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REDUCTION OF A VEHICLE MULTIBODY DYNAMIC MODEL USING HOMOTOPY OPTIMIZATION

Despite the ever-increasing computational power of modern processors, the reduction of complex multibody dynamic models remains an important topic of investigation, particularly for design optimization, sensitivity analysis, parameter identification, and controller tuning tasks, which can require hundreds or thousands of simulations. In this work, we first develop a high-fidelity model of a production sports utility vehicle in Adams/Car. Single-link equivalent kinematic quarter-car (SLEKQ, pronounced "sleek") models for the front and rear suspensions are then developed in MapleSim. To avoid the computational complexity associated with introducing bushings or kinematic loops, all suspension linkages are lumped into a single unsprung mass at each corner of the vehicle. The SLEKQ models are designed to replicate the kinematic behaviour of a full suspension model using lookup tables or polynomial functions, which are obtained from the high-fidelity Adams model in this work. The predictive capability of each SLEKQ model relies on the use of appropriate parameters for the nonlinear spring and damper, which include the stiffness and damping contributions of the bushings, and the unsprung mass. Homotopy optimization is used to identify the parameters that minimize the difference between the responses of the Adams and MapleSim models. Finally, the SLEKQ models are assembled to construct a reduced 10-degree-of-freedom model of the full vehicle, the dynamic performance of which is validated against that of the high-fidelity Adams model using four-post heave and pitch tests.

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1. Introduction

A fundamental objective of model reduction techniques is the systematic formulation of a minimally complex model that represents all behaviour of interest in the original model to a sufficient level of precision [1]. This objective echoes Einstein's famous sentiment that models should be made as simple as possible, but no simpler [2]. A reduced model can be particularly useful in the design and optimization of vehicle suspensions, where bushings and kinematic loops result in stiff or differential-algebraic equations that can be time-consuming to solve.

In this work, we first develop high-fidelity models of the front MacPherson strut and rear semi-trailing-arm multi-link suspensions of a production sports utility vehicle in Adams/Car[1]. A single-link equivalent kinematic quarter-car (SLEKQ, pronounced "sleek") model is then developed in MapleSim[2]. Since bushings and kinematic loops increase the computational complexity of a model, all suspension linkages are lumped into a single unsprung mass at each corner of the vehicle, which is similar to the approach used by CarSim [3]. Unlike the simple quarter-car models traditionally used, which provide only approximations of the true suspension kinematics, each SLEKQ model has the same kinematic behaviour as the analogous full suspension model. In this work, polynomial functions obtained from the high-fidelity Adams model are used to define the kinematic behaviour of each SLEKQ model. One advantage of this approach is that a single multibody model can be used to represent any suspension type simply by adjusting the kinematic curves and model parameters. Furthermore, the performance implications of varying the suspension curves (such as the relationship between camber and vertical displacement) can be evaluated simply by adjusting these curves directly in the model, rather than modifying specific hard point locations.

The accuracy of the dynamic performance of each SLEKQ model relies on the use of appropriate parameters for the nonlinear spring and damper (which include the stiffness and damping contributions of the bushings) and the unsprung mass. The parameters that minimize the difference between the response of the Adams model and that of the MapleSim model are identified using homotopy optimization [4], an optimization technique designed to avoid converging to a local minimum. The use of homotopy optimization in this work is related to the work of Abarbanel et al. [5], and follows the approach presented by Vyasarayani et al. [6]. Once the parameters for the SLEKQ models have been identified, they are used to construct a reduced

[1] Adams is a trademark of MSC.Software Corporation.
[2] MapleSim is a trademark of Waterloo Maple Inc.

10-degree-of-freedom model of the full vehicle, the dynamic performance of which is validated against that of the Adams model.

The development of the high-fidelity Adams model is described briefly in Section 2. The development of the SLEKQ model is presented in Section 3. In Section 4, we discuss the strategy used for identifying the optimal parameters for the reduced model; the parameter identification results are presented and validated in Section 5. Finally, conclusions and future work are outlined in Section 6.

2. High-fidelity Adams model

To aid in the development and tuning of the reduced model, a high-fidelity full vehicle model is first created using Adams/Car 2010, a software tool widely used in the automotive industry for simulating the dynamics of vehicles. The model is developed using measured data and specifications obtained from component drawings. The production sports utility vehicle being studied has a MacPherson strut front suspension and a semi-trailing-arm multi-link rear suspension, as shown in Figure 1.

