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SIMULTANEOUS OPTIMIZATION OF FLOTATION COLUMN PERFORMANCE USING GENETIC EVOLUTIONARY ALGORITHM

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Abstract: Column flotation is a multivariable process. Its optimization guarantees the metallurgical yield of the process, expressed by the grade and recovery of the concentrate. The present work aimed at applying genetic algorithms (GAs) to optimize a pilot column flotation process which is characterized by being difficult to be optimized via conventional methods. A non-linear mathematical model was used to describe the dynamic behavior of the multivariable process. The solution of the optimization problem using conventional algorithms does not always lead to convergence because of the high dimensionality and non-linearity of the model. In order to deal with this process, the use of a genetic evolutionary algorithm is justified. In this way, GA was coupled with the multivariate non-linear regression (MNLR) of the column flotation metallurgical performance as a fitting function in order to optimize the column flotation process. Then, this kind of intelligent approach was verified by using mineral processing approaches such as Halbich's upgrading curve. The aim of the optimization through GAs was searching for the process inputs that maximize the productivity of copper in the Sarcheshmeh pilot plant. In this case, the simulation optimization problem was defined as finding the best values for the froth height, chemical reagent dosage, wash water, air flow rate, air holdup, and Cu grade in rougher and column feed streams. The results indicated that GA was a robust and powerful search method to find the best values of the flotation column model parameters that lead to more reliable simulation predictions at a reasonable time. Based on the grade–recovery Halbich upgrading curve, the MNLR model coupled with GA can be used for determination of the flotation optimum conditions.

Keywords: *flotation column, optimization, genetic algorithm, non-linear regression, upgrading curve*

Introduction

Column flotation is a widely used process for the concentration of low grade ore as well as recycling and solvent extraction. As a consequence of the widespread flotation circuits that have occurred during the past few years, there has been a rapid growth in

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practical knowledge in relation to column design, structure, operation, optimization, and control. Flotation processes are difficult to optimize at a fundamental level, and at present automatic monitoring and control of industrial plants have met with limited success. Practically, these processes are most often controlled by human operators who tend to evaluate the performance of the plant based on their own experience and other heuristic rules. Full potentials of plants are usually not used optimally, owing to lack of experience on the part of the operators, human error, etc. Indeed, considerable variation is sometimes observed between different shifts, or during different times of the day. These operational instabilities are considered to play a significant role in the cost-effective operation of flotation plants.

Some attempts have recently been made at the development of decision support systems that would assist the operator controlling the plant. Although these systems have met with varying degrees of success, it was only with the advent of on-line sensors that effective data-driven development of expert systems could be initiated. Due to the lack of reliable techniques for optimizing variables on-line in the industrial and pilot environments, as well as complexity and difficulty of modeling flotation processes, new methods such as artificial intelligence (AI) must be employed. Mineral processing is a field that has seen limited application of AI although there are certainly some successful examples.

Achieving the maximum grade and recovery of concentration of column flotation is an important research topic that a mineral processing plant is planned to reach by optimization of the operation conditions. This issue is to minimize the distance of a desired metallurgical performance with real observations in the plant. Despite the considerable investment in the study of this issue and the numerous reported studies, there is a still lack of knowledge about operation optimal conditions for the flotation column. The description of the optimum conditions is a complex task. The transient behavior of this process is even more difficult to explain, being the dynamic model constituted by a large number of differential equations with several difficult parameters or impossible estimation (Vieira et al., 2005). To overcome the difficulties in the development of optimizing models, the development of intelligent models based on experimental data is used. One of the most commonly used techniques is called genetic algorithm (GA).

The performance of the flotation column is determined by grade and recovery, but a key determinant is the optimization of the parameters that constitute the flotation operation. Optimization guarantees that the column operation reaches the reference values necessary for the desired recovery and grade of the concentrate stream. According to previous investigations, GA is a powerful tool for optimizing the complex processes such as the flotation column. The metallurgical performance of the column flotation is a function of a broad group of variables present in flotation: chemical dosage, gas holdup, froth height, feed characteristics, solids content, and air, and wash water flow rates, etc., Therefore, the knowledge of the independent variables should be useful at the time of optimizing and controlling the operation of the flotation column.

This paper shows the capability of an evolutionary algorithm which was coupled with the multivariate non-linear regression (MNLR) model in order to optimize the flotation column operation parameters. The multiple regression methods is a powerful modelling technique frequently used to make predictions from several independent variables. The proper selection of regression techniques is one of the most important factors to the success of prediction modeling. Since most of the regression algorithms currently available do not directly consider interaction effects during the modeling process, the interaction terms must be subjectively determined prior to performing a regression analysis. Various non-linear models were fitted to data and for each model residual analysis were performed by plotting predicted vs. observed values.

Researchers based on statistical techniques usually treat, incorrectly, grade and recovery independently. Moreover, generally, engineers would like to see the recovery-grade curves to understand how far from the ideal conditions. Therefore, it is essential to use upgrading curves (Drzymala, 2006; 2007), that are plotted as quality vs. either quantity or quality of separation products (Drzymala et al., 2012). One of the mostly used upgrading curves is the Halbich grade-recovery curve. The Halbich plot is practical and useful as well as has many advantages over other upgrading curves because of using recovery and grade, which are generally used in industrial, liberation, kinetic, and theoretical studies (Drzymala et al., 2012). Therefore, in this paper, the optimization process in terms of mineral processing was analyzed using the upgrading curve.

