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MATHEMATICAL MODELING OF FLUID FLOW IN THE DOUBLY CONNECTED AREA

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Abstract

The article presents the study of the flow of non-Newtonian liquid on the hydrodynamic initial section in doubly connected region. A method which allows solving this problem is to use the principle of conservation of momentum for a moving fluid. The study used the equation Navier-Stokesa for in viscid fluid moving in a two dimensional space. For the solution of the Navier-Stokes equations it is necessary to know the function of cross linking for the velocity distribution in hydrodynamic initial section in doubly connected region. This function was obtained by integrating the Navier-Stokes equations and after substitution in the equation for the distribution of local velocities allows calculating the local velocity in the liquid flow on the hydrodynamic initial section in doubly connected region. In addition to the analysis of the function of cross linking was discussed.
Keywords: non-Newtonian fluid, doubly connected region, local velocity, Navier-Stokes equations.

1. INTRODUCTION

Flow of non-Newtonian liquid on the hydrodynamic initial section in the doubly connected region is of great practical importance. Such flows are explored in aviation, chemical industry, and nuclear power. The computer-aided design (CAD), computational fluid dynamics (CFD) and computational structural mechanics (CSM) commercial software were used as common tools as well as intentionally developed computer programmers were used as basic tools in some tasks, in particular, for aerodynamic optimization, calculation of load and stability of aircraft [1].

The distribution of local velocity and pressure along the length of the hydrodynamic initial section is of great practical importance in order to calculate various process parameters. For non-Newtonian [3, 4] fluids of this type of flow have been sufficiently studied. The study of such flows was doing Leybenzon, Mak-Kelvi, Fridrikson and Bird et al. Research non-Newtonian fluids are approximate and do not take into account the effect of all the conditions of the fluid inlet on the hydrodynamic initial section. For a calculation of local velocity it is necessary to know the function of cross linking. This function is obtained in the present work.

2. THE STUDY

The problem lies in the fact that nowadays we do not know whether the Navier-Stokes equation can be solved uniquely for each meaningful initial condition. Despite almost a century of intensive research over this issue we still do not know whether we can actually predict liquid flow. We do not know whether the initial limited and smooth velocity field can explode at some point, that is, to achieve at some point in the area in question an infinite value, all the time evolving to this point, according to the Navier-Stokes equation. The authors tried to estimate the equation and determine the linking function.

The proposed formula for constructing velocity profiles at given distances from the entrance of the fluid in the pipe within the initial section used in the presence of the dependence function of cross linking on the coordinate z, that meets the functional relationship *b=f(z)*. However, the solution can only be obtained for a given index n of the flow. Substitution of the obtained values b (z) allows one to analyze the process of fluid flow.

Let us take $n = \frac{1}{2}$.

It is known that substituting $n = \frac{q}{2S}$ with $(q, S) = 1$ the expression for only makes sense when $\frac{\partial u}{\partial y} < 0.$

To avoid this, the equation should be written as [2]:

$$-\left(-K\frac{\partial u}{\partial y}\right)^{\frac{2q}{4S}} = -\left[\left(-K\frac{\partial u}{\partial y}\right)^{2q}\right]^{\frac{1}{4S}} = -\left[\left(K\frac{\partial u}{\partial y}\right)^{2}\right]^{\frac{q}{4S}} = -\left[K\frac{\partial u}{\partial y}\right]^{\frac{2q}{4S}} = -\left[K\frac{\partial u}{\partial y}\right]^{\frac{q}{2S}}.$$

Where:

u – the local velocity of the fluid;
y – coordinate perpendicular to the axis of the channel;
q, s – arbitrary variables selected for the analysis of possible variants of substitution of the index movements;
n – the index of the flow of non-Newtonian fluid;
b – joining decisions.

When $n = \frac{1}{2}$, should put $\tau = -K\left[\frac{\partial u}{\partial y}\right]^{\frac{1}{2}}$.

K – grease constant;
τ – shear stresses.