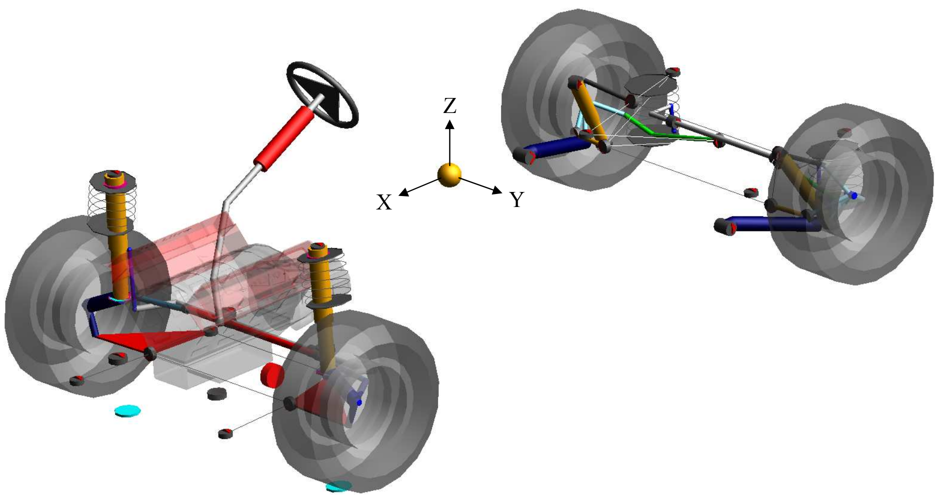

Fig. 1. High-fidelity Adams/Car model

Component specification data is used to determine the linear rates for the suspension coil springs as well as the splines describing the response of the dampers. Bushings are modelled as linear elements with damping rates equal to 1% of the linear stiffness rates, as suggested in [7]. All other chassis components, including control arms, uprights, and subframes, are modelled as rigid bodies with known masses; inertias are estimated from the material properties and approximate component geometries.

3. Single-link equivalent kinematic quarter-car model

A reduced version of the Adams model described in Section 2 is developed in MapleSim 5. Each corner of the suspension in the high-fidelity model is replaced with a reduced model containing a massless suspension joint, which is designed to retain the same kinematic performance as the high-fidelity model (using data obtained from Adams) while eliminating all closed kinematic chains. Once all parameters have been identified for the SLEKQ models, they are used to construct a reduced model of the full vehicle. The symbolic simplification and optimization algorithms available in MapleSim generate dynamic simulation code that is computationally efficient, a highly beneficial characteristic for parameter identification applications. The differentiability of the symbolic expressions can also be advantageous for sensitivity analysis purposes.

The Adams model is used to generate lookup tables or polynomial functions describing the orientation (a function of camber, caster, and toe) of the unsprung mass and its displacements in the X- and Y-directions as functions of its vertical displacement. In this work, second- and third-order polynomials have been used. These kinematic suspension curves can also be obtained either experimentally or, in simple cases, analytically based on the geometry of the suspension. Three body-fixed rotations are used to determine the orientation of the unsprung mass, the first two of which are defined by the suspension toe angle (about the Z-axis) and camber angle (about the rotated X'-axis). The third rotation (about the Y''-axis) can be related to the variation in the steering axis due to the heave motion of the suspension; however, due to differences in the definition of the steering axis for different suspension types, a single function relating this third rotation to the orientation of the steering axis is not possible. MapleSim reports the orientation of a rigid body using 9 direction cosines, which are used to validate the rotational kinematics of the SLEKQ model (see Figure 2); validation of the translational kinematics is trivial.

Once the kinematic behaviour of the SLEKQ model has been validated, its dynamic behaviour must be considered. We must first locate the base of the spring and damper relative to a coordinate frame fixed to the unsprung mass. Correctly locating the base of the spring and damper is necessary to ensure the forces generated by these components are applied along the appropriate line of action. Appropriate spring stiffness and damping coefficient curves must also be determined. Since the objective in this work is to create a simplified, computationally efficient model, the bushings are not modelled individually. Instead, the bushing stiffness and damping effects are lumped into the spring and damper curves, which must be identified. The

