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Sensitivity of transient temperature field in domain of forearm insulated by protective clothing with respect to perturbations of external boundary heat flux

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Abstract. The problem discussed in the paper is numerical modeling of thermal processes in the domain of biological tissue secured by a layer of protective clothing being in thermal contact with the environment. The cross-section of the forearm (2D problem) is treated as non-homogeneous domain in which the sub-domains of skin tissue, fat, muscle and bone are distinguished. The air gap between skin tissue and protective clothing is taken into account. The process of external heating is determined by Robin boundary condition and sensitivity analysis with respect to the perturbations of heat transfer coefficient and ambient temperature is also discussed. Both the basic boundary-initial problem and the sensitivity problems are solved by means of control volume method using Voronoi polygons.

Key words: bioheat transfer, sensitivity analysis, numerical modeling, control volume method, Voronoi diagram.

1. Introduction

The problem of skin tissue heating can be described by the system of partial differential equations (energy equations), the boundary condition given on the external surface of the system, the boundary conditions between skin tissue and protective clothing, the boundary conditions on the surfaces limiting the successive sub-domains of forearm and the initial conditions. The transient temperature field in tissue subdomains is determined by a Fourier-type equation called the Pennes equation [1–5]. This equation contains two additional components (the source functions) connected with the blood perfusion and metabolism. In the case of tissue freezing, the third source function controlling the evolution of latent heat appears [6, 7]. The Pennes equation belongs to the group of the so-called macroscopic tissue models. It should be pointed out that the tissue models can also be described by the Cattaneo-Vernotte equation [8] or the dual phase lag equation [9, 10], but the Pennes approach is, so far, the most commonly used. The forearm domain is a non-homogeneous one and represents the composition of skin tissue, fat, muscle, bone and blood vessels (arteries and veins). The successive subdomains differ in the values of thermal parameters; in this paper the data quoted by Fiala et al. [11] are applied. The parameters of textiles can be found in [12].

2. Mathematical description of the process

The cross section of forearm (middle part) is shown in Fig. 1 [13].

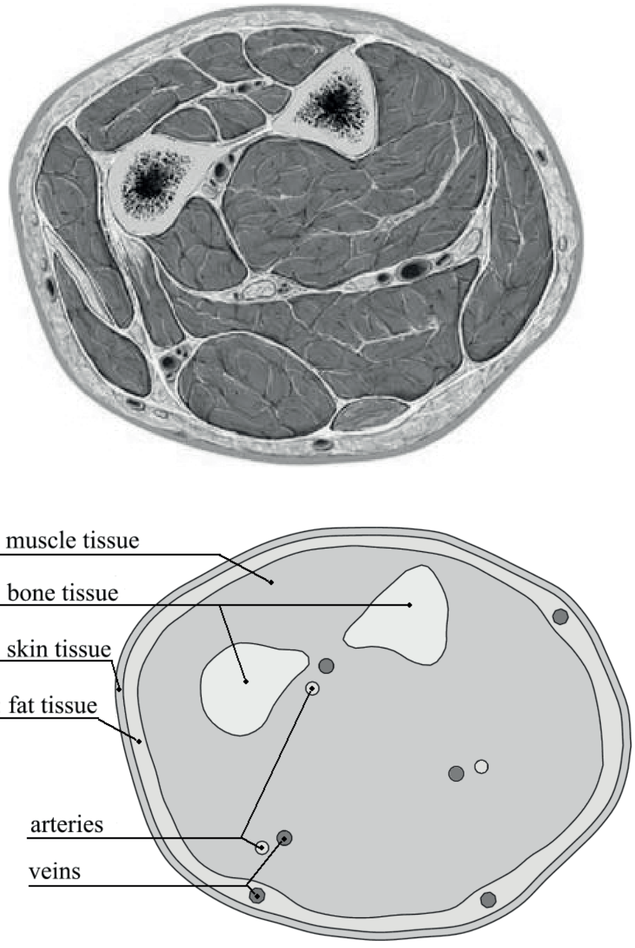

Fig. 1. Forearm cross section and simplified 2D geometrical model

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Heat transfer processes in the domain considered are described by a system of Pennes partial differential equations.