Boundary conditions in the entrance area of the former [6]. Substituting the boundary conditions obtained after differentiation of equation: $\frac{dP}{dz} = \tau(z,1) + \frac{\partial \tau}{\partial z}(z,1)$;

where:

$\frac{dP}{dz}$ – the pressure distribution along the length of the hydrodynamic initial section.

z – coordinate along the channel axis.

All quantities of the equation are dimensionless. Taken in the field λε≤y≤1 speed equation, which is presented below and differentiate this equation [2], [6] we have:

$$\frac{\partial u}{\partial y}=\frac{b(z)+2}{b(z)}-b(z)\left(\frac{y^2-\lambda^2\varepsilon^2}{1-\lambda^2\varepsilon^2}\right)^{b(z)-1}2y\frac{1}{1-\lambda^2\varepsilon^2}=-2[b(z)+2]y\left(\frac{y^2-\lambda^2\varepsilon^2}{1-\lambda^2\varepsilon^2}\right)^{b(z)-1}\cdot\frac{1}{1-\lambda^2\varepsilon^2}=-\frac{2[b(z)+2]y(y^2-\lambda^2\varepsilon^2)^{b(z)-1}}{(1-\lambda^2\varepsilon^2)^{b(z)}}$$

Where $\chi\varepsilon$ the position of the maximum velocity.

Shear stress at $n=\frac{1}{2}$

$$\tau=-\left\{\frac{2[b(z)+2]y[y^2-\lambda^2\varepsilon^2]^{b(z)-1}}{(1-\lambda^2\varepsilon^2)^{b(z)}}\right\}^{\frac{1}{2}},$$

as $y\geq\chi\varepsilon$, converting expressions get

$$\tau=-\frac{\sqrt{2}\sqrt{b(z)+2}\,y^{\frac{1}{2}}(y^2-\lambda^2\varepsilon^2)^{[b(z)-1]/-2}}{(1-\lambda^2\varepsilon^2)^{b(z)/2}}$$

Assuming $y=1$ we have $\tau(z,1)=\frac{\sqrt{2}}{(1-\lambda^2\varepsilon^2)^{1/2}}\sqrt{b(z)+2}$

Compute $\frac{\partial\tau}{\partial y}(z,1)$ $\quad\tau(z,1)=-\left\{\frac{2[b(z)+2]y[y^2-\lambda^2\varepsilon^2]^{b(z)-1}}{(1-\lambda^2\varepsilon^2)^{b(z)}}\right\}^{1/2}$

Differentiate according to y

$$\frac{\partial\tau}{\partial y}=-\frac{\sqrt{2(b(z)+2)}}{(1-\lambda^2\varepsilon^2)^{b(z)/2}}\cdot\frac{1}{2}\left[y(y^2-\lambda^2\varepsilon^2)^{b(z)-1}\right]^{-1/2}.$$

When $y=1$

$$\frac{\partial\tau}{\partial y}(z,1)=-\frac{\sqrt{2(b(z)+2)}}{2(1-\lambda^2\varepsilon^2)^{b(z)/2}}\cdot(1-\lambda^2\varepsilon^2)^{[b(z)-3]/2)}\left[2b(z)-1-\lambda^2\varepsilon^2\right]==-\frac{\sqrt{2[b(z)+2]}}{2(1-\lambda^2\varepsilon^2)^{3/2}}\cdot\left[2b(z)-1-\lambda^2\varepsilon^2\right]$$

The equation for $b(z)$

$$\frac{d}{dz}\int_{\lambda\varepsilon}^{1}yu^2dy=\int_{\lambda\varepsilon}^{1}-\frac{1}{\rho}(\tau_w+\tau_w^1)ydy+\frac{1}{\rho}\int_{\lambda\varepsilon}^{1}\frac{\partial}{\partial y}(y,\tau)dy=\frac{1}{2\rho}\tau_w(1+\lambda^2\varepsilon^2)+\frac{1}{2\rho}(\lambda^2\varepsilon^2-1)\tau_w^1.$$

$$\tau(\lambda\varepsilon)=0.$$

Where ρ – the density of the liquid,
Then ratio

$$\frac{d}{dz}\int_{\lambda\varepsilon}^{1}yu^2dy=\frac{1}{2\rho}(1+\lambda^2\varepsilon^2)\tau_w-\frac{1}{2\rho}(1-\lambda^2\varepsilon^2)\tau_w^1,$$