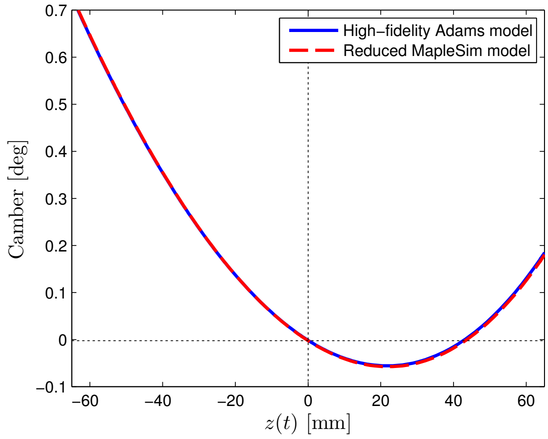

Fig. 2. Validation of camber for front SLEKQ model

contribution of each bushing to the overall system behaviour is very difficult to ascertain analytically, thus representing an ideal opportunity to apply a parameter identification strategy.

The following piecewise linear function is used to define the damper force:

$$F_d(t) = \begin{cases} -d_1 d_2 - d_3\,(v(t) + d_2)\,, & v(t) < -d_2 \\ -d_1 v(t), & -d_2 \le v(t) < 0 \\ -d_4 v(t), & 0 \le v(t) < d_5 \\ -d_4 d_5 - d_6\,(v(t) - d_5)\,, & v(t) \ge d_5 \end{cases} \tag{1}$$

where $v(t)$ is the speed of the damper piston, d_1 and d_4 are the low-speed bump and rebound rates, d_2 and d_5 define speeds of transition between low- and high-speed bump and rebound, and d_3 and d_6 are the high-speed bump and rebound rates. All coefficients d_i are determined using parameter identification. Equation 1 was selected for its ability to accommodate differences between bump and rebound damping, as well as differences between low- and high-speed damping. Note that a second-order Bézier curve is used at each transition in Equation 1 to enforce C^1 continuity. The spring force is defined as follows:

$$F_k(t) = k_3\,\Delta L^3 + k_2\,\Delta L^2 + k_1\,\Delta L + k_0 \tag{2}$$

where ΔL is the amount of spring compression from its undeformed length, and coefficients k_i must be identified. A third-order polynomial was found to provide a sufficient approximation of the spring curve.

The final parameter that must be identified in the SLEKQ model is the unsprung mass. The upright and all components outboard thereof contribute entirely to the unsprung mass; however, the distribution of each control arm and drive shaft between the sprung and unsprung masses is unknown. In practice, various approximations are typically used in the automotive industry, such as splitting the mass of these components equally between the sprung and unsprung masses [8]. To improve the accuracy of the reduced model, however, the unsprung mass is also identified.

4. Parameter Identification

The identification of parameters in a mathematical model can be viewed as an optimization problem in which the error between a set of experimental data and the model response is minimized [9]. The homotopy optimization strategy [4] was selected for its ability to avoid converging to a local minimum, even if only poor initial parameter guesses are known (as is the case in this work). Furthermore, homotopy optimization has already been successfully applied to parameter identification problems for reduced vehicle models [6] and other multibody systems [10]. The homotopy optimization approach involves coupling a high-gain observer to the equations of motion [11]:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{p}, t) + \lambda \boldsymbol{\Gamma} \mathbf{e}(t) \tag{3}$$

where $\mathbf{x}$ is the state vector, $\mathbf{p}$ contains the parameters to be identified, $0 \le \lambda \le 1$ is the coupling parameter, $\boldsymbol{\Gamma}$ contains the observer gains, and $\mathbf{e}(t)$ is the error vector (the difference between the experimental data and the model response). The observer term $\lambda \boldsymbol{\Gamma} \mathbf{e}(t)$ smooths the objective function, which prevents convergence to a local minimum. The parameter identification process begins with $\lambda = 1$, where the observers are fully coupled to the equations of motion. Gains $\boldsymbol{\Gamma}$ must be sufficiently high to force the mathematical model to track the experimental data when $\lambda = 1$. At each stage of the algorithm, λ is decreased by a small amount $\delta\lambda$, which reduces the magnitude of the coupling term. The parameters $\mathbf{p}$ are then optimized, using the optimal parameter values from the previous stage as initial guesses. This process is continued until $\lambda = 0$ and the original equations of motion are recovered. Provided $\delta\lambda$ is sufficiently small, the initial parameter guesses at each stage of the algorithm will be close to the global optimum, thereby avoiding convergence to a local minimum [6].