In this study, the proposed method was validated using data from a case study done on the samples of the flotation column at the Sarcheshmeh copper pilot plant. The rest of this paper was organized as follows: one section discusses GA concepts and its related studies in mineral processing. A next section briefly reviews the experimental work and introduces the Sarcheshmeh pilot plant. Finally, the results of the proposed approach and conclusions of this study were taken into consideration.

Background

GAs have been applied successfully in many manufacturing and engineering areas such as economics, control, optimization, electrical machining process etc. (Goldberg et al., 1989; Holland, 1975; Deb, 1995; Chang et al., 2006; Victorino, et al., 2007). It consists of a population of artificial agents imitating the animals’ behavior in the real world. GAs are inspired by evolutionary biology. Since GAs have the global searching ability, and they can also be easily implemented, they are widely used in many areas (Chen et al., 2011). In recent years, GA methods have been successfully presented by several researchers for the optimization of crushing, grinding, and flotation plants (Barone et al., 2002; Karr, 1993, 1996, 1997; Venter et al., 1997; While et al., 2004; Hasanzadeh and Farzanegan, 2011).

Venter et al. (1997) proposed an approach from the field of GA for flow sheet design. The strength of the work was that circuits could be assembled with the GA approach. However, full process optimization of the assembled circuit remained elusive. Karr et al. (1996, 1997) used a combination of fuzzy logic and GA for three different applications in mineral processing, a grinding process and size separation process for hydrocyclone and froth recovery maximization for the flotation circuits. The process parameters were controlled using a fuzzy logic based model and GA was used to determine the necessary condition for the optimal performance of the processes. Svedensten and Evertsson (2005) presented a successful GA for the optimization of crushing plant operation. In this work, a novel method for the modelling and optimization of crushing plants was shown based on the structural modeling of crushing plants and parameter optimization. Structural modelling was performed by utilizing mathematical models of the different production units, rock materials, and economics of the crushing plant. The efficiency of the proposed algorithm was demonstrated concerning a crushing plant which contained a small number of machines. Gupta et al. (2007) developed a plant optimization technique using GA to maximize the overall revenue generated by a coal preparation plant by searching the best possible combination of overall yield and multiple product quality constraints.

Over the years, considerable progresses have been made in various aspects of design of flotation circuits, especially in finding the optimal number of flotation cells. Guria et al. (2005, 2006) used GA for the optimization of the performance of flotation circuits. In these studies, GA allowed the evolution of an initial population of circuit solutions by numerical operations which simulated the probabilities of reproduction, crossing or mutation. An extensive review by Mendez et al. (2009) on the conceptual design of flotation circuits provided a solid base to compare various approaches taken to solve this multi-objective optimization problem. Rezende et al. (2008) employed GAs to optimize an industrial chemical reactor. The results illustrated that the GAs were successfully used in the process optimization. Ghobadi et al. (2011) used GA for the optimization of the performance of flotation circuits. The algorithm was applied for two optimization examples with the objective of achieving a desired concentrate grade within a specific total cells volume. The comparison of the results with the published data indicated that the proposed oriented GA reduced the calculation time by 1/60 for a two-stage flotation system, and provided a simpler circuit with a similar performance.

Genetic algorithm

Recent advances in computer hardware and software allowed researchers to develop new search strategies to be used in function optimization problems. Therefore, it is now possible to better integrate optimization algorithms into simulation packages such as GA. Nowadays, the optimization has been used in many applications, including transportation, biological and medical sciences, business, computer science, engineering, and social science to solve real process problems. GA has also found various applications in mineral processing, including process control, circuit design, pattern recognition of multivariate data, optimization of parameters, crushing, and comminution. GAs are global optimization search algorithms inspired by Darwin's theory of survival of the fittest because it makes an analogy with biological evolution (Wang, 2005). Holland (1975) introduced an optimization procedure that mimicked the process observed in natural evolution called GAs. The GAs approach starts with a random population of chromosomes that are a set of solutions for the optimization.

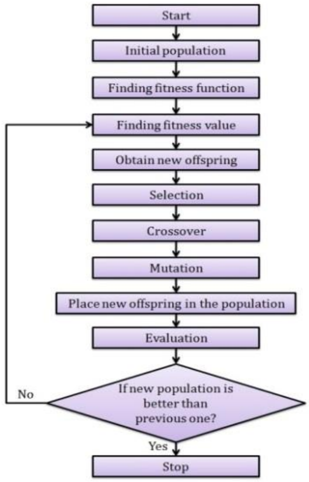

Fig. 1. Flowchart of GA procedure

Selection

While performing the selection operations, the system uses either the tournament selection or the roulette wheel selection to select chromosomes with higher fitness values into the gene pool to perform the crossover operation. The purpose of the selection operation is that the chromosomes with larger fitness values are chosen to participate in the production of the next generation (Baker, 1985).