Where τ_W and $\tau^1{}_W$ – shear stresses in y = 1 $\left(\tau_w^1 = \dfrac{d\tau}{dy}\right)$

Let us calculate the intergral

$$\rho\int_{\lambda\varepsilon}^{1} y\frac{(b(z)+2)^2}{b(z)^2}\left[1+\frac{(y^2-\lambda^2\varepsilon^2)^{2b(z)}}{(1-\lambda^2\varepsilon^2)^{2b(z)}}-\frac{2(y^2-\lambda^2\varepsilon^2)^{b(z)}}{(1-\lambda^2\varepsilon^2)^{b(z)}}\right]dy=\frac{[b(z)+]^2(1-\lambda^2\varepsilon^2)\rho}{[2b(z)+1][b(z)+]}.$$

Find the equation for the value $b(z)$
Differentiate this expression

$$\rho\frac{d}{dz}\left\{\frac{[b(z)+2]^2(1-\lambda^2\varepsilon^2)}{[2b(z)+1][b(z)+1]}\right\}=\frac{1}{2}(1+\lambda^2\varepsilon^2)-\frac{\sqrt{2}}{(1-\lambda^2\varepsilon^2)^{1/2}}\sqrt{b(z)+2}-\frac{1}{2}(1-\lambda^2\varepsilon^2)$$

$$\left\{-\frac{\sqrt{2}\sqrt{b(z)+2}}{2(1-\lambda^2\varepsilon^2)^{3/2}}[2b(z)-1-\lambda^2\varepsilon^2]\right\}=\frac{\sqrt{b(z)+2}}{\sqrt{2}(1-\lambda^2\varepsilon^2)^{1/2}}[b(z)-3/2(1+\lambda^2\varepsilon^2)].$$

Put $w_1=\dfrac{3}{2}(1+\lambda^2\varepsilon^2)$. then $\rho\dfrac{d}{dz}\left\{\dfrac{(1-\lambda^2\varepsilon^2)[b(z)+2]^2}{[2b(z)+1][b(z)+1]}\right\}=\dfrac{\sqrt{2}\sqrt{b(z)+2}}{2(1-\lambda^2\varepsilon^2)^{1/2}}[b(z)-w_1]$

Take $E_1=(1-\lambda^2\varepsilon^2)^{3/2}\sqrt{2}\rho$ then $E_1\dfrac{d}{dz}\left\{\dfrac{[b(z)+2]^2}{[2b(z)+1][b(z)+1]}\right\}=\sqrt{b(z)+2}[b(z)-w_1]$

Take the integral

$$\int_{bo(z)}^{b(z)}\frac{1}{\sqrt{b(z)+2}[b(z)-w_1]}\frac{d}{db(z)}\left\{\frac{[b(z)+2]^2}{[2b(z)+1][b(z)+1]}\right\}db(z)=\frac{[b(z)+2]^2}{[2b(z)+1][b(z)+1]\sqrt{b(z)+2}[b(z)-w_1]}\Bigg|_{b_0(z)}^{b(z)}+$$

$$+\frac{3}{2}\int_{bo(z)}^{b(z)}\frac{[b(z)+2]^2\left[b(z)+\dfrac{4-w_1}{3}\right]}{[b(z)+2]^{3/2}[b(z)-w_1]^2[2b(z)+1][b(z)+1]}db(z)$$

Put

$$I_1=\int\frac{\sqrt{b(z)+2}\left[b(z)+\dfrac{2c-w_1}{3}\right]}{[b(z)-w_1]^2[2b(z)+1][b(z)+1]}db(z).$$

Here $C = 2$

Put also $w_2=\dfrac{2c-w_1}{3}=\dfrac{4-w_1}{3}$

Then $I_1=\displaystyle\int\frac{\sqrt{b(z)+2}\,[b(z)+w_2]}{[b(z)-w_1]^2[2b(z)+1][b(z)+1]}db(z).$

