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SIMULATION OF THE STRESS-STRAIN STATE OF ECCENTRICALLY COMPRESSED AND TENSIONED REINFORCED CONCRETE BEAMS

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Abstract: The paper deals with the working peculiarities of the support zones of reinforced concrete elements subject to bending with due account of the eccentric compression and tension. The authors performed simulation of the stress-strain behaviour of the indicated structures with the aid of “Lira” software which results are shown in the graphical and tabulated form. The performed simulation allowed of tracing the work of the studied sample beams till collapse. Such approach made it possible to single out and generalize the main collapse patterns of the inclined cross-sections of the reinforced concrete elements subject to bending on which basis the authors developed the improved method to calculate their strength (Karpiuk et al., 2019).
Keywords: reinforced concrete, eccentric compression, eccentric tension, inclined section, stress.

1. INTRODUCTION

The analysis of literature has shown that a lot of such works were devoted to a study of the bearing capacity of the normal cross-sections of reinforced concrete structures. At the same time, the bearing capacity of the inclined cross-sections still remains understudied despite the fact that collapse of the reinforced concrete structures in the inclined, plane or spatial cross-sections is very hazardous and, consequently, highly undesirable. Besides, the action of longitudinal tension forces in the structures leads to reduction of their strength and cracking resistance in the support zones by several times (Selejdak et al., 2019). Along with that, the bearing capacity of the support zones of the eccentrically compressed reinforced concrete elements is 50-70% higher (Klymenko et al., 2019), and that of the eccentrically tensioned is lower than the strength of common non-overreinforced elements, which has to be taken into account when designing new and making appraisal of the existing span structures.

2. METHODOLOGY OF RESEARCH

Simulation of the stress-strain behaviour of the studied elements was accomplished in a non-linear mode with the aid of a widely used finite element software complex “Lira” on the basis of the general theory of reinforced concrete containing cracks which was developed by Karpenko, 1996 and his followers. Considering that the concrete and the reinforcement work together and have different physical and mechanical characteristics (Kolchunov et al., 2016) that display non-linear dependencies versus stresses and deformations, in order to implement the finite element models of the studied reinforced concrete elements by means of said software complex the widely applied concrete deformation diagrams SE SRIBS (Golyshev and Bambura, 2004; Davydenko et al., 2007) and bilinear diagrams illustrating deformation of A500C reinforcement class were added.

The concrete strength limit of the complex non-uniformly stressed studied samples was determined with the aid of the phenomenological strength criterion of Geniyev et al., 1974, which was input in said software complex. Taking into account that the studied elements are symmetrical, the calculation was made for one half of the beam only; the beam was conventionally divided into spatial eight-node isoparametric finite elements No.236 of 1x1x1 cm so as to make it convenient to simulate the reinforcement and take into account the crushed stone size and the dimensions of the beam cross-section.

The number of elements in the beam model was, as a rule, 18,320. Step by step method and incremental-iterative algorithm were applied together with the piecewise-linear dependence of No.14 library with the appropriate algorithm. Same as in the real test, the longitudinal compressing or tensioning forces were applied in steps, and afterwards the transverse force was applied until the beam collapsed.

3. RESULTS

The results of numerical accomplishment of the real tests with common (test task) eccentrically tensioned and compressed reinforced concrete beams are shown in Figs. 1 and 2 as well as in Table 1.

Analysis of the simulation results obtained when testing the stress-strain behaviour of the studied reinforced concrete elements indicated that the use of the non-linear finite element calculation for that purpose (Gorodetskiy et al., 2013), which is based on the general mechanics of the reinforced concrete (Karpenko, 1996), (Iakovenko and Kolchunov, 2017) with the use of the phenomenological strength criterion (Geniyev et al., 1974), enables to reflect the results of the performed real and numerical tests with the sufficient for practical purposes accuracy (variation ratio υ = 9 ... 18%, Table 1). It is evident that in order to improve the calculation accuracy it is desirable to apply more refined strength criteria. Still, the use of the indicated non-linear finite element calculation made it possible to simulate the stress-strain behaviour of the studied elements at all stages of their work, including collapse. Sequential analysis of the isofields of stress, movements and forces in the real structure material enabled to reliably evaluate the influence of the studied external structural factors upon their bearing capacity and predict the nature of the subsequent deformation and the ultimate collapse.

