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SYSTEM FOR VEHICLE ROUTING PROBLEM ALGORITHMS ANALYSIS

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Abstract. *Paper concerns the software system supporting the analysis of different cases of solving VRP by various algorithms. VRP has been characterised and application structure has been presented. Illustrative experimental results show the usefulness of the system.*

Keywords: vehicle routing problem, simulated annealing, optimization

SYSTEM DO ANALIZY ALGORYTMÓW PLANOWANIA DOSTAW

Streszczenie. *Artykuł przedstawia oprogramowanie wspomagające analizę różnych przypadków rozwiązywania planowania dostaw (ang. Vehicle Routing Problem, VRP) przez różne algorytmy. Zaprezentowano w artykule problem VRP oraz strukturę omawianego systemu. Pokazano również wyniki eksperymentów, które pokazują użyteczność systemu.*

Słowa kluczowe: planowanie dostaw, symulowane wyżarzanie, optymalizacja

Introduction

One of the most extensively studied optimization problems is undoubtedly the Traveling Salesman Problem (TSP). It was first mentioned in the 19th century by Kirkman and Hamilton, but it was defined by Menger [6]. The simplest variant of the TSP consists in finding the shortest route between n cities, starting and finishing in the same city.

The generalized form of this problem is described as the Vehicle Routing Problem (VRP). The VRP was first formulated mathematically in 1959 in the work by Dantzig and Ramser, who developed the first algorithmic solution and applied it to a real-world problem of fuel deliveries [2]. The VRP consists in creating a set of paths (routes), one route for each vehicle, starting and ending at a single warehouse. All those routes should meet the customer demand, satisfy all the operating constraints imposed and minimize the total cost of transportation. Depending on the set of elements, such as the road network, customers, warehouses, vehicles and drivers, a specific variant of the VRP, related to a particular research or real-world problem can be obtained [7].

The paper is organized as follows. In Section 1 placed the definition of Vehicle Routing Problem. Section 2 presents the concept of application that allows to analyze any algorithms for solving VRP. Section 3 describes the simulated annealing algorithm, in short. Section 4 reports the results of experiments. Finally, Section 5 presents the conclusions.

1. Mathematical Model of Vehicle Routing Problem

The road network used for the transport of goods is most often represented in the form of a graph, the edges of which represent the roads, while the vertices represent the warehouse and the customers – figure 1. Such special vertices (warehouses) are also characterized by the number and type of vehicles and the total number of goods which they are able to distribute. Customers can be characterized also by time windows within which the delivery should be accomplished. The fleet of vehicles used for delivering goods may be homogeneous or heterogeneous. The size of the fleet can be fixed or adjusted to customer demand.

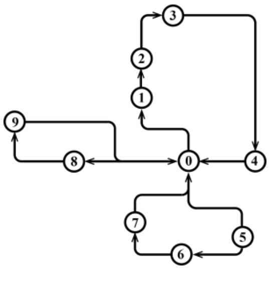

Fig. 1. Vehicle Routing Problem

The VRP is to minimize the goal function [7]:

$$\sum_{i\in V}\sum_{j\in V} c_{ij}\sum_{k=1}^{K} x_{ijk} \to \min \tag{1}$$

with constraints:

$$\sum_{k=1}^{K} y_{ik} = 1 \qquad \forall i \in V\setminus\{0\}, \tag{2}$$

$$\sum_{k=1}^{K} y_{0k} = K \tag{3}$$

$$\sum_{j\in V} x_{ijk} = \sum_{j\in V} x_{jik} = y_{ik} \qquad \forall i\in V, k=1,...,K, \tag{4}$$

$$\sum_{i\in V} d_i y_{ik} \le C \qquad \forall k=1,...,K, \tag{5}$$

$$x_{ijk}\left(w_{ik}+s_i+t_{ij}-w_{jk}\right)\le 0 \qquad \forall i, j\in V, \forall k=1,...,K, \tag{6}$$