Crossover

Crossover operator consists of picking up two chromosomes as parents from the mating pool at random and exchanging some portion of the solutions between themselves. The system compares a randomly generated number with a predefined crossover rate between 0 and 1. If the randomly generated number is smaller than or equal to the crossover rate, then the system randomly chooses a crossover point and exchanges the genes after the crossover point to generate two offsprings. Then, the system puts these two offsprings into the gene pool, where the size of the gene pool is equal to the size of the population. Otherwise, the system puts these two chromosomes back into the gene pool (Costa et al., 2005). The crossover operation will be performed repeatedly until the gene pool is full. The goal of the crossover operation is to spend the solution space in order to search for better solutions (Chen and Chien, 2011). Previous experiences showed that a good crossover rate usually was set around 0.7–0.8 (Shopova et al., 2006).

Mutation

After the crossover operation, the system chooses a certain number of chromosomes from the gene pool to carry out the mutation operation. The system compares a randomly generated number with a predefined mutation rate between 0 and 1. If the randomly generated number is smaller than or equal to the mutation rate, then the system randomly selects some genes of the selected chromosome to mutate, i.e., to alter the values of the genes. Then, it puts the mutated chromosome back into the gene pool. Otherwise, it keeps the chromosome unchanged, and puts it back into the gene pool. After the mutation operation, the chosen chromosomes will be put back into the gene pool. It should be noted that not every generation will do the mutation. Mutation is forced for some newly formed children in order to prevent all solutions from converging to their particular local optima (Chen and Chien, 2011).

Calculate the fitness value

After the crossover and the mutation processes, the GA will calculate the fitness value of each chromosome by a fitness function, where chromosomes with higher fitness values will be selected from the gene pool for reproduction in the next generation. The GA stops if the terminated condition or the maximum number of generations is achieved. Otherwise, it will return to the selection process (Chen and Chien, 2011).

Experimental work

Description of pilot plant

The Sarcheshmeh copper ore body, which may be ranked as the third or fourth largest in the world, contain 1 petagram of ore averaging 0.90% copper and 0.03% molybdenum (Nakhaei et al., 2012). The Sarcheshmeh pilot plant can process 1.6 Mg/h of ore. The main responsibility of the pilot plant is to find the optimum operating conditions (i.e., reagent type and dosage, pH value, grind size, etc.) in the processing of various ore types, and to evaluate any changes in the circuit before implementing in the plant. The rougher flotation bank consists of 14 cells (35 dm^3 each) in three units, and the regrind mill is a 76.2 cm by 137.2 cm ball mill. The scavenger banks have 6 cells (30 dm^3 each). The single-stage flotation column operation employed in the cleaner circuit was composed of a column with 26 cm internal diameter and 540 cm height. Figure 2 illustrates the flotation circuit examined in this study. The pilot flotation column was equipped with flow meters for feed, wash water, and air as well as with a conductivity profile. Local control loops were implemented to regulate feed, tails, wash water, and air flow rates. Two 23 cm long spargers were used, which were made of PVC tubes. The holes of 1.5 mm in diameter in a grid with dimensions of 2.5 cm×2 cm were drilled. The air flow rate was measured by a mass flow meter and controlled by a pneumatic control valve of globe type. The feed was transferred to the column 1.8 m down from the top through a 3.75 cm diameter pipe which was connected to the middle of the column.

Figure 3 illustrates a schematic diagram of the flotation column employed in this study. The wash water flow rate was measured using an electromagnetic flow meter, and was controlled by a pneumatic valve. Additionally, the air flow rate was measured by a mass flow meter, and was controlled manually using a flow meter. The pulp feeding was controlled via a pneumatic valve, and was measured by an electromagnetic flow meter. The pulp-froth interface position was measured using a semi-analytical method according to the conductivity profile along the column. The conductivity profile sensor consisted of 30 ring electrodes mounted on a 1.8 m long stainless steel tube with a diameter of 1.5 cm. The non-floated flow rate was also controlled by a variable-speed peristaltic pump driven by a frequency inverter. The pressure measurements were used to calculate the values of the air holdup and of the froth layer height. The data-acquisition system was also connected via a port to a microcomputer.

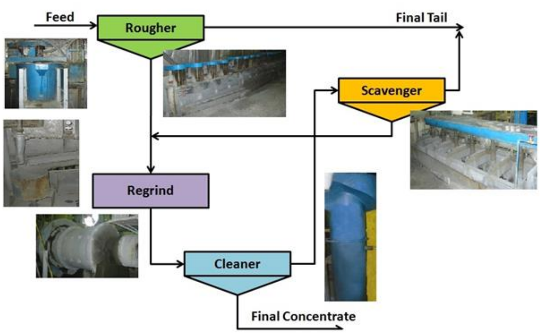

Fig. 2. Flow sheet of the flotation circuit of the Sarcheshmeh pilot plant

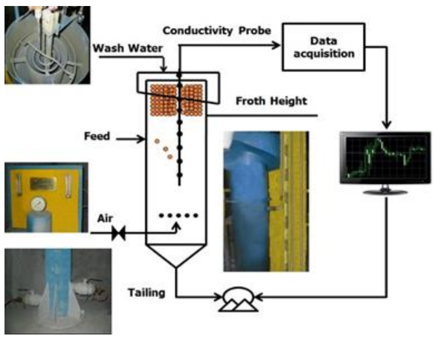

Fig. 3. Schematic diagram of the flotation column employed in this study

Experimental data

The flotation experiments were carried out in the Sarcheshmeh pilot plant (Fig. 2). The experimental work (four tests) was conducted under different operational conditions. Each experiment consisted of a series of step disturbances of several variables. The most important step in the developing of an MNLR model was to collect the data that could be employed for the modelling. Therefore, a series of reliable pilot data was collected over a period of 13 min based on RTD (Nakhaei et al., 2012) in order to cover the fluctuations in all the measured variables related to the metallurgical performance prediction the flotation column. A total of 90 data pairs were selected from the experimental database. The simultaneous measured variables were chemical reagents dosage, froth height, air and wash water flow rates, gas holdup, Cu grade in the rougher feed and column feed streams as independent variables as well as Cu grade and recovery in the final concentration stream as dependent variables. Similarly, the column tail stream was measured for the calculation of recovery, simultaneously. The ranges of the input and output variables for the metallurgical performance of mathematical formulation (MNLR model) of the 90 samples are shown in Table 1.