$$c_e(T)\frac{\partial T_e(x,t)}{\partial t}=\nabla\left[\lambda_e(T)\nabla T_e(x,t)\right]+ +Q_{per\,e}(x,t)+Q_{met\,e}(x,t), \quad e=1,...,4 \tag{1}$$

where $e = 1, …, 4$ corresponds to the successive tissue sub-domains (skin, fat, muscle and bone, respectively), c_e is the volumetric specific heat, λ_e is the thermal conductivity, Q_{per} and Q_{met} are the capacities of volumetric internal heat sources connected with the blood perfusion and metabolism [W/m^3], T, $x = \{x_1, x_2\}$, t denotes the temperature, spatial co-ordinates and time. The perfusion heat source is given by the formula

$$Q_{per\,e}(x,t)=c_b G_{be}(T)\left[T_b-T_e(x,t)\right], \quad T_b=\left(T_{b\,artery}+T_{b\,vein}\right)/2 \tag{2}$$

where G_{be} is blood perfusion [m^3 blood/(s m^3 tissue)], c_b is blood volumetric specific heat and $T_{b\,artery}$ and $T_{b\,vein}$ are arterial and vein blood temperatures. Metabolic heat source Q_{met} (x, t) can be treated as a constant value.

Equation describing the transient temperature field in the domain of fabric takes the following form:

$$c_0(T)\frac{\partial T_0(x,t)}{\partial t}=\nabla\left[\lambda_0(T)\nabla T_0(x,t)\right] \tag{3}$$

while the boundary condition between skin surface and fabric is

$$x\in\Gamma_{0,1}: \quad -\lambda_0\frac{\partial T_0(x,t)}{\partial n}= =\frac{T_0(x,t)-T_1(x,t)}{R(x,t)}=-\lambda_1\frac{\partial T_1(x,t)}{\partial n} \tag{4}$$

where R is thermal resistance of the air gap.

On the contact surface between the tissue sub-domains, the continuity of temperature and heat fluxes are assumed:

$$x\in\Gamma_{e,e+1}:\begin{cases}-\lambda_e\dfrac{\partial T_e(x,t)}{\partial n}=-\lambda_{e+1}\dfrac{\partial T_{e+1}(x,t)}{\partial n}\\ T_e(x,t)=T_{e+1}(x,t)\end{cases}, \quad e=1,2,3 \tag{5}$$

where $\partial / \partial n$ is temperature derivative in normal direction.

On the outer surface of the fabric, the Robin boundary condition is taken into account:

$$x\in\Gamma_{out}: \quad -\lambda_0\frac{\partial T_0(x,t)}{\partial n}=\alpha_{out}\left[T_0(x,t)-T_{amb}\right] \tag{6}$$

where α_{out} is heat transfer coefficient, T_{amb} is ambient temperature. The same type of boundary conditions is given on the surfaces between blood vessels and soft tissue sub-domains, in particular

$$x\in\Gamma_{artery}: \quad -\lambda_3\frac{\partial T_3(x,t)}{\partial n}=\alpha_{artery}\left[T_3(x,t)-T_{b\,artery}\right] \tag{7}$$

and

$$x\in\Gamma_{vein}: \quad -\lambda_e\frac{\partial T_e(x,t)}{\partial n}= =\alpha_{vein}\left[T_e(x,t)-T_{b\,vein}\right], \quad e=2,3 \tag{8}$$

The initial conditions are also given

$$t=0: \quad T_e(x,t)=T_{steady}(x), \qquad e=0,1,...,4 \tag{9}$$

where T_{steady} is temperature distribution corresponding to steady state conditions in the tissue – fabric domain for the initial ambient temperature given and initial external heat transfer coefficient.

