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## MODELLING OF VIBRATION WITH ABSORBER

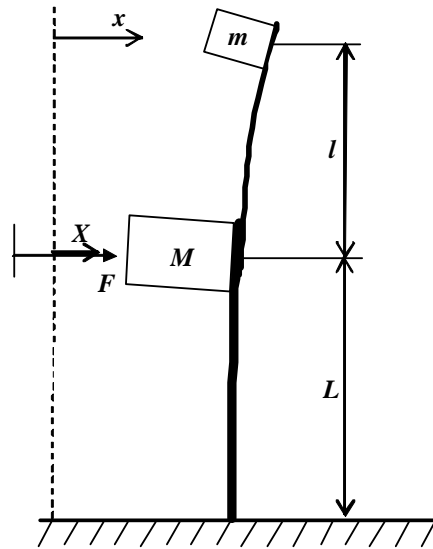
Machine vibration occurs as a result of unbalanced revolving masses, loading forces and moments, starting and coasting of driving motors and other effects. We try to eliminate these undesirable vibrations by suitable mounting of the mechanical system. Another possibility of suppression of the vibration machine is to use an absorber. A model of a vibration mechanical system with a vibration absorber is presented in the paper. The model consists of a fixed-end vibrating beam with primary vibrating mass  $M$  and with stiffness  $K$  and damping  $B$ . The harmonic force  $F$  with frequency  $\omega$  and amplitude  $A$  acts on the primary mass  $M$  as well as on the beam with stiffness  $k$  and damping  $b$ , where the secondary mass  $m$  of the absorber is mounted. The task of the absorber is to minimize the movement of the mass  $M$ . A model (Fig. 1) has two degrees of freedom  $X$  and  $x$ . Two equations describe the movement of the primary mass  $M$  and the secondary mass  $m$  of the absorber. The model is solved first numerically by using Simulink. In numerical solution it is possible to set up parameters of the mechanical system with respect to the vibration reduction. According to the mechanical requirements for the mechanical system (output trajectory, velocity, forces and torques acting on the individual parts of the system), it is also possible to design input parameters (driving torques and other parameters of the mechanical system). A experimental model will be designed in order to verify results, where a voice-coil will be used as the actuator of the force  $F$ . Numerical and experimental model allows changing parameters i.e. masses  $M$  and  $m$ , length  $L$  and  $l$  of the beams and frequency  $\omega$  of the excitation force  $F$ . The results from the numerical solution are then possible to compare with the measured values of the experimental model. It is assumed that the experimental model will be equipped with accelerometers enabling the measuring of vibration, which will be placed on the masses  $M$  and  $m$ .

### 1. MODEL OF THE SYSTEM WITH ABSORBER

The model (Fig. 1) shows vibrating primary beam (length  $L$ , thickness  $t_p$ , width  $b_p$ ) fixed at bottom end with primary mass  $M$  and vibrating secondary beam (length  $l$ , thickness  $t_s$ , width  $b_s$ ) with secondary mass  $m$  of the absorber. The masses of both beams are small as compared to the masses  $M$  and  $m$ . A harmonic force  $F$  (frequency  $\omega$ , amplitude  $A$ ) acts on the primary mass  $M$  [1].

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Fig. 1. Model of the vibrating mass  $M$  with absorber

Dynamical behaviour of the mechanical system [2],[3] with two degrees of freedom  $X$  and  $x$  (Fig. 1) describes two equations as follows:

$$\begin{aligned} M\ddot{X} + B\dot{X} - b\dot{x} + KX - kx &= F, \\ m\ddot{x} - b\dot{X} + b\dot{x} - kX + kx &= 0. \end{aligned} \quad (1)$$

The parameters of the model are:

Table 1. Parameters of the model

Property	Primary	Absorber	Units
Mass $M, m$	0.800	0.396	kg
Length $L, l$	300	80	mm
Thickness $t_p, t_s$	4.00	0.71	mm
Width $b_p, b_s$	51	23.2	mm
Modulus $E$	73.1	200.0	GPa
Mass density	2768	7860	kg/m <sup>3</sup>