Table 1. Maximum and minimum of variables used in MNLR

Variables	Range	Mean	Std.	Index
Froth height (cm)	35-120	83.61	20.52	X_1
Collector dosage (g/Mg)	36-40	38	1.64	X_2
Frother dosage (g/Mg)	32-36	34	1.64	X_3
Air holdup (%)	71-92	82.36	4.1	X_4
Air flow rate (cm/s)	0.63-1.72	1.1	0.25	X_5
Wash water flow rate (cm/s)	0.11-0.4	0.27	0.08	X_6
Cu grade in the rougher feed (%)	0.77-0.93	0.82	0.04	X_7
Cu grade in the flotation column feed (%)	6.95-11.96	8.89	1.22	X_8
Cu grade in the flotation column concentrate (%)	15.93-25.21	21.13	2.12	F_c
Cu recovery in the flotation column (%)	83.34-91.27	87.33	1.75	R_e
Cu grade in the flotation column tail (%)	1.05-2.68	1.81	0.45	-

In all tests, the rougher feed flow rate was kept at 1.6 Mg/h. The particle size characterization and solid percent data of the sample are presented in Table 2. Before the flotation, the pulp was first conditioned with Z11 (sodium isopropylxanthate) and Nascol 451 (a mixture of mercaptobenzothiazole and sodium di-n-butyldithio-phosphate) as collectors, and MIBC (methylisobutylcarbonol), Dowfroth 250 (methyl -terminated polypropyleneglycols) as frothers. The pH was adjusted to 11.8 with lime. The frothers and collectors were added into the rougher cells and ball mill (before the flotation circuit), respectively. The chemical analysis and mineralogical composition of the samples showed that the ore contained 1.78% $CuFeS_2$, 0.27% Cu_2S, and 0.083% MoS_2.

Table 2. Flotation conditions used in the experiments (pH=11.8)

Parameter	Rougher feed	Column feed	Final concentrate	Final tail
Solid (%)	27.0	14.0	14.5	28.0
–0.044 mm (# –325 mesh), %	48.0	85.0	74.0	54.0

Results and discussion

Multivariate non-linear regression

In this study, the MNLR models were used. A method that is suitable for this procedure is called the iterative nonlinear least squares fitting. This process minimizes the value of the squared sum (SS) of the difference between data and fit. However, it is different from the linear regression which is an iterative or cyclical process. This involves making an initial estimate of the parameter values. The initial parameter estimates should be based on prior knowledge of the data or a sensible guess based on the function used to fit the data. The first iteration involves computing the SS based on the initial parameter values. The second iteration involves changing the parameter values by a small amount and recalculating the SS. This process is repeated several times to ensure that changes in the parameter values resulted in the lowest possible value of SS. The various non-linear models were fitted to data, and for each model residual analyses were performed by plotting predicted vs. observed values by SPSS 19. The performances of the models developed in this study were assessed using various standard statistical performance evaluation criteria. The statistical measures considered were the correlation coefficient (R) as seen in Eq.1:

$$R = \frac{n(\sum XY) - (\sum X)(\sum Y)}{\sqrt{(n\sum X^2 - (\sum X)^2)(n\sum Y^2 - (\sum Y)^2)}} \tag{1}$$

where X stands for predicted values, Y experimental values, and n is the number of datasets. Ultimately, the model with the highest values of the correlation coefficient and the least error between observed and predicted values was selected as the target model. A total of 90 sets of data were used in the metallurgical performance prediction of the flotation column by MNLR. In the first stage, 60 data sets were employed for arranging equations stage. When the arrangement was completed, the empirical models were validated for its generalization capabilities. The validation for its generalization ability was carried out by investigating its capability to predict the output sets that were not included in the arranging process. For this purpose, about 30 new databases were selected. MNLR equations were developed for the prediction of the Cu grade and recovery, which were employed for the fitness function in GA by 60 data set as the following equations:

$$Fc = \frac{(0.236 \cdot \exp(X_1^{0.199})) + (X_8^{0.209} \cdot X_4^{0.165} \cdot \exp(X_6 \cdot 0.909))}{(0.008 \cdot \exp(X_7) \cdot 0.762 \cdot X_5^{-0.608}) + (X_2^{-0.385} + X_3^{0.1})^{-2.288}} \tag{2}$$

$$\mathrm{Re} = \frac{(4.267 \cdot \exp(X_5^{0.062}) \cdot (X_4^{0.25} \cdot X_7^{0.05} \cdot \exp(X_8^{0.006}))}{(0.01 \cdot ((\exp(X_1^{0.213}) + 0.5 \cdot (X_6^{0.5}))) + (X_2^{-0.067} \cdot X_3^{0.058})} \tag{3}$$

These models were validated using the analysis of variance (Table 3) at a confidence level of 95%.