Function

$$\frac{b(z)+w_2}{[b(z)-w_1]^2[2b(z)+1][b(z)+1]}=\frac{A_1}{b(z)-w_1}+\frac{A_2}{[b(z)-w_1]^2}+\frac{B}{2b(z)+1}+\frac{C}{b(z)+1},$$

We calculate the coefficients A_1, A_2;

$$\left.\begin{array}{l} b(z)=w_1 \\ b(z)=1/2 \\ b(z)=-1 \\ b(z)^3 \end{array}\right| \quad w_1+w_2=A_2(2w_1+1)(w_1+1), \qquad A_2=\frac{w_1+w_2}{(2w_1+1)(w_1+1)},$$

$$-1/2+w_2=B(-1/2-w_1)(1/2), \quad B=\frac{4w_2-2}{2w_1+1},$$

$$w_2-1=C(w_1+1)^2(-1), \qquad C=\frac{1-w_2}{(w_1+1)^2},$$

$$0=A_1(2)+B+2C, \qquad A_1+-\frac{2C+B}{2}$$

Because $c > 0$ and $w_1 > 0$ and $w_3=\sqrt{2+w_1}$

$$I_{1,2}=\int\frac{\sqrt{b(z)+2}}{[b(z)-w_1]}db(z)=\int\frac{2\chi^2 d\chi}{(\chi^2-w_1)^2},$$

$$\mathrm{I}_{1,3}=\sqrt{b(z)+2}-\frac{1}{\sqrt{2}}arctg\sqrt{2}\sqrt{b(z)+2}$$

and finally

$$I_{1,4}=\int\frac{\sqrt{b(z)+2}}{b(z)+1}db(z)=2\sqrt{b(z)+1}-arctg\sqrt{b(z)+2}$$

Where b is a function of cross-linking in the input section.

Given that: $3A_1+3/2B+3C=0$, we get:

$$\frac{1}{E_1}Z=\frac{[b(z)+2]^{3/2}}{[2b(z)+1][b(z)+1][b(z)-w_1]}-\frac{[b_0(z)+2]^{3/2}}{[2b_0(z)+1][b_0(z)+1][b_0(z)-w_1]}-$$
$$-\frac{3A_2}{4}[b(z)-b_0(z)]\left\{\frac{1}{[b(z)-w_1][b_0(z)-w_1]}+\frac{1}{[b(z)+w_1][b_0(z)+w_1]}\right\}+$$

$$+\frac{3A_1w_1}{2}\ln\left[\frac{(\sqrt{b(z)+2}-w_3)(\sqrt{b_0(z)+2}+w_3)}{(\sqrt{b(z)+2}+w_3)(\sqrt{b_0(z)+2}-w_3)}\right]+\frac{3A_2}{4w_1}\ln\left[\frac{(\sqrt{b(z)+2}-w_1)(\sqrt{b_0(z)+2}+w_1)}{(\sqrt{b(z)+2}+w_1)(\sqrt{b_0(z)+2}-w_1)}\right]-$$
$$-\frac{3B}{2\sqrt{2}}arctg\left[\frac{\sqrt{2}(\sqrt{b(z)+2}-\sqrt{b_0(z)+2})}{1+2\sqrt{b(z)+2}\sqrt{b_0(z)+2}}\right]-\frac{3C}{2}arctg\left[\frac{\sqrt{b(z)+2}-\sqrt{b_0(z)+2}}{1+\sqrt{b(z)+2}\sqrt{b_0(z)+2}}\right].$$

After substituting in the last equation the accepted values w_1, w_2, w_3, E, we finally obtain the expression. Value A_1, A_2, B and C are presented above in the text [5].