Based on the results of the performed simulation it became possible to single out and generalize the main collapse patterns of the inclined cross-sections of the reinforced concrete elements subject to bending (Karpiuk et al., 2019):

- *A-1/$N_в$* or *A-2/$N_н$* in normal cross-sections near the support;
- *B/M* in the inclined cross-sections subject to the prevailing action of the bending moment;
- *C/V* in the inclined cross-sections subject to the prevailing action of the transverse force;
- *Д//см* in the inclined compressed concrete strip between the concentrated load (force) and the support;
- *F/V* which results from pressing through above the middle support of the continuous beams.

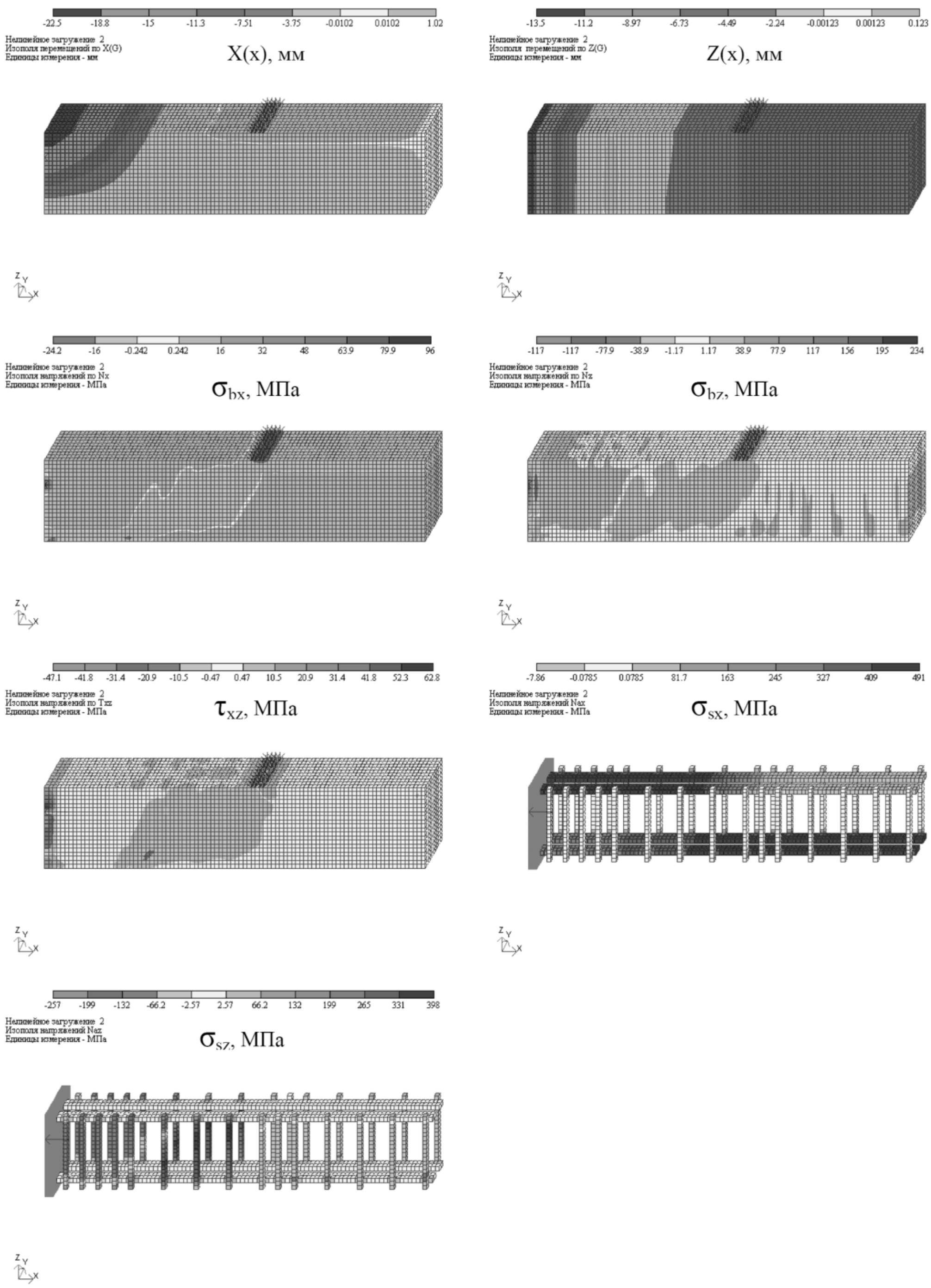

Fig. 1. Isofields of the movements and stresses in the material of the eccentrically tensioned beam which collapses because of yielding of the top erection bars

Hence, the analysis of the simulated average values of movements and stresses in concrete and reinforcement in the characteristic cross-sections of the plain stress studied elements has confirmed the earlier obtained data (Karpiuk, 2014) that the collapse of the eccentrically tensioned beams according to *A-1/Nв* pattern begins