Table 3. ANOVA obtained from the SPSS output

	Fc		
Source	Sum of squares	*df*	Mean squares
Regression	40499.14	10	4049.9
Residual	72.92	80	0.91
Uncorrected total	40572.06	90	
Corrected total	400.54	89	
	Re		
Source	Sum of squares	*df*	Mean squares
Regression	686682.62	11	62425.69
Residual	84.49	79	1.07
Uncorrected total	686767.11	90	
Corrected total	274.66	89	

The correlations between observed and predicted values via the use of the proposed mathematical models at arranging equations stages are shown in Fig. 4. The distributions of difference between predicted Cu grade and recovery, and actual amounts at arranging equations stages are shown in Fig. 5. The results of agreement between measured and predicted values, and prediction error values at validation stage are shown in Figures 6 and 7.

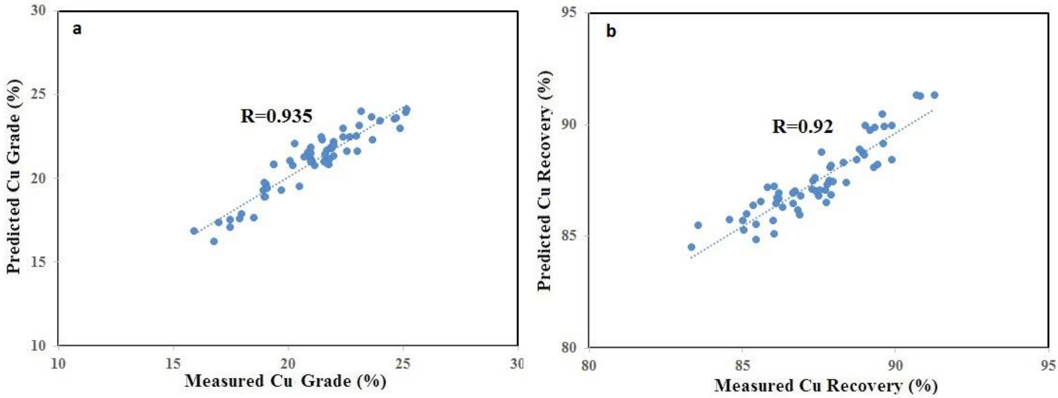

Fig. 4. Correlation between estimated and observed values at arranging equations stage (a) Cu grade (b) Cu recovery

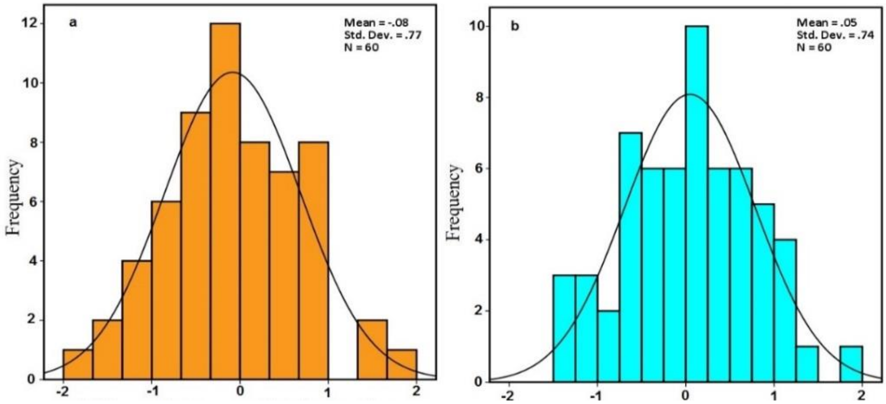

Fig. 5. Distribution of the difference between predicted (a) Cu grade and (b) recovery and actual values at validation stage using new data

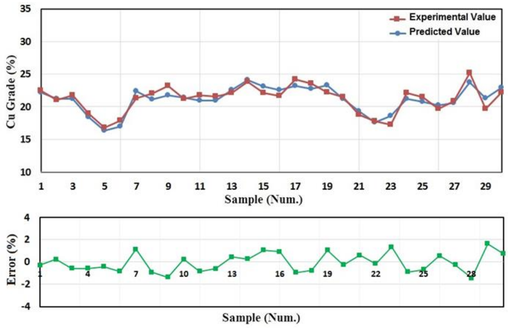

Fig. 6. Comparison of predicted Cu grade versus actual values and estimation error values by MNLR in validation process

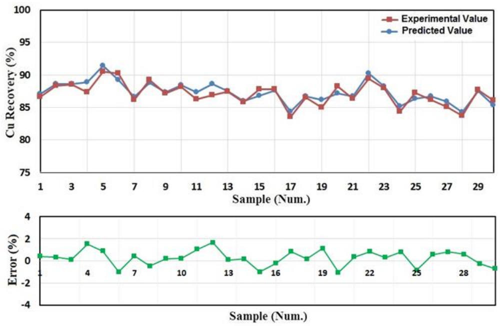

Fig. 7. Comparison of predicted Cu recovery versus actual values and estimation error values by MNLR in validation process