3. THE CHARACTERISTICS OF THE FLOW VELOCITY AT THE INNER TUBE

Next, let us consider the branch profile of the speeds related to the surface of the inner tube, i.e., the area $\varepsilon \le y \le \chi\varepsilon$

The curve equation in general form takes the form of:

$$u(z,y) = \frac{b(z)+2}{b(z)}\left[1-\left|\frac{(\lambda\varepsilon)^2 - y^2}{\varepsilon^2(1-\lambda^2)}\right|^{b(z)}\right]$$

At $y=\varepsilon$ get the original equation in the form:

$$-\rho\frac{dP}{dz}+\frac{1}{\rho}\frac{1}{\varepsilon}\frac{\partial}{\partial y}(y,\tau)\Big|_{y=\varepsilon}=0.$$

Then

$$\frac{dP}{dz}=\frac{1}{\varepsilon\rho^2}\left(\tau+y\frac{\partial\tau}{\partial y}\right)_{y=\varepsilon} \text{ and } \frac{dP}{dz}=\frac{1}{\varepsilon\rho^2}\left[\tau(z,\varepsilon)+\varepsilon\frac{\partial\tau}{\partial y}(z,\varepsilon)\right]$$

find $\frac{\partial u}{\partial y}$. get $\frac{\partial u}{\partial y}=\frac{2[b(z)+2](\lambda^2\varepsilon^2-y^2)^{b(z)-1}}{[\varepsilon^2(1-\lambda^2)]^{b(z)}}$.

So as y $\le \varepsilon\lambda$ and $\chi \le 1$ in case $b(z)+2>0$

$$\left|\frac{\partial u}{\partial y}\right|=\frac{\partial u}{\partial y}$$

$$\tau(z,y)=-\frac{\sqrt{2}[b(z)+2]^{1/2}(\lambda^2\varepsilon^2-y^2)^{\frac{b(z)-1}{2}}}{[\varepsilon^2(1-\lambda^2)]^{b(z)/2}}$$

If $y=\varepsilon$ get:

$$\tau=\tau(z,\varepsilon)=-\frac{\sqrt{2}[b(z)+2]^{1/2}}{\varepsilon\sqrt{1-\lambda^2}}. \qquad \frac{\partial\tau}{\partial y}=\frac{\sqrt{2}[b(z)+2]^{1/2}[b(z)-1]}{(1-\lambda^2)^{3/2}\varepsilon^2}$$

So, according to the equations above we receive:

$$\frac{dP}{dz}=\frac{\sqrt{2}[b(z)+2]^{1/2}[b(z)-\lambda^2]}{\varepsilon\rho^{22}(1-\lambda^2)^{3/2}}.$$

Let us calculate the integral $\int_{\varepsilon}^{\lambda\varepsilon} y\cdot u^2\cdot dy$,

$$\int_{\varepsilon}^{\lambda\varepsilon} y\left[\frac{b(z)+2}{b(z)}\right]^2\left\{1-\left[\frac{\lambda^2\varepsilon^2-y^2}{\varepsilon^2(1-\lambda^2)}\right]^{b(z)}\right\}^2 dy.$$

Designating by L we get:

$$L=\int y\left[\frac{b(z)+2}{b(z)}\right]^2\left\{1-2\left[\frac{\lambda^2\varepsilon^2-y^2}{\varepsilon^2(1-\lambda^2)}\right]^{b(z)}+\left[\frac{\lambda^2\varepsilon^2-y^2}{\varepsilon^2(1-\lambda^2)}\right]^{2b(z)}\right\}^2=$$

$$=\left[\frac{b(z)+2}{b(z)}\right]^2\cdot\left\{\frac{y^2}{2}-2\frac{1}{[\varepsilon^2(1-\lambda^2)]^{b(z)}}\cdot\frac{1}{-2[b(z)+1]}\cdot(\lambda^2\varepsilon^2-y^2)^{b(z)+1}+\right.$$
$$\left.\frac{1}{[\varepsilon^2(1-\lambda^2)]^{2b(z)}}\cdot\frac{1}{[2b(z)+1]\cdot(-2)}\cdot(\lambda^2\varepsilon^2-y^2)^{2b(z)+1}\right\}$$

or

$$L=\left[\frac{b(z)+2}{b(z)}\right]^2\left\{\frac{y^2}{2}+\frac{1}{[\varepsilon^2(1-\lambda^2)]^{b(z)}}\cdot\frac{1}{b(z)+1}\cdot(\lambda^2\varepsilon^2-y^2)^{b(z)+1}-\frac{1}{2[\varepsilon^2(1-\lambda^2)]^{2b(z)}}\cdot\frac{1}{2b(z)+1}\cdot(\lambda^2\varepsilon^2-y^2)^{2b(z)+1}\right\}.$$