3. Sensitivity model

The sensitivity analysis presented in this paper concerns the changes of transient temperature field in domain considered due to perturbations of parameters α_{out} and T_{amb} appearing in the external Robin boundary condition (6). As it is well known, the sensitivity function U_k (x, t) is defined in the following way:

$$U_k(x,t)=\lim_{\Delta p_k\to 0}\frac{T(x,t,p_1,...,p_k+\Delta p_k,...,p_n)-T(x,t,p_1,...,p_k,...,p_n)}{\Delta p_k}= =\frac{\partial T(x,t)}{\partial p_k} \tag{10}$$

Thus, the sensitivity model can be created by the differentiation of energy equations and boundary-initial conditions with respect to the parameter considered (a direct approach – e.g. [14–17]). Differentiation of the Pennes equation with respect to the external heat transfer coefficient gives

$$\frac{\partial}{\partial\alpha_{out}}\left[c_e\frac{\partial T_e(x,t)}{\partial t}\right]=\frac{\partial}{\partial\alpha_{out}}\left[\lambda_e\nabla^2T_e(x,t)\right]- -\frac{\partial}{\partial\alpha_{out}}\left[c_bG_{be}T_e(x,t)\right], \ e=1,...,4 \tag{11}$$

or, using the Schwarz theorem about mixed partial derivative,

$$c_e\frac{\partial U_e(x,t)}{\partial t}=\lambda_e\nabla^2U_e(x,t)-c_bG_{be}U_e(x,t), \ e=1,...,4 \tag{12}$$

where $U_e(x, t) = \partial T_e(x, t) / \partial\alpha_{out}$. A similar equation determines sensitivity function in the domain of fabric:

$$c_0\frac{\partial U_0(x,t)}{\partial t}=\lambda_0\nabla^2U_0(x,t) \tag{13}$$

One can see that the values of thermal conductivities both for tissue sub-domains and the fabric are assumed to be constant. Differentiation of internal boundary conditions with respect to the external heat transfer coefficient leads to the following equations:

– between forearm sub-domains

$$x \in \Gamma_{e,e+1}: \begin{cases} -\lambda_e \dfrac{\partial U_e(x,t)}{\partial n} = -\lambda_{e+1} \dfrac{\partial U_{e+1}(x,t)}{\partial n}, \\ U_e(x,t) = U_{e+1}(x,t) \end{cases} \quad (14)$$

$e = 1,2,3$

– between skin surface and fabric

$$x \in \Gamma_{0,1}: \quad -\lambda_0 \frac{\partial U_0(x,t)}{\partial n} = \frac{U_0(x,t) - U_1(x,t)}{R(x,t)} = = -\lambda_1 \frac{\partial U_1(x,t)}{\partial n} \quad (15)$$

– between tissue and blood vessels

$$x \in \Gamma_{artery}: \quad -\lambda_3 \frac{\partial U_3(x,t)}{\partial n} = \alpha_{artery} U_3(x,t) \quad (16)$$

and

$$x \in \Gamma_{vein}: \quad -\lambda_e \frac{\partial U_e(x,t)}{\partial n} = \alpha_{vein} U_e(x,t), \quad e = 2,3 \quad (17)$$

To obtain conditions (16) and (17) in the form analogous to Robin conditions (8) and (9), in the basic model one can write

$$x \in \Gamma_{artery}: \quad -\lambda_3 \frac{\partial U_3(x,t)}{\partial n} = \alpha_{artery} \left[U_3(x,t) - U_{b\,artery} \right] \quad (18)$$

and

$$x \in \Gamma_{vein}: \quad -\lambda_e \frac{\partial U_e(x,t)}{\partial n} = \alpha_{vein} \left[U_e(x,t) - U_{b\,vein} \right], \quad (19)$$

$e = 2,3$

while blood sensitivities $U_{b\,artery}$ and $U_{b\,vein}$ are equal to zero. The forms presented above and below will allow for using the same computation procedure simultaneously with the basic model and the sensitivity model.

On the external surface of the system one has

$$x \in \Gamma_{out}: -\lambda_0 \frac{\partial U_0(x,t)}{\partial n} = T_0(x,t) - T_{amb} + \alpha_{out} U_0(x,t) = \alpha_{out} \left[U_0(x,t) - U_{amb}(x,t) \right] \quad (20)$$

Introduced in the contractual way, ambient sensitivity

$$U_{amb}(x,t) = \frac{T_{amb} - T_0(x,t)}{\alpha_{out}} \quad (21)$$

leads to condition (20) in the same form as Robin boundary condition. One can see that the sensitivity model and the basic one are coupled. To determine ambient sensitivity the knowledge of temporary temperature field is necessary.