The initial population will be modified to reach a better answer. At each step, the GA selects individuals (chromosomes) from the current population (parents) randomly, and uses them to produce the children for the next generation. After several generations, according to essence of the GA, it tries to move to the best solution. At each step, the GA uses three main types of rules to create the next generation from the current population. The GA procedure performs four operations, i.e., selection operations, crossover operations, mutation operations, and evaluation operations to search the near optimal solution (Shopova et al., 2006; Abedini et al., 2011). The selection operator chooses the best solutions in the population. The selection occurs with a given probability on the base of fitness functions. The fitness function plays a role of the environment to distinguish between good and bad solutions. A particular group of parents is selected from the population to generate offspring by defined genetic operations of cross-over and mutation. The fitness of all the offspring is then evaluated using the same criterion, and the chromosomes in the current population are then replaced by their offsprings based on a certain replacement strategy (Chen and Chien, 2011). This procedure is repeated through iterations (or generations) until a termination criterion is satisfied. The main operations of GA are shown in Figure 1.

Table 4 depicts the prediction results of the model in the validation process. The MNLR equations predicted the Cu grade and recovery with the correlation coefficients of 0.907 and 0.898, respectively, at the validation stage using new data.

Table 4. Statistical performance evaluation criteria for the proposed model performance in validation process

Predicted variable	Cu grade				Cu recovery			
Parameter	R	RMSE	Max	Min	R	RMSE	Max	Min
MNLR	0.907	0.870	1.650	-1.440	0.890	0.835	1.680	-1.040

According to the above significant results, it can be concluded that the proposed multiple nonlinear regression formulas yield significant predictions of the grade and recovery.

Calculation of fitness function

The principal goal of this pilot plant circuit was to generate a product of the desired grade and recovery, i.e. 25% and 89%, respectively. This goal could obviously be achieved relatively simple by optimizing the flotation column. The issue described above was well suited to an evolutionary computation approach. The problem couldn't easily be described analytically, but a GA model was available that could be used to find out the optimum conditions. The search space was too large for an exhaustive search, and there was little to guide an engineer in determining good condition for a given scenario. The fitness function must be defined by the user as a normal MATLAB file, and its handle is entered into GA. As mentioned previously, a general MNLR equation was proposed using 8 independent variables in order to optimize condition operation of the flotation column.

In the present work, the optimization of a flotation column was performed in order to achieve desired values of copper grade and recovery. Solving Eq. (4) provided desired values of Cu grade and recovery of the concentration in the flotation column. Therefore, the simplified objective function could be defined as seen in Eq. 4:

$$\text{Fitness function} = \left[(25 - F_C)^2 + (90 - \mathrm{Re})^2\right] . \tag{4}$$

As the rule of thumb, optimal Cu grade and recovery are set at level 25 and 90%, respectively, in the Sarcheshmeh pilot plant. In other words, the optimization problem was defined as: what should be the operation conditions to the circuit, so that the Cu grade and recovery of final concentration will be equal to 25 and 90%, respectively? Applying algorithms based on traditional gradient to optimize problems with several local optimums may result in getting trapped in local optima. In these methods, finding the appropriate initial estimates of the parameters which lead to the convergence of the global optimum could be difficult. To overcome these limitations, various approaches based on evolutionary optimization algorithms have been developed. A reliable optimization method which was used in this study was GA. To check the claim that GA is suitable for optimization in this case, the convexity of fitness function was investigated. It is very hard (if not impossible) to solve most non-convex problems exactly in a reasonable time. Hence, the idea of using heuristic algorithms, which produce desired solutions, is offered. In mathematics, the Hessian matrix is a square matrix of second-order partial derivatives of a function. It describes the convexity of a function of many variables. If the function is twice differentiable and the Hessian is positive semi-definite in the entire domain, then the function is convex. On the other hand, if the Hessian matrix has a negative eigenvalue at a point in the interior of the domain, then the function is not convex.

The results of the second partial derivative test show that *Re* and *Fc* functions are non-convex functions, so that: $\frac{\partial^2 Re}{\partial X_1^2} < 0 \ \ and \ \frac{\partial^2 Fc}{\partial X_1^2} < 0$. Therefore, the Eq. 4 is not convex and it has a local optimal point.

Study of GA parameters

Computer simulation of column flotation operation is an active research field in mineral processing. Most of the processing units used for the flotation column have been mathematically modeled by many researchers during past decades, and a number of commercial simulation software packages have been introduced to industrial users. However, most of these packages lack numerical search capabilities for simulation optimization. One of the important challenges in the column flotation is to find the optimum operating conditions to achieve a high-quality product. In such a case, optimization is a complicated concept that can be defined differently. There are several methods for solving the optimization problem. These methods are either based on gradient evaluation or evolutionary methods. In the present work, we implemented a GA to optimize the column flotation parameters shown in Fig. 8. In this case, the optimization problem was defined as finding the best values for the froth height, chemical reagent dosage, wash water, air flow rate, air holdup, and Cu grade in rougher and column feed streams. The above-mentioned variables are important operating variables, which can normally be manipulated or modified in mineral processing plants. The lower and upper bounds values of all parameters that used in GA are presented in Table 5.

The algorithms used in this research were designed in MATLAB Version 2009b. The parameters in the GA procedures must be decided upon the population size, the number of generation for terminating the search, the ratio of crossover, and the ratio of mutation. The values of these parameters used in this study are listed in Table 6. In this study, two-point cross-over and uniform mutation operators were used, and the probability of the cross-over and mutation operators was adjusted at 0.8 and 0.07, respectively. Likewise, the GA started with 300 randomly generated chromosomes. Figures 9 to 11 display a number of plots generated by the GA during its execution. These plots give detailed information about various aspects of the executed optimization algorithm.