The solution takes the form:

$$\int_{\varepsilon}^{\lambda\varepsilon} yu^2dy=\frac{\varepsilon^2(1-\lambda^2)[b(z)+2]^2}{[b(z)+1][2b(z)+1]}.$$

Take the integral of the right side of the original equation multiplied by y and denoted by K. Then

$$K=\int_{\varepsilon}^{\lambda\varepsilon}\left[-\rho y\frac{dP}{dz}+\frac{1}{\rho}\cdot y\cdot\frac{1}{y}\frac{\partial}{\partial y}(y,\tau)\right]dy=$$
$$\rho\frac{dP}{dz}\cdot\frac{\lambda^2\varepsilon^2-\varepsilon^2}{2}+\frac{1}{\rho}\lambda\varepsilon\tau(z,\varepsilon)-\frac{1}{\rho}\varepsilon\tau(z,\varepsilon).$$

$$K=\rho\frac{\sqrt{2}}{\varepsilon^2\rho^2(1-\lambda^2)^{3/2}}\left[b(z)+2\right]^{1/2}[b(z)-\lambda^2]\frac{\lambda^2\varepsilon^2-\varepsilon^2}{2}+\frac{1}{\rho}\varepsilon\left[-\frac{\sqrt{2}\left[b(z)+2\right]^{1/2}}{\varepsilon\sqrt{1-\lambda^2}}\right]=$$
$$=\frac{\sqrt{2}}{\rho\sqrt{1-\lambda^2}}\cdot\frac{\left[b(z)+2\right]^{1/2}}{2}\left\{[b(z)-\lambda^2]+2\right\}=\frac{1}{2\rho}\frac{\sqrt{2}}{(1-\lambda^2)^{1/2}}\left[b(z)+2\right]^{1/2}\cdot\left[b(z)+2-\lambda^2\right].$$

Next, when $y=\varepsilon$

$$\frac{d}{dz}\int_{\varepsilon}^{\lambda\varepsilon} yu^2dy=\int_{\varepsilon}^{\lambda\varepsilon} y\left[-\rho\frac{dP}{dz}+\frac{1}{\rho}\cdot\frac{1}{y}\cdot\frac{\partial}{\partial y}(y,\tau)\right]dy.$$

$$\frac{d}{dz}\left\{\varepsilon^2(1-\lambda^2)\frac{[b(z)+2]^2}{[b(z)+1][2b(z)+1]}\right\}=\frac{1}{2\rho}\cdot\frac{\sqrt{2}}{(1-\lambda^2)^{1/2}}[b(z)+2]^{1/2}[b(z)+2-\lambda^2].$$

Take $$\frac{1}{E_1}=M=\frac{2}{\varepsilon^2(1-\lambda^2)^{3/2}2\rho}.$$

Then $$\frac{1}{M}\cdot\frac{d}{dz}\frac{[b(z)+2]^2}{[b(z)+1][2b(z)+1]}=[b(z)+2]^{1/2}\cdot[b(z)-w_1],$$

and $$E_1\frac{d}{dz}\left\{\frac{[b(z)+2]^2}{[b(z)+1][2b(z)+1}\right\}=\sqrt{b(z)+2}[b(z)-w_1].$$

Comparing the obtained equation with the equation belonging to the interval $\chi\varepsilon\le y\le 1$ and previously investigated profile are the same equation. Therefore, further calculation is similar to the replacement value w_1 on $w_1 = 2-\lambda^2$ and E_1 on $E_1=\rho\sqrt{2}E^2(1-\lambda^2)^{3/2}$.