The only difference in the sensitivity model concerning the ambient temperature is the form of condition (20), namely

$$U_{amb}(x,t) = 1 \quad (22)$$

4. Control volume method

At the stage of numerical computations the control volume method (CVM) is applied; in other words, the domain considered is divided into a certain number of small cells and the governing equations in the integral form are applied individually to each one of them. This procedure guarantees, *a priori*, the conservation of physical quantities like mass, momentum and energy. It is also extremely flexible and conceptually simple. In this paper, 2D control volumes corresponding to Voronoi polygons (also called the Thiessen or Dirichlet cells in two dimensions) [18] have been used. Such a version of CVM was in details discussed by Ciesielski and Mochnacki in [19, 20]. Here, only basic information concerning this variant of CVM will be presented. So, the domain analyzed is divided into N volumes (Fig. 2) and the algorithm presented allows for finding the transient temperature field at the set of nodes corresponding to central points of the control volumes.

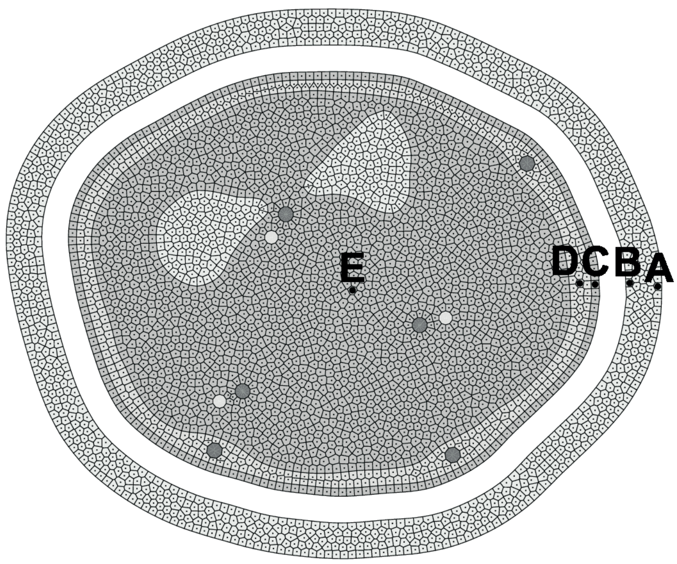

Fig. 2. Tissue and fabric sub-domains

The polygon that contains the point x_i (central point) is denoted by CV_i (Fig. 3). All of the Voronoi regions are convex

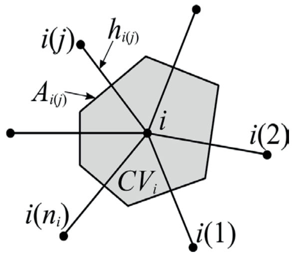

Fig. 3. Control volume CV_i

polygons, and each polygon is defined by lines bisecting sectors between the central point and neighbouring points. The bisecting lines and the connection lines are perpendicular to each other (it is very convenient at the stage of CVM equations construction). Let us consider control volume CV_i with the central node x_i.

It should be pointed out that the mathematical model concerning temperature $T_e(x, t)$ and sensitivity models are practically the same. Therefore, CVM equations concern both $T_e(x, t)$ and $U_e(x, t)$ – the searched distributions of these functions will be denoted as $W_e(x, t)$. Below it is assumed that the thermal capacities and capacities of the internal heat sources are concentrated at the nodes representing elements, while thermal resistances are concentrated on the sectors joining the nodes. Additionally, the constant values of the parameters of successive sub-domains are taken into account. The CVM equations result from the integration of equations (1) and (12) with respect to time and volume CV_i. Let us consider the time interval $\Delta t = t^{f+1} - t^f$.

$$\int_{t^f}^{t^{f+1}} \int_{CV_i} c_e \frac{\partial W_e(x,t)}{\partial t} \mathrm{d}V\mathrm{d}t = \int_{t^f}^{t^{f+1}} \int_{CV_i} \lambda_e \nabla^2 W_e(x,t) \mathrm{d}V\mathrm{d}t + \int_{t^f}^{t^{f+1}} \int_{CV_i} Q_e(x,t) \mathrm{d}V\mathrm{d}t \tag{23}$$