To demonstrate the use of homotopy optimization for parameter identification, we consider the simple quarter-car model shown in Figure 3.

In Section 5, the same strategy described below is applied to a nonlinear SLEKQ model. The unsprung mass m_{unsp} and behaviour of the spring and damper are identified by fixing the sprung mass m_{sp} to the ground and applying a vertical force $F_Z(t)$ at the tire contact patch, as in a kinematics and compliance test. To further simplify the present discussion, we shall assume

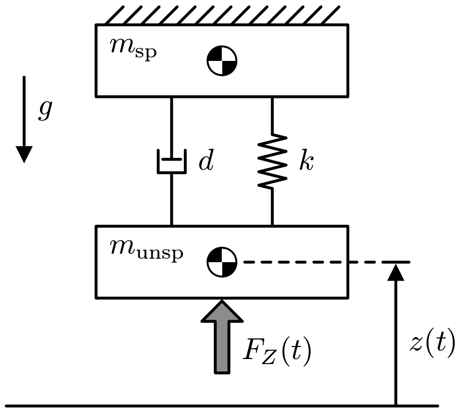

Fig. 3. Simple linear mass-spring-damper system

the damper and spring exhibit purely linear behaviour; in the MapleSim model, these components are described by Equations 1 and 2, respectively. The dynamics of this mass-spring-damper system are governed by the following second-order ordinary differential equation:

$$m_{\mathrm{unsp}}\ddot{z}(t) = F_Z(t) - d\dot{z}(t) - kz(t) - m_{\mathrm{unsp}}g \tag{4}$$

where d and k are the damping and stiffness coefficients. The homotopy coupling term is now added to the right-hand side of Equation 4. We use the errors on the position, velocity, and acceleration level in the forcing term, which then takes the form of a proportional-integral-derivative (PID) controller:

$$\lambda\left\{G_I N_I(z_{\mathrm{e}}(t) - z(t)) + G_P N_P(\dot{z}_{\mathrm{e}}(t) - \dot{z}(t)) + G_D N_D(\ddot{z}_{\mathrm{e}}(t) - \ddot{z}(t))\right\} \tag{5}$$

where $z_{\mathrm{e}}(t)$, $\dot{z}_{\mathrm{e}}(t)$, and $\ddot{z}_{\mathrm{e}}(t)$ are the experimental position, velocity, and acceleration data we wish to track using Equation 4, and N_I, N_P, and N_D are used to normalize the errors. Appropriate PID gains G_P, G_I, and G_D must be determined such that the mathematical model tracks the experimental data when $\lambda = 1$. The MapleSim implementation of the homotopy coupling term is shown in Figure 4.

The final MapleSim model is exported as an S-function, and the parameter identification procedure is performed in Matlab[3]. The `fminsearch` routine, which is based on the Nelder–Mead simplex method [12], is used to minimize the following objective function at each value of λ:

$$J = w_1 \int_0^{t_f} (z_{\mathrm{e}}(t) - z(t))^2 dt + w_2 \int_0^{t_f} (\dot{z}_{\mathrm{e}}(t) - \dot{z}(t))^2 dt + w_3 \int_0^{t_f} (\ddot{z}_{\mathrm{e}}(t) - \ddot{z}(t))^2 dt \tag{6}$$

[3] Matlab is a registered trademark of The MathWorks, Inc.