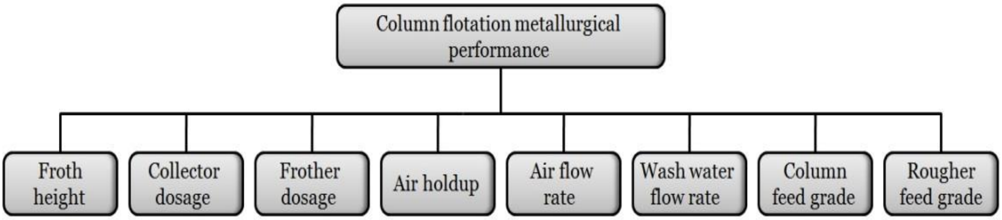

Fig. 8. Column flotation modelling (model must contain at least eight different variables)

Table 5. Lower and upper bounds of variables used in GA model

Variables	Lower bound	Upper bound
Froth height (cm)	30	130
Frother dosage (g/Mg)	32	42
Collector dosage (g/Mg)	30	40
Air holdup (%)	70	95
Air flow rate (cm/s)	0.5	1.8
Wash water flow rate (cm/s)	0.1	0.5
Cu grade in the rougher feed (%)	0.7	0.95
Cu grade in the flotation column feed (%)	5	12.5

Table 6. Parameters of the GA

Parameter	Setting
Population size	300
Fitness scaling	Rank
Selection function	Roulette
Reproduction elite count	2
Crossover fraction	0.8
Mutation function, ratio of mutation	Adaptive feasible, 0.07
Crossover function	Two point
Migration direction, fraction and interval	Forward, 0.2, 20

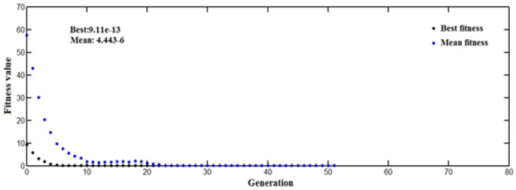

Fig. 9. Convergence of best and mean fitness value

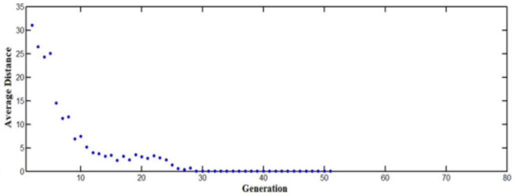

Fig. 10. Average distance between individuals during GA execution

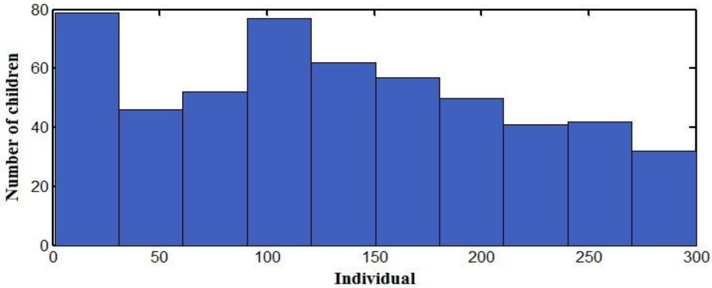

Fig. 11. GA execution information on selection function

Optimization procedure

In order to optimize the flotation column, GA was coupled with the MNLR model. The outcome of the optimization must be a solution that fulfils operator demands, which can be achieved via the use of above-described models (Eqs. 2 and 3). The models used in the software are fully automated so that the user needs very little knowledge of exactly how the modelling works. The optimization process starts by searching the models for the optimization parameters of the flotation column. These parameters are then included in the optimization routine, which finds the best solution within the existing constraints. The simulation results obtained from the mentioned GA-based optimization model are listed in Table 7. The results revealed that GAs were robust and efficient to find out optimal conditions. It is also worth stressing that the GA model demanded around 30 sec in a Pentium 5, 2 GHz Core2Duo CPU, 2 GHz RAM computer to find the best steady-state productivity. This is an important aspect in real time applications of the GA optimization technique coupled with the high non-linear and multivariable model. In the light of the above comparative results, it is concluded that the proposed GA approach may serve well to optimize other mineral processing plant performances.

In summary, a novel plant optimization technique was developed using GAs to maximize the overall grade and recovery by searching the best possible combination of operational conditions. The unique features of GA in comparison with other techniques, including faster convergence, coverage of a wider search space, the ability to get out of local extreme using mutation technique and the optimization of numerous variables at the same time. Therefore, the GAs based optimization approach can be applied to address quite for many copper and mineral processing related applications.

Determining the optimal conditions by the Halbich upgrading curve.