$$a_i\sum_{j\in V} x_{ijk} \le w_{ik} \le b_i \sum_{j\in V} x_{ijk} \qquad \forall i \in V, k=1,...,K, \tag{7}$$

$$a_0 \le w_{ik} \le b_{n+1} \qquad \forall k=1,...,K, i\in\{0,n+1\} \tag{8}$$

$$\sum_{i\in S}\sum_{j\in S} x_{ijk} \le |S|-1 \quad \forall S\subseteq V\setminus\{0\}, |S|\ge 2, k=1,...,K, \tag{9}$$

$$y_{ik}\in\{0,1\} \quad \forall i\in V, k=1,...,K, \tag{10}$$

$$x_{ijk}\in\{0,1\} \quad \forall i, j \in V, k=1,...,K, \tag{11}$$

where:

$$y_{ik} = \begin{cases} 1 & \text{when demand for customer i fulfills the vehicle k} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ijk} = \begin{cases} 1 & \text{when the vehicle k overcomes the route from the customer i to j} \\ 0 & \text{otherwise} \end{cases}$$

Next conditions require to the solution the following restrictions: each client is assigned to exactly one vehicle (2), K vehicle leaves the depot (3), on route form i to i client is assigned the same vehicle (4), the capacity of any vehicle belonging to the fleet cannot be exceeded (5), the customer must be handled in the time window defined for him (6)–(8). The last condition (9) eliminates the possibility of creating routes unconnected with the depot.

2. Application

The specialized software system has been developed for the purpose of conduct algorithms researches. It consists of three independent subsystems: Data Loader, Algorithm (Test) Runner and Solution Analyser. As Figure 2 shows, all subsystems interact by middle database.

Entry data are process by Data Loader subsystem and saved in database.

Due to fact that developed system accepts wide variety of data forms Data Loader transforms the entry data to common form used within the system. However, the application enables to use original data in particular algorithm.

Algorithm Runner subsystem loads data from database and runs given algorithm fed by received data. Algorithms used in this subsystem can implement any logic if only it could extend abstract interface recognized by the algorithm executor module. As a result the application creates aggregated set of routes with ordered points (clients) to visit based on rough data set or a previously generated solution.

Last step of data processing is performed by Solution Analyser subsystem. It helps in further solutions analysing by giving statistical information, comparing given solutions or visualising of particular solution.

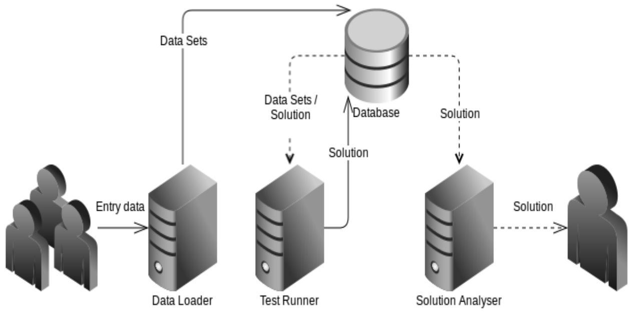

Fig. 2. Context diagram of application

3. Simulated Annealing

Simulated annealing is a probabilistic method for finding the global minimum of the cost function, proposed independently by Kirkpatrick, Gelett and Vecchi [3] and Cerny [1]. A characteristic feature of the algorithm is the possibility of leaving a local minimum (by adopting a worse state as current solution) – the smallest value in a range of areas of study the cost function, higher than the minimum value of the function in the whole area (actual global minimum). This is done with the predetermined probability linearly dependent on the temperature – the algorithm in the initial stages (when the temperature is high) is easier consents to worse solution, while during iterations and temperature decrease such skips are less likely, driving the algorithm to optimization in a particular minimum.