If the natural frequency of the absorber equals to the excitation frequency  $\omega$ , then (in case that  $B=b=0$ ) the movement of the primary mass  $M$  will equal to zero:

$$\omega = \sqrt{\frac{k}{m}}. \quad (2)$$

On the Fig. 2 is shown the basic Simulink diagram of the solution of equations (1). The stiffness  $K$ ,  $k$  of the primary beam and secondary beam (Tab. 1) are

$$K = \frac{1}{4} E_p b_p \left(\frac{t_p}{L}\right)^3 = 2209.2 \text{ N/m}, k = \frac{1}{4} E_s b_s \left(\frac{t_s}{l}\right)^3 = 810.9 \text{ N/m}. \quad (3)$$

The Fig. 3 shows numerical solution of equations (1) with Simulink.

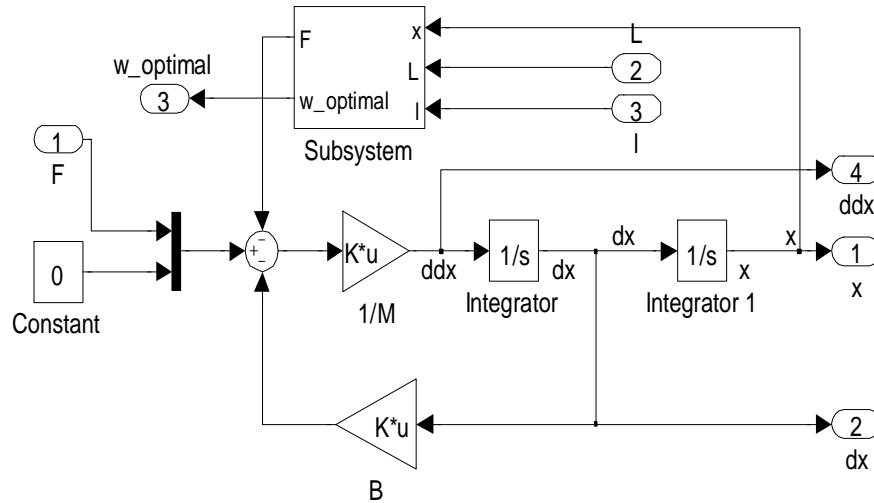


Fig. 2. Simulink diagram of the solution of equations (1)

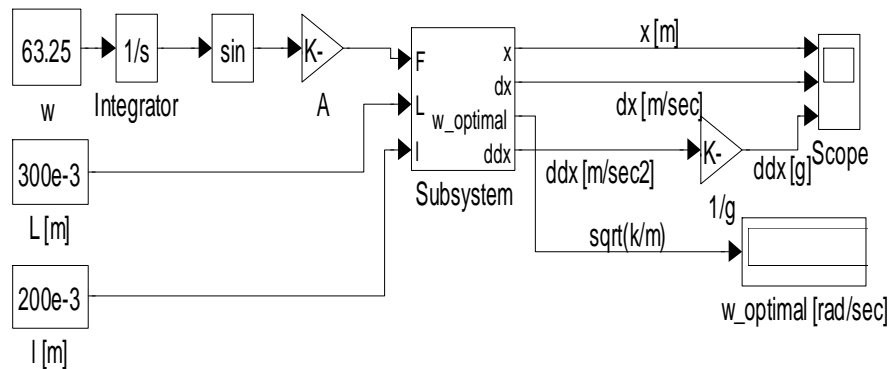


Fig. 3. Simulink diagram of the input properties of the modelled vibration absorber

## 2. EXPERIMENTAL MODEL

Based on the numerical solution was constructed the experimental model for verification of dynamic damping (Fig. 5), allowing simulation of the force  $F$ , masses  $M$  and  $m$ , length  $L$  and  $l$ . Dynamic forcing is applied to the primary mass  $M$  of the experimental model using a voice-coil actuator. A voice-coil actuator is made from permanent magnet

and cylindrical coil of magnet wire with  $N$  turns of diameter  $D$ . When a controlled electrical current  $i(t)$  is applied to the coil, a force  $F(t)$  is induced in the coil. It uses mass-produced cylindrical solenoid T3864 Series, supplied by Transmotec [4] with the manufacturer's parameters: maximum stroke 20mm, maximum power of 4,700 grams.