Table 7. Results obtained with GA

Optimization variables	Best value
Froth height (cm)	99.25
Frother dosage (g/Mg)	34.8
Collector dosage (g/Mg)	39.1
Air holdup (%)	92.3
Air flow rate (cm/s)	1.5
Wash water flow rate (cm/s)	0.41
Cu grade in the rougher feed (%)	0.84
Cu grade in the flotation column feed (%)	11.64

It should be noticed that this kind of intelligent approach should be verified by using mineral processing approaches. Therefore, the results must be approved and assessed in terms of mineral processing. The grade-recovery curves are frequently used because they are practical, and indicate several characteristic features of separation results. In this study, the new flotation tests (apart from the tests mentioned in this section) were carried out by using the full factorial center point repeated experimental design. Three important parameters: froth height, air, and wash water rates were chosen as independent variables (design factors), and two levels of these variables with their base points were used to generate data for 2^3 factorial design. The variables and their levels are given in Table 8. It should be noted that other variables were considered constant (frother dosage: 40 g/Mg, collector dosage: 34 g/Mg, Cu grade in the rougher feed: 0.8%). Moreover, Cu grade in the flotation column feed was about 10.8-11%.

Table 8. Variables and their levels flotation factors

Variable	Low level (–1)	Mid Level (0)	High Level (+1)
A, froth height, cm	35	70	100
B, air rate, cm/s	0.7	1.1	1.5
C, wash water rate, cm/s	0.1	0.2	0.35

The factorial design which consists of the factors, levels, and values are also given in Table 9. In the design matrix, the higher level was designated as "+1" whereas the lower one and mid-point were designated as "–1" and "0", respectively.

Table 9. Factorial design matrix and responses

Run	A	B	C	Grade	Recovery
1	–1	–1	–1	17.42	92.29
2	1	–1	–1	19.7	86.36
3	–1	1	–1	17.05	96.31
4	1	1	–1	20.74	90.5
5	–1	–1	1	19.84	90.67
6	1	–1	1	22.13	86.26
7	–1	1	1	20.22	93.8
8	1	1	1	23.08	90.84
9	0	0	0	21.02	91.33
10	0	0	0	20.91	91.89
11	0	0	0	19.97	93.01

Statistical software was applied to specify the optimal conditions by considering data and calculations achieved from the factorial design. The recovery and grade together were taken as criteria and the possible highest points selected for numerical determination of the optimal condition. According to the experimental study, 90.84% of recovery and 23.08% of grade were obtained at the high level of variables. It is known that the best separation results for a set of experiments can be determined by using the upgrading curves, which consist of the grade-recovery plot known as the Halbich curve (Drzymala, 2006). The reason for this is that the Halbich curve has some advantages over other upgrading curves because it considers two essential parameters of separation results, which are recovery and grade. The Halbich plot for all 11 experimental points is presented in Table 9, and seen in Fig. 12. Each straight line connects data points obtained for a constant level of both air and wash water rates, and increasing of froth height. The line L_b approximates the data points obtained for experiments conducted three times at the zero level of all parameters. The best results are those forming an upgrading line which is the closest to the ideal separation line. This occurs at the L_1 line which represents flotation results obtained for runs 7 and 8 conducted at the increasing amount of the froth height and constant (higher) level of air and constant (higher) level of the wash water rates.

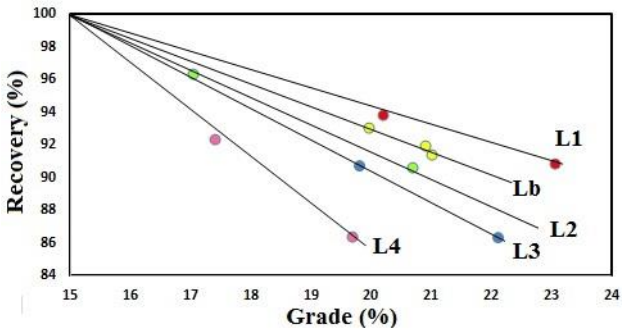

Fig 12. The Halbich upgrading curve with the results of all experiments

The same conclusion can be achieved considering the optimum level of the investigated parameters via the use of not only the direct flotation results, but also the data generated with Eqs. 1 and 2, and plotted on the Halbich curve. It means that these Eqs. coupled with the GA can be useful for finding optimum flotation conditions provided that a proper criterion for the optimum conditions is applied. The optimum results obtained from the GA optimization is close to the points on line L1 in the Halbich curve.

Conclusions

In this paper, a new numerical approach based on genetic algorithm (GA) for optimization of column flotation was investigated. The multivariate non-linear regression model (MNLR) used to describe column flotation metallurgical performance was non-linear and of high dimensionality. Such characteristics motivated the use of GA, an evolutionary method, to optimize the model, since deterministic methods have proved to be unsuccessful in dealing with models of high non-linearity and dimensionality. The GA model was used coupled with the MNLR model (as a fitness function) of the column flotation. In this work, the optimization problem was defined as finding the best values for the froth height, chemical reagent dosage, wash water, and air flow rates, air holdup, and Cu grade in rougher and column feed streams. The GA parameters fitted values led to the final optimization run, which found the best eight input process variables values that conduct to the maximal productivity of flotation column. The proposed GA found accurately the best values of flotation column model variables with error $9.11 \cdot 10^{-13}$. Then, the optimal conditions for the flotation were determined by taking into account both recovery and grade simultaneously using the Halbich plot. According to the plot, the best results for the optimization were close to the ones obtained with the GA. The high computational time-demand was not observed in the present application, and this act made the GA appropriate for a real-time implementation. The case study of optimizing variables for a column flotation process using the GA showed that the GA is a powerful and robust multivariable search method which can be used effectively to find the best values of operating parameters. Regarding the importance of process optimization, incorporating GA algorithms into mineral processing simulators can be a great help to possible users at control plant.

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