For the basic model

$$Q_e(x,t) = c_b G_{be}\left[T_b - T_e(x,t)\right] + Q_{met\,e}(x,t) \tag{24}$$

while for the sensitivity one

$$Q_e(x,t) = -c_b G_{be} U_e(x,t) \tag{25}$$

Using the Gauss-Ostrogradsky's theorem, one obtains

$$\int_{t^f}^{t^{f+1}} \int_{CV_i} c_e \frac{\partial W_e(x,t)}{\partial t} \mathrm{d}V\mathrm{d}t = \int_{t^f}^{t^{f+1}} \int_{A_i} \lambda_e \mathbf{n} \cdot \nabla W_e(x,t) \mathrm{d}A\mathrm{d}t + \int_{t^f}^{t^{f+1}} \int_{CV_i} Q_e(x,t) \mathrm{d}V\mathrm{d}t \tag{26}$$

where A_i is the surface (perimeter) limiting CV_i. The integration of the left-hand side of equation (26) gives

$$\int_{t^f}^{t^{f+1}} \int_{CV_i} c_e \frac{\partial W_e(x,t)}{\partial t} \mathrm{d}V\,\mathrm{d}t \cong c_i\left(W_i^{f+1} - W_i^f\right)\Delta V_i \tag{27}$$

In a similar way one can approximate the last component in equation (26), namely

$$\int_{t^f}^{t^{f+1}} \int_{CV_i} Q_e(x,t) \mathrm{d}V\mathrm{d}t \cong Q_i^f\, \Delta V_i \Delta t \tag{28}$$

The term determining the fluxes between ΔV_i and its neighbourhoods can be written in the form

$$\begin{aligned} &\int_{t^f}^{t^{f+1}} \int_{A_i} \lambda_e \mathbf{n} \cdot \nabla W_e(x,t)\, \mathrm{d}A\mathrm{d}t = \\ &= \int_{t^f}^{t^{f+1}} \left(\sum_{j=1}^{n_i} \int_{A_{i(j)}} \overline{\lambda_{i(j)}}\, \mathbf{n}_{i(j)} \cdot \left[\nabla W(x,t)\right]_{i(j)} \mathrm{d}A_{i(j)} \right) \mathrm{d}t \cong \\ &\cong \int_{t^f}^{t^{f+1}} \left(\sum_{j=1}^{n_i} \overline{\lambda_{i(j)}}\, \mathbf{n}_{i(j)} \cdot \left[\nabla W(x,t)\right]_{i(j)} A_{i(j)} \right) \mathrm{d}t \cong \\ &\cong \int_{t^f}^{t^{f+1}} \left(\sum_{j=1}^{n_i} \overline{\lambda_{i(j)}} \frac{W_{i(j)} - W_i}{h_{i(j)}} A_{i(j)} \right) \mathrm{d}t = \\ &= \int_{t^f}^{t^{f+1}} \left(\sum_{j=1}^{n_i} \frac{W_{i(j)} - W_i}{R_{i(j)}} A_{i(j)} \right) \mathrm{d}t = \sum_{j=1}^{n_i} \frac{W_{i(j)}^f - W_i^f}{R_{i(j)}} A_{i(j)} \Delta t \end{aligned} \tag{29}$$

where $\overline{\lambda_{i(j)}}$ is mean thermal conductivity between two central points of two adjoining control volumes with nodes i and $i(j)$, while $R_{i(j)}$ is thermal resistance between these nodes; in the case of internal control volumes it is equal to

$$R_{i(j)} = \frac{h_{i(j)}}{\overline{\lambda_{i(j)}}} = \frac{0.5h_{i(j)}}{\lambda_i} + \frac{0.5h_{i(j)}}{\lambda_{i(j)}} \tag{30}$$

wherein $h_{i(j)}$ is distance between the nodes i and $i(j)$ – see Fig. 3.