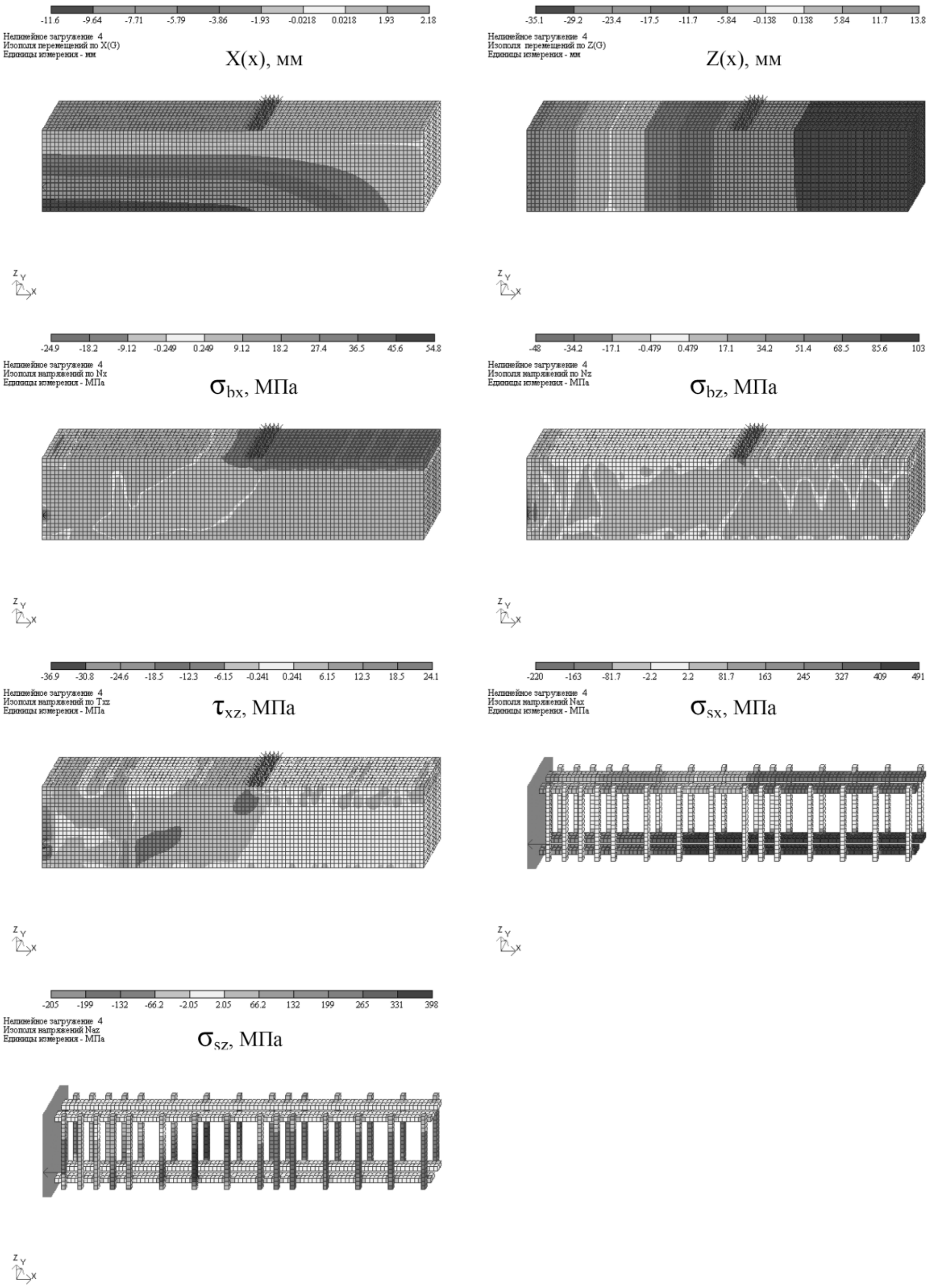

Fig. 2. Stress-strain behaviour of the eccentrically tensioned beam that collapses because of yielding of the longitudinal working reinforcement in the mouth of a hazardous inclined crack which reaches the yielding limit in the transverse rods

with an extraordinary high level of stress in the top erection bars (0.95...0.98f_{yk}) above the end support if their number is insufficient. It just might occur that the *A-2/Nн* collapse pattern can take place in case of the inadequate number of the lower (working) bars. At that, the nature of formation (across the entire height of the cross-section) and opening of normal cracks indicates that the so-called 2nd case of the eccentric tension takes place, i.e., the case of small eccentricities.

Table 1
Results of simulation of the stress-strain behaviour of the eccentrically tensioned and compressed reinforced concrete beams with the aid of “Lira” software prior to collapse

Test No.	**Deflections in characteristic cross-section of a beam**		**Averaged stresses in concrete and reinforcement, MPa**								**Collapse pattern**
	-Z, мм		$-\sigma_{cx}$	$-\sigma_{cz}$	max $-\tau_{cxz}$	σ_{sx}		σ'_{sx}		max $-\sigma_{swz}$	
	x=a	*x=l/2*	*x=a*	*x=0…a*	*x=0…a*	*x=0*	*x=a*	*x=0*	*x=a*	*x=0…a*	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Series III-A. Real two-factor experiment at $N_{tension}$										
1	25,1	41,9	15,9	16,4	5,8	409	491	245	163	260	*B/M*
2	4,50	13,5	16,00	20,0	5,5	327	491	491	245	398	*A-1/N*$_{в}$
3	4,80	7,21	22,7	8,09	6,1	228	456	39,7	-75,9	398	*C/V*
4	5,25	7,90	20,70	9,8	9,77	240	480	80	-160	398	*C/V*