Figure 3 shows a flow diagram for the proposed solution by focusing on optimizing the first solution. The algorithm is a loop whose stop condition is based on the temperature. After generating the initial solution it permits the transfer to the optimization module (which implements the simulated annealing algorithm) [4, 5].

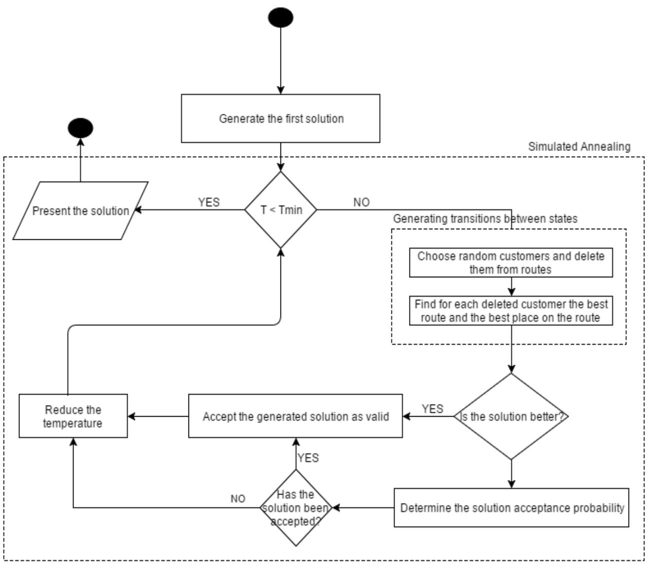

Fig. 3. Flow diagram of simulated annealing algorithm for VRP with time window

4. Experiments

The study was conducted for the Solomon test data set [8] to illustrate the performance of developed system. The geographical data are randomly generated in problem sets R, clustered in problem sets C, and a mix of random and clustered structures in problem sets by RC (Figure 4). The experiment consisted in finding the solution for three versions of the classic VRP taking into account the customers' time windows. The created routes fit the constant capacity of vehicles for the whole fleet as well as limitations regarding the size of orders. Using the proposed application the impact of various parameters of simulated annealing algorithm on the obtained results was effectively examined. In particular three parameters of the algorithm were analysed: initial temperature, temperature delta ΔT, percentage of random orders. The initial values of above parameters were 1000, 0.05 and 0.2 respectively.