The CVM equation written in the convention of "explicit" scheme takes the form

$$c_i\left(W_i^{f+1} - W_i^f\right)\Delta V_i = \sum_{j=1}^{n_i} \frac{W_{i(j)}^f - W_i^f}{R_{i(j)}} A_{i(j)} \Delta t + Q_i^f\, \Delta V_i \Delta t \tag{31}$$

from which

$$W_i^{f+1} = W_i^f + \frac{\Delta t}{c_i \Delta V_i} \sum_{j=1}^{n_i} \frac{W_{i(j)}^f - W_i^f}{R_{i(j)}} A_{i(j)} + \frac{\Delta t}{c_i} Q_i^f \tag{32}$$

or

$$W_i^{f+1} = W_i^f + \sum_{j=1}^{n_i} G_{i(j)}\left(W_{i(j)}^f - W_i^f\right) + \frac{\Delta t}{c_i} Q_i^f \tag{33}$$

where

$$G_{i(j)} = \frac{A_{i(j)}\, \Delta t}{c_i R_{i(j)}\, \Delta V_i} \tag{34}$$

In order to determine the stability condition of the explicit differential scheme (33), the sum of the coefficients at W_i^f must be positive. Hence, this condition for each node i can be written in the following form:

$$1 - \sum_{j=1}^{n_i} G_{i(j)} - \frac{\Delta t\, G_{bi} c_b}{c_i} > 0 \tag{35}$$

The factor $-c_b G_{be}$ is enclosed in the definition of formula Q_i^f. From inequality (35) one can determine the critical time step Δt

$$\Delta t < \min_{i=1,\dots,N} \frac{c_i}{G_{bi} c_b + \dfrac{1}{\Delta V_i} \sum_{j=1}^{n_i} \dfrac{A_{i(j)}}{R_{i(j)}}} \tag{36}$$

In the case of external CV when the boundary $A_{i(j)}$ of CV_i covers the surface Γ_{out}, Γ_{artery} or Γ_{vein}, the implementation of boundary conditions (6–8) and (18–20) must be introduced. For this reason the following approximation of appropriate term in equation (29) is used:

$$\overline{\lambda_{i(j)}}\, \mathbf{n}_{i(j)} \cdot \left[\nabla W(x,t)\right]_{i(j)}^f A_{i(j)} \cong \frac{W_a - W_i^f}{R_{i(j)}} A_{i(j)} \tag{37}$$

where W_a is the value of ambient temperature or "ambient sensitivity" and $R_{i(j)}$ corresponds in this case to the thermal resistance between central point of CV_i and its environment in $i(j)$ direction. It is defined as follows:

$$R_{i(j)} = \frac{0.5 h_{i(j)}}{\lambda_i} + \frac{1}{\alpha} \tag{38}$$

wherein $\alpha = \{\alpha_{out}, \alpha_{artery}, \alpha_{vein}\}$, respectively.

In the case of control volumes bordering the layer of trapped air (between the skin surface and protective clothing), thermal resistance (30) should be increased by thermal resistance of the air gap.

5. Example of computations

The fabric-forearm domain stays in thermal contact with the environment at temperature equal to $T_{amb} = 20$°C, while heat transfer coefficient $\alpha_{out} = 3.7$ W/m^{2}K. Thermal resistance of trapped air is assumed to be a constant value $R = 0.077$ m^{2}K/W. The authors have developed the procedure of the exact computation of temperature-dependent thermal resistance R, but the preliminary calculations show that the constant value proposed is quite acceptable. Additionally, the following blood temperatures are assumed: $T_{b\,artery} = 36$°C, $T_{b\,vein} = 35$°C, while $\alpha_{artery} = \alpha_{vein} = 5000$ W/m^{2}K.

Thermophysical parameters of the successive sub-domains are assumed to be constants and are presented in Table 1. Parameters of tissues and textile are taken from [11, 12].

The initial temperature distribution is found using Gauss method (simple iteration method). It corresponds, practically, to the use of the basic computer program for the optional initial condition and continuation of computations until the temperature field becomes stabilized. At the moment $t = 0$, ambient temperature increases to $T_{amb} = 70$°C (heat transfer coefficient $\alpha_{out} = 100$ W/m^{2}K). Numerical simulation concerns the process of tissue heating. The basic solution (in the form of heating curves at the selected set of points – see Fig. 2) are shown in Fig. 4.