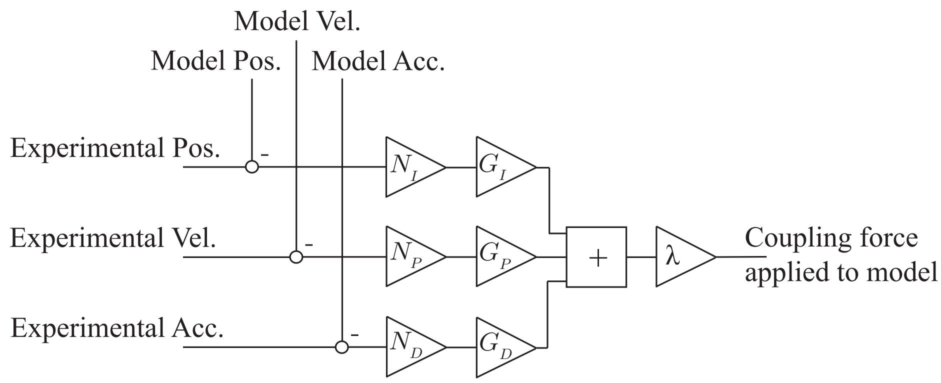

Fig. 4. MapleSim implementation of homotopy coupling term

where $z(t)$, $\dot{z}(t)$, and $\ddot{z}(t)$ are obtained from Equations 4 and 5, and w_i are weights. Experimental data is generated from the high-fidelity Adams model using a swept-sine force input $F_Z(t) = F_0 + 1.0\sin(\omega t)$ kN, where the offset F_0 is the corner weight of the vehicle. A frequency range of $0 \leq \omega \leq 6$ Hz ensures that the input signal is sufficiently rich to capture the behaviour of the suspension encountered in typical highway driving conditions [13]. Note that the undeformed length of the spring in the MapleSim model is computed such that the ride height of the reduced model matches that of the Adams model. An initial estimate for m_{unsp} is determined using the approach presented by Milliken and Milliken [8], where the mass of each suspension component is equally distributed between the sprung and unsprung masses. The initial estimate for the spring curve is determined using ride rate data from the Adams model. Since the contribution of the bushings to the suspension damping characteristics is unknown, the initial estimate for the damper curve is determined solely based on the damper in the Adams model.

5. Results and Discussion

The homotopy optimization approach described in Section 4 is used to identify parameters for two SLEKQ models: one representing the front suspension of the high-fidelity model, the other representing the rear suspension. The responses of each of these models using the initial parameter guesses and final identified parameters are shown in Figures 5 and 6; the homotopy coupling gains used for each model are given in Table 1.

Note that the front SLEKQ model tracks the experimental data more successfully than the rear. This discrepancy can be attributed to two properties unique to the rear suspension that cannot be adequately represented by the current SLEKQ model. In particular, as shown in Figure 1, the rear damper is not nearly as vertical as that on the front suspension. Furthermore, the rear

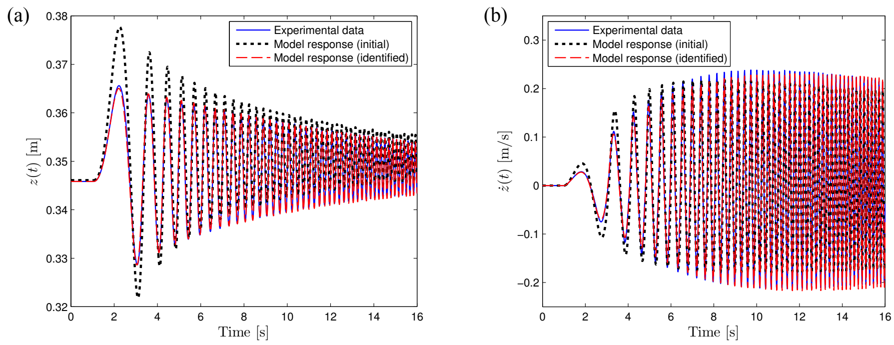

Fig. 5. Comparison between experimental data and model response for front suspension using initial parameter guesses and final identified parameters: (a) displacement and (b) velocity of unsprung mass

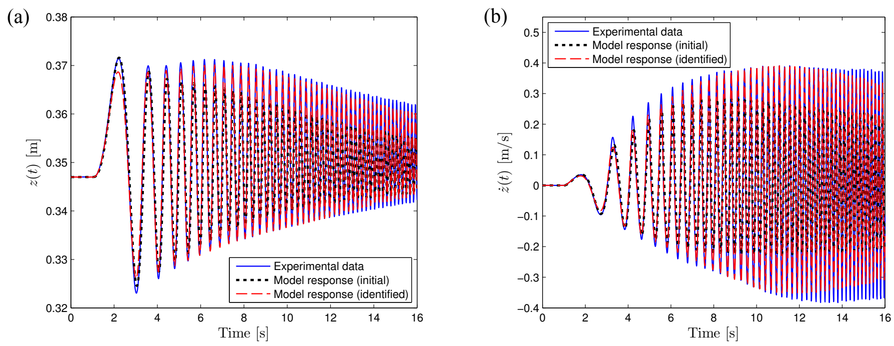

Fig. 6. Comparison between experimental data and model response for rear suspension using initial parameter guesses and final identified parameters: (a) displacement and (b) velocity of unsprung mass

Table 1.

