natural frequencies and the axial load of the cable, evaluated by modal analysis method – cable no 2

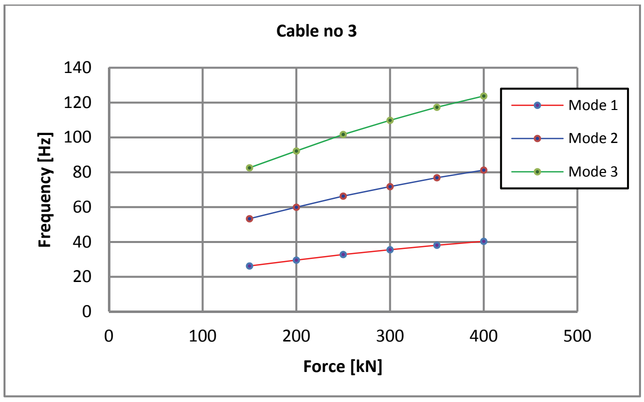

Fig. 10. Relationship between the natural frequencies and the axial load of the cable, evaluated by modal analysis method – cable no 3

The measured characteristics of $f_n = h(F)$ of the three cable samples were used to determine the values of bending rigidity of the cable *EI*, a parameter not usually specified in cable datasheets. Taking into account that the skidding machine is mounted horizontally, it has been assumed that the relatively short cable sections during measurements are in a situation where the cable has no slope($\phi = 0$) and is practically not deflected($d \approx 0$). This makes it possible to use in the calculations a relation describing a relatively simple model of cable i. e. model 2 (Formula 2), because errors resulting from model limitations are insignificant in this case.

The values of coefficients a and b of the linear function were determined using the linear regression method for the set of measurement data, which are characteristics of $f_n = h(F)$:

$$\left(\frac{f_n}{n}\right)^2 = a \cdot F + b, \tag{14}$$

where: $a = \frac{1}{4mL^2}$ (15)

$$b = \frac{n^2\pi^2}{4mL^4} \cdot EI \tag{16}$$

Therefore, the bending rigidity of the cable is given by the formula:

$$EI = b \cdot \frac{4mL^4}{n^2\pi^2} \tag{17}$$

The results of the calculations, for the second modal frequency ($n = 2$), for the three cable samples are given in Table 4.

Tab. 4. Values of the *EI* parameter determined on the basis of measurement data using the linear regression method

		Cable no 1	Cable no2	Cable no3
EI	kN·m^2	334.9	47.8	21.5
U(*EI*)	kN·m^2	4.2	1.1	0.3
$U_r(EI)$	%	1.3	2.3	1.4

In view of the planned construction of a portable cable tension measurement device, cable vibration measurements were performed using a piezoelectric transducer at one measurement point. The values of the natural vibration frequencies were determined on the basis of peak-piking vibration spectrum analysis. Tables 5, 6 and 7 show the results of comparison of eigenfrequency values obtained from modal analysis and spectrum analysis using the peak-piking method of cable vibrations. The given results are average values of the series of approximately 20 measurements. Modal analysis results were treated as reference results.

The results of the comparison indicate that the modal frequency values obtained by using different methods of measuring cable vibrations and different signal processing methods are highly consistent. In particular, these results indicate the possibility of using a simpler method for determining the intrinsic frequency of a cable, which is the peak-piking method, in the design of a portable measuring instrument. The method is simpler from the point of view of performing cable vibration measurements and vibration signal processing algorithm.

Tab. 5. Results of comparison of values of natural frequencies obtained by modal analysis method and by peak-piking method of spectrum analysis – cable no 1

f_{1_mod}	$f_{1\_peak\text{-}piking}$	Błąd	f_{2_mod}	$f_{2\_peak\text{-}piking}$	Error
Hz	Hz	%	Hz	Hz	%
10.441	10.50	0.6	27.369	27.92	2.0
13.067	13.00	–0.5	32.135	32.00	–0.4
15.757	15.92	1.0	36.844	36.78	–0.2
17.872	18.00	0.7	40.611	40.50	–0.3
19.727	20.00	1.4	43.872	44.00	0.3
21.789	21.50	–1.3	46.858	47.00	0.3

Tab. 6. Results of comparison of values of natural frequencies obtained by modal analysis method and by peak-piking method of spectrum analysis – cable no 2

f_{1_mod}	$f_{1\_peak\text{-}piking}$	Błąd	f_{2_mod}	$f_{2\_peak\text{-}piking}$	Error
Hz	Hz	%	Hz	Hz	%
13,503	–2.1	2.1	29.291	29.75	16
17,928	–3.2	3.2	38.180	38.75	1.5
20,697	–1.5	1.5	43.292	43.75	1.1
23,505	–1.6	1.6	49.032	49.50	1.0
26,315	–1.1	1.1	53.686	54.00	0.6
28,525	–0.8	0.8	57.962	58.09	0.2
30,668	–1.2	1.2	62.220	62.76	0.9
32,486	–0.8	0.8	65.790	66.21	0.6
35,922	–0.2	0.2	72.540	72.68	0.2

Tab. 7. Results of comparison of values of natural frequencies obtained by modal analysis method and by peak-piking method of spectrum analysis – cable no 3

f_{1_mod}	$f_{1\_peak\text{-}piking}$	Błąd	f_{2_mod}	$f_{2\_peak\text{-}piking}$	Error
Hz	Hz	%	Hz	Hz	%
26.235	16.75	1.0	53.361	34.71	0.7
29.585	22.50	1.3	59.935	46.00	0.5
32.824	26.50	0.5	66.330	53.78	–0.5
35.571	26.50	0.9	71.777	53.75	–0.4
38.158	29.98	0.5	76.899	60.25	0.4
40.380	33.00	1.5	81.255	66.00	1.2

4. Summary and conclusions

In the study of cables, the impulse method of excitation of cable vibrations and the modal analysis method of the cable response to this stimulation were used. As a result, the frequency dependencies of different forms of modal vibrations on the tensioning force of the cable were determined for three types of cables.

Based on the measured dependence of modal frequencies on the tensioning force of the cable, a method of determining a numerical value of cable bending rigidity (*EI*) using a method of linear regression in the processing of obtained measurement data was developed.

On the basis of the results of comparative tests with the modal analysis method, a simpler method of determining modal frequencies of the cable on the basis of spectrum analysis by using the peak-piking method was found more useful.

The use of the modal hammer tip (grey) in the impulse method of cable excitation to vibrations limited the frequency analysis band to approx. 90 Hz. This allowed to determine the first two modal frequencies of the tested cables. For the tested types of cables and the model used in the calculations, the number of modal frequencies was sufficient. For other types of cable or where more modal frequencies have to be determined, it is necessary to increase the frequency analysis bandwidth of the cable vibration signals by using a modal hammer tip of a higher hardness.

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