5	25,60	38,40	15,90	23,9	5,0	245	491	491	163	398	*A-1/N*$_{в}$
6	4,60	6,90	20,3	8,1	7,9	218	437	72,8	-146	398	*C/V*
7	23,40	35,10	18,2	17,10	6,15	327	491	163	-81,7	398	*B/M*
8	4,08	6,13	24,1	9,3	6,08	216	432	288	-71,9	398	*C/V*
9	4,44	6,66	18,2	12,3	5,33	316	475	237	-79	394	*B/M*
Series III-B. Numerical five-factor experiment at $N_{compress}$											
1	25,1	41,9	15,9	16,4	5,8	409	491,0	245,0	82	260	*B/M*
2	3,0	6,0	15,9	16,7	5,8	409,0	491,0	124	82	134	*B/M*
3	2,9	5,8	23,3	22,9	13,1	231	396,0	66	-66	313	*D//см*
4	6,1	9,2	15,0	11,3	6,4	163,0	491,0	81,7	-245,0	402	*C/V*
5	2,4	4,8	16,0	15,4	5,6	381,0	457,0	152,0	76,2	119	*B/M*
6	15,0	22,5	8,0	16,7	5,8	409,0	491,0	245,0	81,7	236	*B/M*
7	11,2	13,6	26,3	10,6	6,6	163,0	491,0	81,7	-245	231	*C/V*
8	32,3	40,3	11,2	10,1	5,9	368	491,0	327,0	-204	388	*B/M*
9	1,8	5,3	8,2	18,8	5,4	245	327,0	491,0	327	80	*A-1/N*$_{в}$
10	1,4	2,8	8,0	19,9	5,4	245	327,0	491,0	245,0	202	*A-1/N*$_{в}$
11	4,2	6,4	20,2	6,2	5,8	158,0	475,0	2,38	-237,0	251	*C/V*
12	7,4	11,1	10,0	7,5	6,1	368	491,0	81,7	-204	402	*B/M*
13	3,3	-6,6	24,3	20,8	11,1	33	448,0	38	-112	268	*B/M*
14	24,7	41,1	10,0	8,8	5,2	328	491,0	43	-368	310	*B/M*
15	0,9	9	8	23,0	5,5	163	163	491,0	409	-2,4	*A-1/N*$_{в}$
16	2,5	5,0	8,1	20,3	5,5	327	325	491,0	409	318	*A-1/N*$_{в}$
17	6,3	9,4	16,9	12,3	5,9	327	491,0	205,0	-116	290	*B/M*
18	4,1	6,2	13,7	12,1	13,3	309,0	465,0	232,0	-77,3	398	*B/M*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
19	9,2	13,8	18,3	11,0	7,9	245,0	491,0	163,0	-81,7	344	B/M
20	4,9	7,3	12,2	12,3	8,0	281,0	482,0	241,0	-120,0	398	*B/M*
21	4,4	6,6	13,7	12,3	8,0	317,0	475,0	237,0	-79,0	300	*B/M*