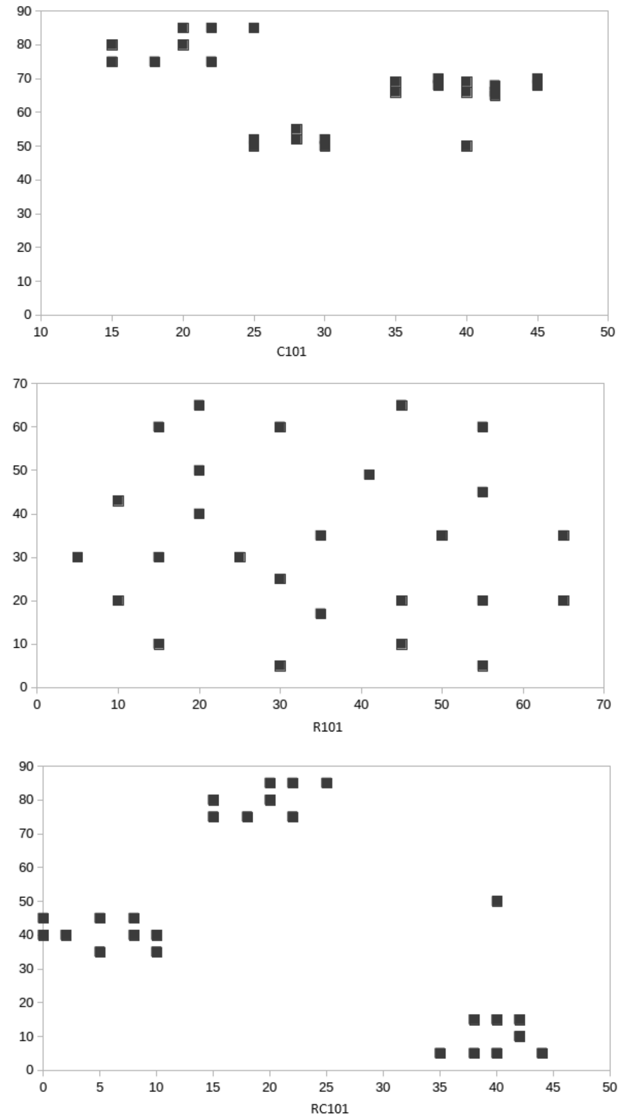

Fig. 4. Data representation of problem sets

4.1. Analysing of the ΔT parameter impact

Temperature of n-th loop of the algorithm is calculated using the formula: $T_n = (1-\Delta T)\cdot T_{n-1}$. The following range of ΔT was analysed: 0.01–0.185, step: 0.005, and obtained results are shown in Figure 5, Figure 6 and Figure 7 for representations of each group of problem sets. The monotonic dependence between the cost of the solution of the VRP and the ΔT parameter can be observed for data represented in clusters (Figure 5). For these data, the smaller value of ΔT the lower cost of the route.

These results goes along with the expected feature of the algorithm due to fact that lowering ΔT make the whole optimization process longer so that better solution can be found. The results for R set tests (Figure 6) also show dependency between the total cost of the solution and value of temperature delta parameter. The trend is not so clear, however. In Figure 7 the results, just like in R type, show trend, due to which the lower temperature delta is, the better results are receive. RC101 test, however stay stable (likely because quite good local minimum).