Homotopy coupling gains used to optimize each SLEKQ model

	G_I	G_P	G_D
Front suspension	0.1	2.0	1.0
Rear suspension	0.5	1.5	0.8

damper is attached to the chassis with a relatively soft bushing. The effect of these two properties is that, at high piston velocities, the damper force has a large longitudinal component, which deforms the relatively soft bushing at the damper-to-chassis mount and has a significant effect on the system response. Although the damping curve has been identified, this phenomenon cannot be reproduced entirely, as evinced by the discrepancy between the experimental data and model response in Figure 6 at higher frequencies.

As shown in Table 2, the identified unsprung masses for the front and rear suspensions are both within physically reasonable ranges (i.e., including between 0% and 100% of the mass of the suspension linkages).

Table 2.
Parameter identification results for unsprung masses

	Lower bound [kg]	Upper bound [kg]	Initial guess [kg]	Identified value [kg]
Front suspension	58.5	63.7	61.1	59.1
Rear suspension	63.9	76.1	70.0	64.1

Note that, in both cases, the suspension linkages contribute nearly all of their mass to the sprung mass. The identified spring curves for the front and rear suspensions, shown in Figure 7, both characterize springs that are only marginally stiffer than those used in the high-fidelity Adams model.

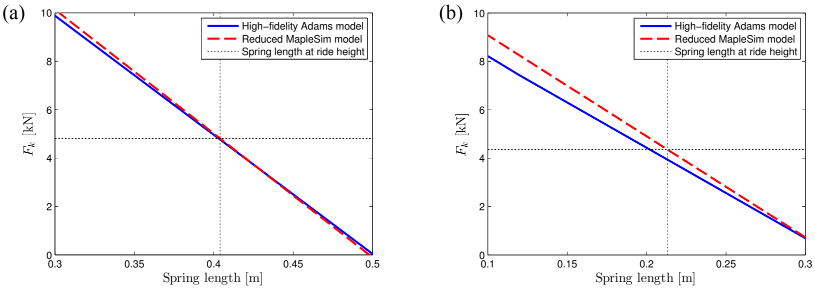

Fig. 7. Force-displacement curves for springs in Adams and MapleSim models on (a) front and (b) rear suspensions

The identified damper curves are shown in Figure 8.

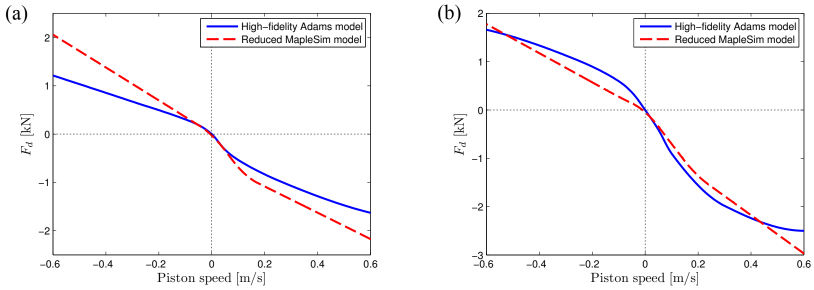

Fig. 8. Force-velocity curves for dampers in Adams and MapleSim models on (a) front and (b) rear suspensions

In both cases, the identification procedure stiffens the damper in the high-speed range. Note that the bump damping (negative damper piston speed) is nearly linear in both the front and rear suspensions.

To validate the parameter identification results, the front and rear suspensions are used to assemble a full vehicle model for performing four-post tests. Four SLEKQ models are attached to a single rigid body, representing the sprung mass of the vehicle. The current model is assumed to be laterally symmetric. Furthermore, since only the heave and pitch motions of the vehicle are currently being investigated, the only additional parameters that appear in the four-post model are the mass and pitch inertia of the sprung mass, the location of the sprung mass center of gravity, and the wheelbase. These additional parameters are obtained directly from the high-fidelity model. Heave and pitch tests are performed, the former of which involves exciting the front and rear suspensions in phase, and the latter of which involves exciting the front and rear suspensions 180° out of phase. In both cases, a 30 mm/s constant-velocity sinusoidal sweep is used, with a frequency range of 0 to 6 Hz. As before, the tests are repeated on the high-fidelity Adams model for comparison. The results are shown in Figure 9, and confirm that appropriate parameters have been identified for the SLEKQ models.