22	4,2	6,3	13,7	12,2	5,34	335	446,0	223,0	-74,3	402	*B/M*
23	2,3	3,5	8,1	1,3	5,0	286	409,0	409,0	163,0	155	*A-1/Nв*
24	4,6	6,9	20,2	12,2	7,9	218,0	437,0	38,0	-218	398	*C/V*
25	4,4	6,7	16,2	17,2	9,2	286	490,0	41,0	-104	339	*B/M*
26	4,1	6,1	18,2	9,3	6,1	216,0	432,0	360,0	-36,0	398	*B/M*
27	4,4	6,7	13,7	12,3	5,3	317,0	475,0	237,0	-79,1	392	*B/M*
Series III-V. Real five-factor experiment at $N_{compress}$											
1	4,4	6,7	28,3	21,1	12,6	-163	81,7	-42	-122	29	*Д//см*
2	2,5	5,1	28,5	24,7	15,9	188,0	226,0	19,9	-113	398	*Д//см*
3	2,9	6,7	28,3	21,0	12,7	82	163	-163	-409	3,4	*
4	7,4	11,1	25,5	13,8	6,0	81,7	245	-41,0	-200	2,2	*, **
5	3,6	7,3	28,3	20,8	12,6	163,0	245,0	-81,7	-409	66	*Д//см*
6	18,8	28,3	28,2	24,4	12,9	163,0	245,0	-163	-491	35	*, **
7	8,3	12,5	28,3	21,3	12,7	-81,7	281,7	-5	-200	34	*, **
8	5,1	7,6	29,0	25,6	13,9	311,0	373,0	33	-186	398	*Д//см*
9	2,7	5,5	28,3	20,9	12,6	77,5	103,0	-1,0	-387	398	*Д//см*
10	5,2	8,4	30,5	17,1	7,9	-4,2	163	-81,7	-163	30	*, **, *Д//см*
11	9,4	14,2	28,3	10,8	6,7	81,7	245	-163	-409	66,2	*, **
12	6,3	9,4	27,9	24,5	13,9	409,0	491,0	-81,7	-245	398	*Д//см, B/M*
13	3,2	4,8	28,3	20,7	12,6	1,1	51	-51,0	-204	398	*Д//см*
14	6,4	12,9	25,5	17,0	7,9	-4,9	245	-4,9	-245	33	*, **
15	13,5	21,9	28,2	22,6	12,9	81,7	245	-163	-409	33	*, **, *Д//см*
16	5,5	8,3	25,5	25,6	11,9	245,0	359,0	-81,7	-409	398	*Д//см*
17	9,1	13,7	19,3	13,6	7,8	-4,9	245	-81,7	-245	33	*, **
18	10,1	12,1	29,0	27,4	7,9	163	323,0	-81,7	-409	398	*Д//см*
19	5,6	8,4	28,3	22,0	12,2	-4	327,0	-81,7	-491	66,2	*Д//см*
20	6,4	9,6	25,5	20,2	11,8	163,0	409	-81,7	-368	221	*Д//см*
21	5,1	7,6	29,0	25,3	11,9	3,5	245,0	-43	-327,0	308	*Д//см*
22	8,8	13,0	29,0	25,3	11,9	81,7	409,0	-81,7	-409,0	132	*, *Д//см*
23	6,7	10,0	29,0	20,3	11,8	81,7	327,0	-81,7	-409,0	328	*Д//см*
24	8,0	12,0	29,0	20,3	11,7	81,7	409,0	-81,7	-409,0	132	*, *Д//см*
25	5,6	8,4	29,0	20,3	11,8	81,7	327	-43	-409,0	325	*Д//см*
26	9,1	13,6	29,0	20,4	11,8	81,7	409,0	-81,7	-409,0	132	*Д//см*
27	7,8	11,7	29,0	20,3	11,8	81,7	409,0	-81,7	-409,0	199	*Д//см*