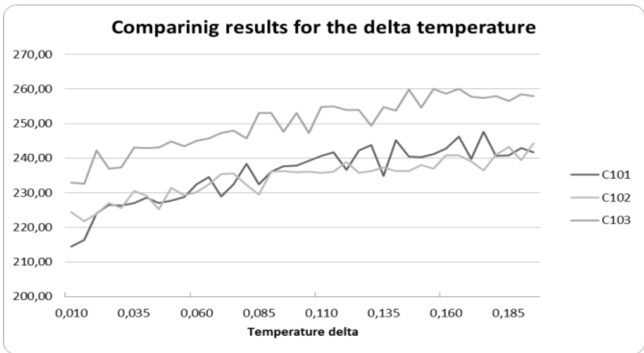

Fig. 5. The delta temperature results for problem sets C

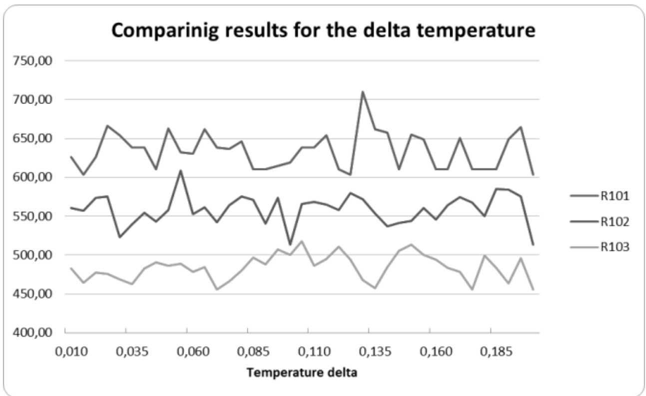

Fig. 6. The delta temperature results for problem sets R

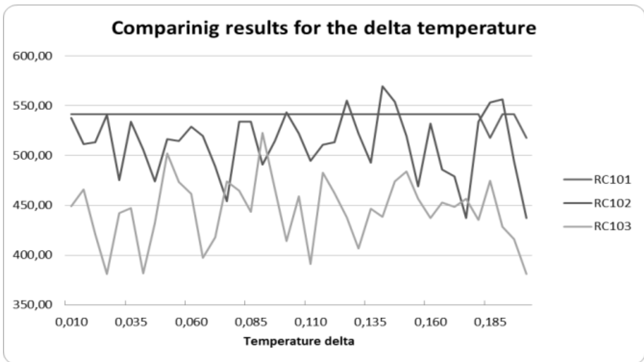

Fig. 7. The delta temperature results for problem sets RC

4.2. Analysing of the initial temperature impact

The initial temperature value in the algorithm was varied in the range of 200–5000, step: 100. In Figure 8, Figure 9 and Figure 10 the obtained results are shown.

In general C type tests results (Figure 8) make it like others types – remain stable regardless initial temperature value. In this particular type it could be observe that C103 test total cost decrease together with increasing initial temperature. This decreasing is not significant, however.

Results for data presented in Figure 9 do not show the significant influence of the initial temperature on the final solution cost. One can argue that higher values of initial temperature should result in lower cost of solution but on the other hand with higher temperature algorithm accepts worse solution with high probability in order to avoid local minimum.

RC type tests results (Figure 10) – just as R and C type results – remain stable despite changing value of initial temperature.

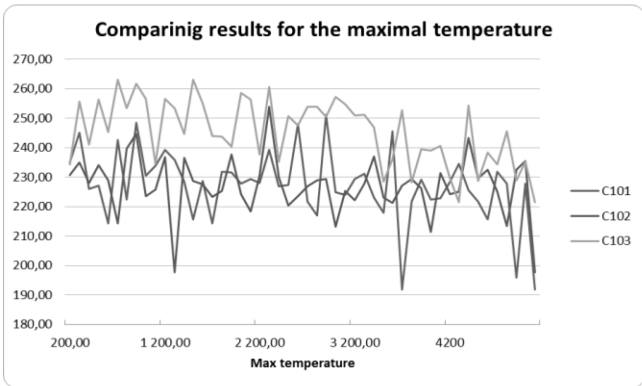

Fig. 8. The maximal temperature results for problem sets C

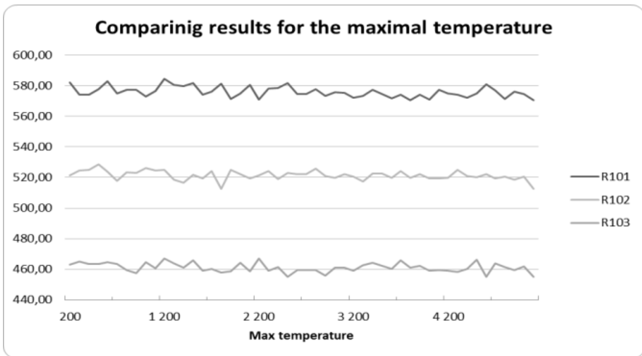

Fig. 9. The maximal temperature results for problem sets R

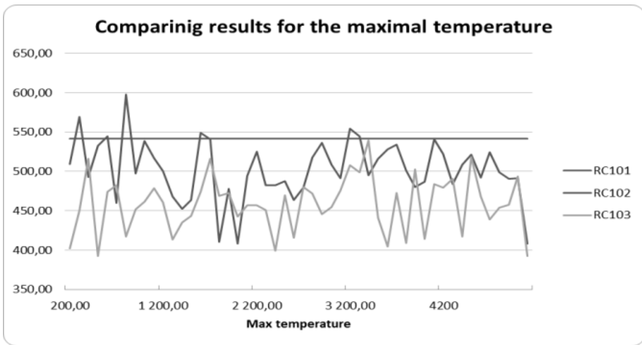

Fig. 10. The maximal temperature results for problem sets RC

4.3. Analysing of the random order influence

The influence of the percentage of random orders (points on routes) to change between routes was analysed. Tested parameter range was chosen as: 2%–98%, step: 2%. The results are presented in Figure 11, Figure 12 and Figure 13.