Fig. 4 Experimental model for verification of dynamic damping

The motion of the primary mass  $M$  and secondary mass  $m$  of the absorber are measured using accelerometers STEVAL-MKI022V1 (8 bits) and STEVAL-MKI024V1 (12 bits) delivered by MEMS Micro-Electro-Mechanical System [5]. The accelerometers transducers their acceleration to the voltage signals, which is proportional to the accelerations  $\ddot{X}(t)$  of the primary mass  $M$  and  $\ddot{x}(t)$  of the secondary mass  $m$  of the absorber.

For the experimental model (Fig. 4) two equations (1) describe the motion of the primary mass  $M$  and the absorber mass  $m$ . If force  $F$  in the first equation (1) is sinusoidal

$$F = F(t) = A \sin(\omega t), \quad (4)$$

Then the response tends also to be sinusoidal.

### 3. RESULT AND CONCLUSION

The course of the deflection of the primary mass  $X$  and secondary mass of the absorber  $x$  was first identified by a numerical solution using Simulink (Fig. 2, Fig. 3).

The Fig. 5 shows the resonance curves of the primary mass  $M$  without absorber determined by a numerical solution (solid line) with Simulink (Fig. 2, Fig. 3) and

measured values (dashed line) in an experimental model (Fig. 4) for the parameters of the model (Tab. 1).