** – bearing of concrete in the pure bending zone;*

*** – yielding of the working reinforcement.*

Source: (Karpiuk, 2014).

The entire transverse cross-section is tensioned, and through transverse cracks are being formed in the boundary condition of the beam (by strength) and along the element. Therefore, we will assume that in this case the concrete does not take part in the work of the element while the boundary force in the normal cross-sections that coincide with the cracks is taken up by reinforcement only. Collapse of the element takes place when the stress in the longitudinal lower or upper reinforcement N_{s34} or N_{s12} reaches the boundary values, i.e., the characteristic (normative) support resistance (indices 12 and 34 mean “referencing” of the longitudinal reinforcement gravity centres to the coordinate axes of beams *X*, *Y* that are directed rightwards and up).

Collapse of the eccentrically tensioned beams according to *B/M* pattern (caused by the prevailing action of the bending moment) occurs in pure form in 48%, and is accompanied by achievement of the tension stresses in the working reinforcement in the mouth (at the beginning) of the hazardous inclined crack reaching, on the average, $0.68f_{yk}$, while in the transverse reinforcement in the shear span equals $0.82\ f_{ywk}$. In the eccentrically tensioned beams such collapse pattern occurs in the mixed form with other patterns in 4% of the cases.

Collapse on accordance with *C/V* pattern (resulting from the prevailing action of the transverse force) is characteristic of the somewhat overreinforced (ρ_f=2.3%) eccentrically tensioned elements (30%) and is accompanied by normal compressing stresses above the apex of the hazardous inclined crack at the level of $\sigma_{cx}=0.90f_{ck}$; $\sigma_{cz}=0.45f_{ck}$, as well as by tangential stresses $\tau_{xz}=0.33f_{ck}$. At that, the stress in the transverse reinforcement reaches $0.89f_{ywk.}$

Collapse of the support zones of the studied plain-stressed eccentrically tensioned (55%) reinforced concrete beams behind the inclined compressed concrete strip (*D//cm*) is typical for somewhat overreinforced ($\rho_f=1.76...2.3\%$) studied elements with small or medium ($a/h_0\leq2$) shear spans, and is characterized by average tension stresses in concrete $\sigma_{cx}\cong0.96f_{ck}$; $\sigma_{cz}\cong0.87f_{ck}$; $\tau_{xz}\cong0.46f_{ck}$ and by tension stresses in the transverse reinforcement $\sigma_{swz}\cong0.52f_{ywk}$.

4. CONCLUSIONS

Having analysed the presented information, the following has been established:

1. The analysis of the results of simulation of the stress-strain behaviour of the studied elements confirmed and enhanced the earlier obtained test data on the nature of their deformation and collapse.
2. Simulation of the stress-strain behaviour of the studied structures making use of the non-linear finite element calculations and use of the real diagrams illustrating condition of the materials and current phenomenological strength criteria with the aid of the field-proven software complex “Lira” enables to numerically describe the test results and make a reliable forecast first of all of their strength.
3. In order to determine strength of the inclined cross-sections of the elements subject to bending it is recommended to choose the most probable collapse pattern.

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