Received results in Figure 11 clearly show that cost function of each test has global minimum. This global minimum exists because too small amount of random orders leads to significant limitation of receiving solutions. On the other hand the more percent of random orders increase the less possibility that algorithm would optimize current solution (due to receiving too different results).

In R type tests (Figure 12) – as distinct from types C or RC – trend is not so visible. Global minimum still can be observed, but cost does not sharply increase above this minimum.

The results (Figure 13) – as well as results in RC type – have global minimum and also show that above this minimum (on percent of randomly choose orders) cost function constantly rise.

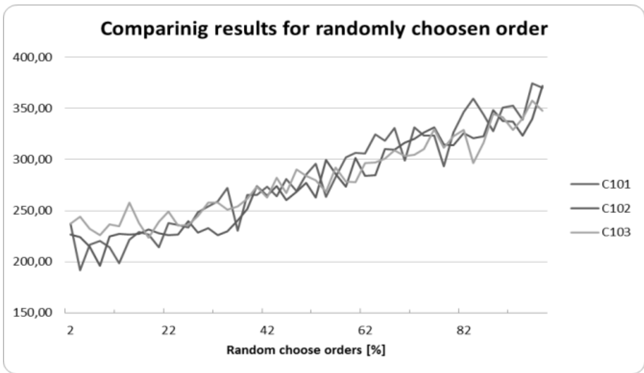

Fig. 11. The percent of randomly choose order results for problem sets C

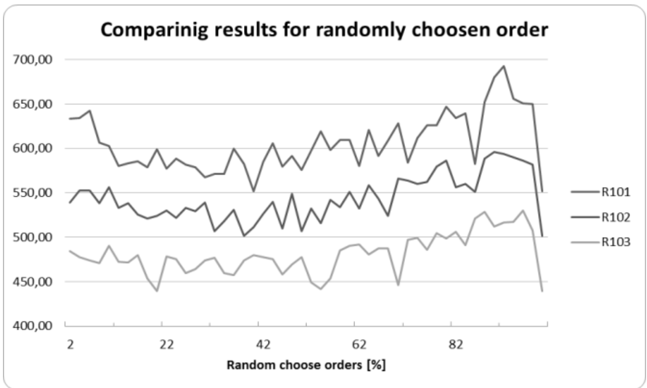

Fig. 12. The percent of randomly choose order results for problem sets R

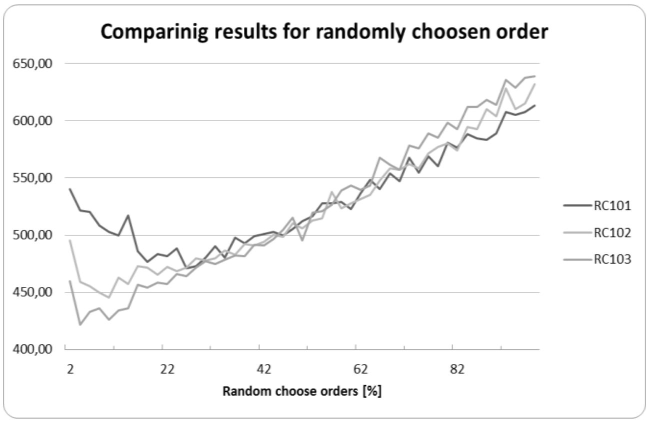

Fig. 13. The percent of randomly choose order results for problem sets RC

5. Summary

The developed software system supporting research on methods solving Vehicle Routing Problem along with theoretical aspects of VRP has been presented. The system was designed in such a way that it enables effective analysis of wide range of VRP cases including various benchmark data sets as well as various algorithms. An example of application of simulated annealing algorithm to Solomon VRP data set and the analysis of the parameters influence illustrated the usefulness of the system.

In general tests results are repetitive through all test types. This is evidence that algorithm behave similarly regardless input data.

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