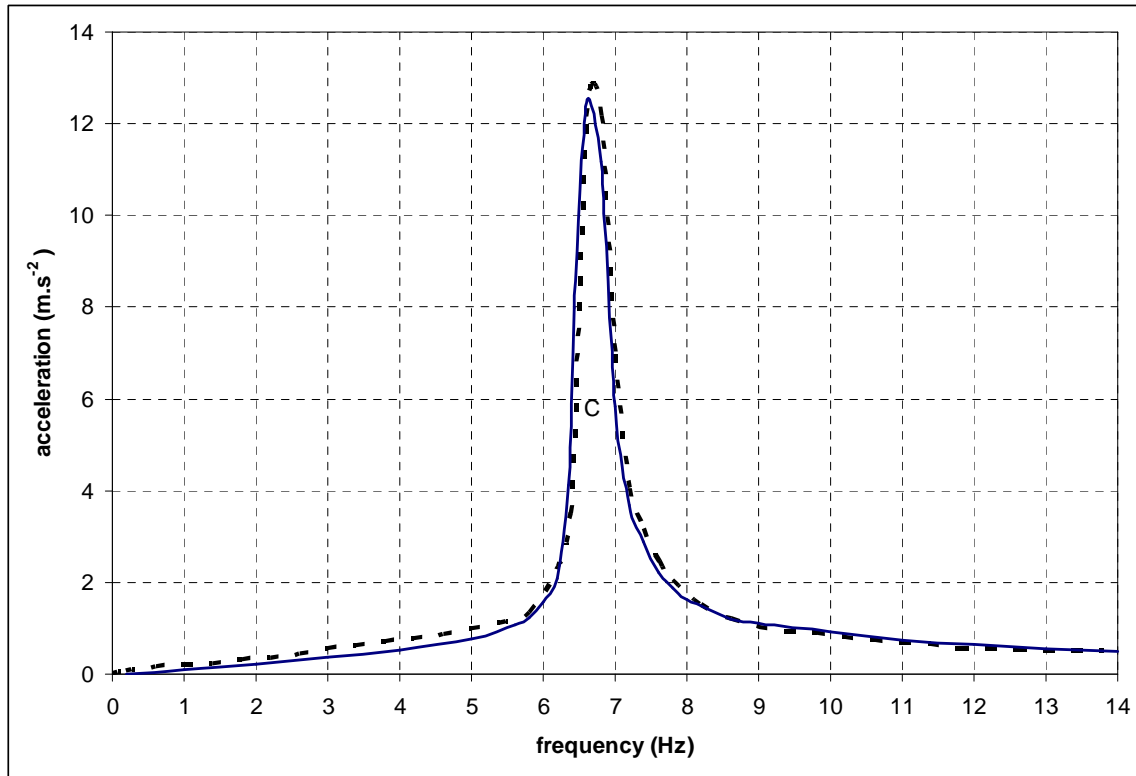


Fig. 5. Resonance curves of the primary mass  $M$  without absorber-numerical solution: solid line, measurement: dashed line

The Fig. 6 shows the resonance curves of the primary mass  $M$  with absorber determined by a numerical solution (solid line) with Simulink (Fig. 2, Fig. 3) and measured values (dashed line) in an experimental model (Fig. 4) for the parameters of the model (Tab. 1).

Comparing the resonance curve for the numerical solution (solid line) and measurement of the experimental model (dashed line) without absorber (Fig. 5) and with absorber (Fig. 6) is the agreement of both solutions. Use of the absorber is evident when comparing the curves in Fig 5 and Fig. 6, which shows that there, is significant vibration reduction main mass  $M$ . Created program for the Simulink numerical solution and proposed an experimental model suitable for modeling and simulation suppress unwanted vibrations of mechanical systems, such as equipment, machinery etc.

It is suitable for use in teaching compared numerical solutions with experimental measurements. Certain variations in the measured curves compared with the numerical solution are caused by inaccurate parameters of the experimental model. Further research will consider other parameters of the experimental model (e.g. component primary mass  $Mg \sin \varphi$  and secondary mass  $mg \sin \varphi$ , where  $\varphi$  is the deflection of beams  $L$  and  $l$ , etc.)

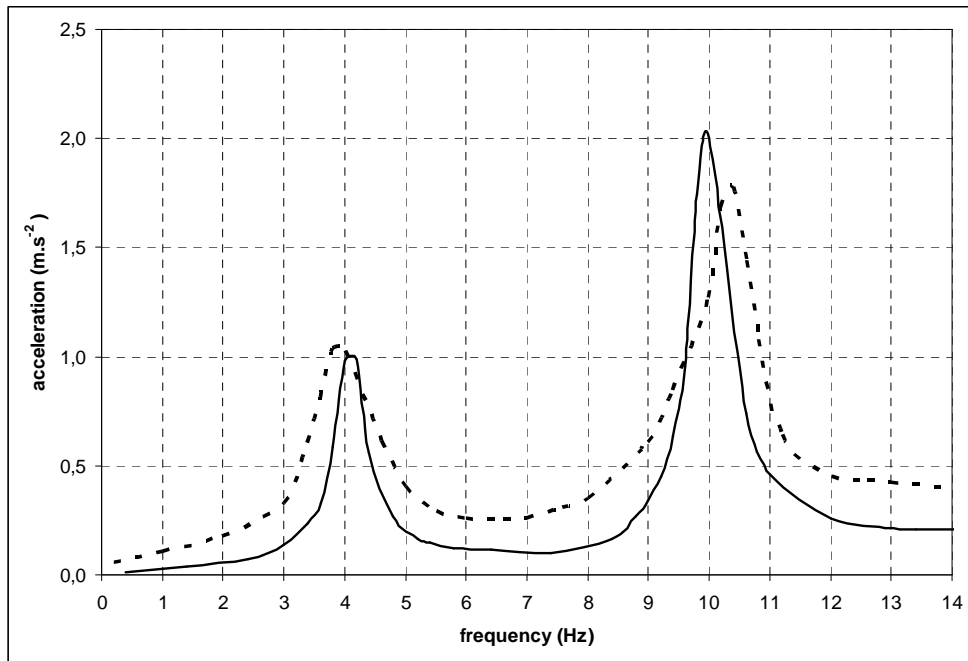


Fig. 6. Resonance curves of the primary mass  $M$  with absorber-numerical solution: solid line, measurement: dashed line

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