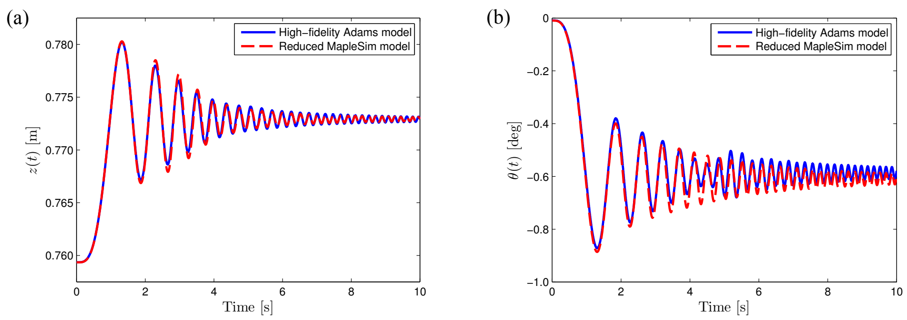

Fig. 9. Results from four-post validation tests: (a) chassis vertical displacement during heave test, and (b) chassis pitch during pitch test

6. Conclusions and Future Work

The development of a single-link equivalent kinematic quarter-car MapleSim model using a massless suspension joint has been presented. Homotopy optimization has been used to identify the unsprung mass and coefficients for the spring and damper curves used in this model so that its response matches that of a MacPherson strut front suspension modelled in Adams/Car. The same model has been used to match the response of a semi-trailing-arm multi-link rear suspension, demonstrating the versatility of the reduced model. A full vehicle model was assembled using these reduced suspension models, and four-post tests were performed to validate the identified parameters.

A remaining area of investigation is the addition of compliance to the reduced model, particularly to account for the relatively soft bushing between the damper and chassis on the rear suspension. Chassis stiffness and the validity of the assumption that the vehicle is laterally symmetric will also be investigated. Finally, the track widths and the roll and yaw inertias of the chassis will be identified using roll and warp four-post tests.

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Redukcja wieloczłonowego dynamicznego modelu pojazdu przy zastosowaniu optymalizacji homotopowej

S t r e s z c z e n i e

Pomimo stale rosnącej mocy obliczeniowej współczesnych procesorów, redukcja złożonych, wieloczłonowych modeli dynamicznych pozostaje ważnym tematem badań, zwłaszcza dla optymalizacji projektowania, analizy wrażliwości, identyfikacji parametrów i optymalizacji sterowników, które mogą wymagać setek lub tysięcy symulacji. W pierwszej części pracy autorzy przedstawiają model o wysokiej wierności opracowany dla seryjnie produkowanego samochodu sportowo-użytkowego (SUV) przy pomocy oprogramowania Adams/Car (MSC.Software Corporation). Następnie w środowisku MapleSim (Waterloo Maple Inc.) zostały opracowane równoważne ćwiartkowe (quarter-car) modele kinematyczne o jednym połączeniu, typu SLEKQ, dla zawieszenia przedniego i tylnego. By uniknąć komplikacji obliczeniowych związanych z wprowadzeniem tulejowania lub pętli kinematycznych, wszystkie układy przenoszące zawieszenia zostały zastąpione pojedynczymi skupionymi nieresorowanymi masami w każdym rogu pojazdu. Zaprojektowane modele typu SLEKQ odtwarzają właściwości kinematycznych modelu kompletnego zawieszenia wykorzystując przy tym tablice przeglądowe lub funkcje wielomianowe, które zostały wcześniej wyznaczone za pomocą wysokiej wierności modelu typu Adams. Zdolność predykcyjna każdego modelu SLEKQ zależy od użycia właściwych parametrów opisujących nieliniowe sprężyny i amortyzatory, które uwzględniają sztywność i wpływ na tłumienie drgań wnoszony przez tulejowanie i nieresorowane masy. Optymalizację homotopową zastosowano w celu identyfikacji tych parametrów, które minimalizują różnicę między odpowiedziami uzyskanymi w modelach typu Adams i SLEKQ. Ostatecznie, z modeli SLEKQ zostaje złożony zredukowany model o dziesięciu stopniach swobody reprezentujący cały pojazd. Właściwości dynamiczne tego modelu są poddane walidacji przez porównanie z właściwościami wysokiej wierności modelu typu Adams w czterokolumnowych testach kołysania i pochylania.