Then similarly:

$$\frac{z}{\rho\sqrt{2}(1-\lambda^2)^{3/2}}=\frac{[b(z)+2]^{3/2}}{[b(z)+1][2b(z)+1][b(z)-(2-\lambda^2)]}-$$
$$-\frac{[b_0(z)+2]^{3/2}}{[2b_0(z)+1][2b_0(z)+1][b_0(z)-(2-\lambda^2)]}-\frac{3A_2}{4}[b(z)-b_0(z)]\cdot$$
$$\cdot\left\{\frac{1}{[b(z)-(2-\lambda^2)][b_0(z)-(2-\lambda^2)]}+\frac{1}{[b(z)+(2-\lambda^2)][b_0(z)+(2-\lambda^2)]}\right\}+$$
$$\frac{3A_1(2-\lambda^2)}{2}\ln\left\{\frac{(\sqrt{b(z)+2}-\sqrt{4-\lambda^2})(\sqrt{b_0(z)+2}+\sqrt{4-\lambda^2})}{(\sqrt{b(z)+2}+\sqrt{4-\lambda^2})(\sqrt{b_0(z)+2}-\sqrt{4-\lambda^2})}\right\}+$$
$$+\frac{3A_2}{4(\lambda^2+2)}\ln\left\{\frac{[(\sqrt{b(z)+2}-(2-\lambda^2)][\sqrt{b_0(z)+2}+(2-\lambda^2)]}{[(\sqrt{b(z)+2}+(2-\lambda^2)][\sqrt{b_0(z)+2}-(2-\lambda^2)]}\right\}-$$
$$-\frac{3B}{2\sqrt{2}}arctg\left[\frac{\sqrt{2}(\sqrt{b(z)+2}-\sqrt{b_0(z)+2})}{1+2\sqrt{b(z)+2}\sqrt{b_0(z)+2})}\right]-\frac{3C}{2}arctg\left[\frac{(\sqrt{b(z)+2}-\sqrt{b_0(z)+2})}{1+\sqrt{b(z)+2}\sqrt{b_0(z)+2})}\right].$$

The analytical solution of equations for $b(z)$ allows you to analyze this dependency and to build the curve $b(z)$, by asking values $b(z)$ for areas $\varepsilon\le y\lambda\varepsilon$ and $\varepsilon\chi\le y\le\varepsilon$, at different flow indices. The joining allows determining the distribution of local velocities, stresses and distribution of pressure along the length of the hydrodynamic initial section in the doubly connected region.

The derived equation to analyze the flow of non-Newtonian fluid in the clearances of the pipes is a bit complicated, however, it can be used for practical calculations [7]. A simple equation for an analysis of fluid flow can be obtained by n=1.

4. CONCLUSION

The results of theoretical analysis of fluid flow non-Newtonian liquid, in the hydraulic lines, especially at the interface connections require verification of an experimental research. Confronting measurements of the analytical method for determining the flow profile [7] allowed drawing conclusions on the convergence of the results of experimental and analytical. The resulting error between the analytical method described above and the results obtained in the experimental studies is on the level of several percent. Analytical determination of the characteristics of the flow velocity in connections of hydraulic lines is meant to optimize the technological process.

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MATEMATYCZNE MODELOWANIE PRZEPŁYWU CIECZY W DWUWYMIAROWYM OBSZARZE

Streszczenie

W artykule przedstawiono analizę przepływu cieczy nienewtonowskiej na styku początkowym podwójnie połączonego odcinka hydraulicznego. Sposób, który pozwala na rozwiązanie tego problemu to jest zastosowanie zasady zachowania pędu do poruszającego się płynu. W badaniu analitycznym zastosowano równanie Naviera-Stokesa dla lepkiego płynu poruszającego się w przestrzeni dwuwymiarowej. Do układu równań Naviera-Stokesa należy wprowadzić funkcję sieciowania dla rozkładu prędkości na początku przepływu hydrodynamicznego w dwuwymiarowym obszarze. Funkcję otrzymano przez całkowanie równań Naviera-Stokesa. Po podstawieniu do równań dla rozkładu lokalnych prędkości umożliwiło to obliczenie lokalnej prędkości w cieczy w dwuwymiarowym przepływie hydrodynamicznym.
Słowa kluczowe: Płyn nieniutonowski, przestrzeń dwuwymiarowa, prędkość lokalna, równanie Naviera-Stokesa.