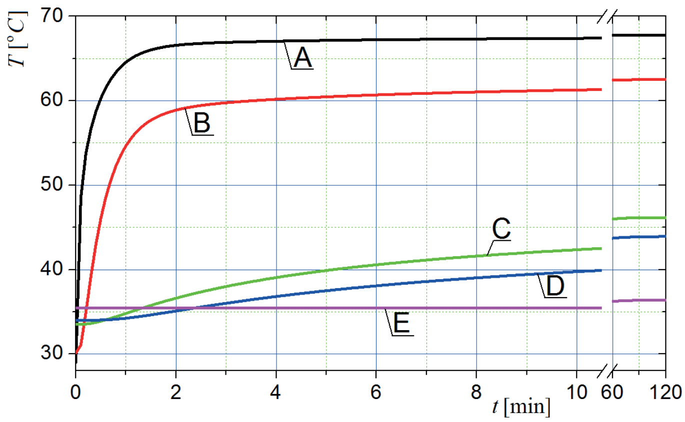

Fig. 4. The basic solution

The next figures show the numerical solution of sensitivity problems. In particular, changes of sensitivity function

Table 1
Thermal parameters of sub-domains

Ω_i		λ[W/mK]	c [J/kg m^3] = c_p [J/kg K] · ρ [kg/m^3]	G_b [1/s]	Q_{met} [W/m^3]
0	*Fabric*	0.17	240000	–	–
1	*Skin*	0.47	3680 · 1085	$1.1000 \cdot 10^{-3}$	368
2	*Fat*	0.16	2300 · 850	$0.0036 \cdot 10^{-3}$	58
3	*Muscle*	0.42	3768 · 1085	$0.5380 \cdot 10^{-3}$	684
4	*Bone*	0.75	1700 · 1357	$0.0000 \cdot 10^{-3}$	0
	Blood	–	3650 · 1069	–	–

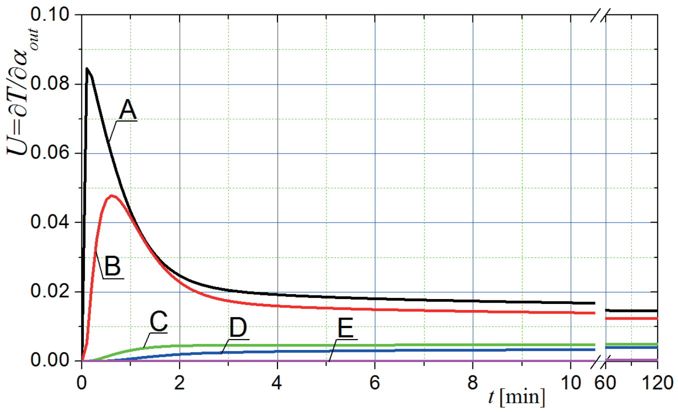

Fig. 5. Sensitivity with respect to heat transfer coefficient

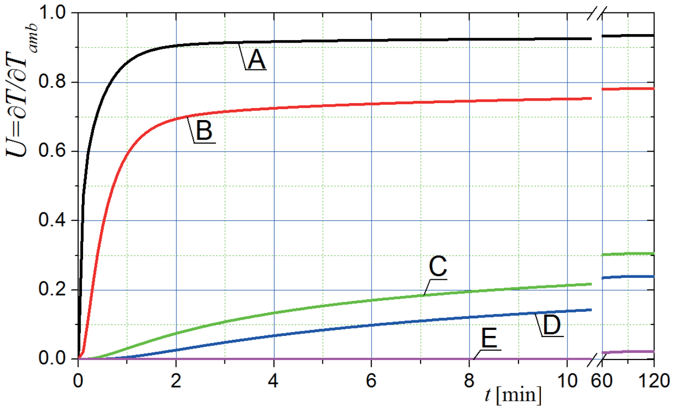

Fig. 6. Sensitivity with respect to ambient temperature

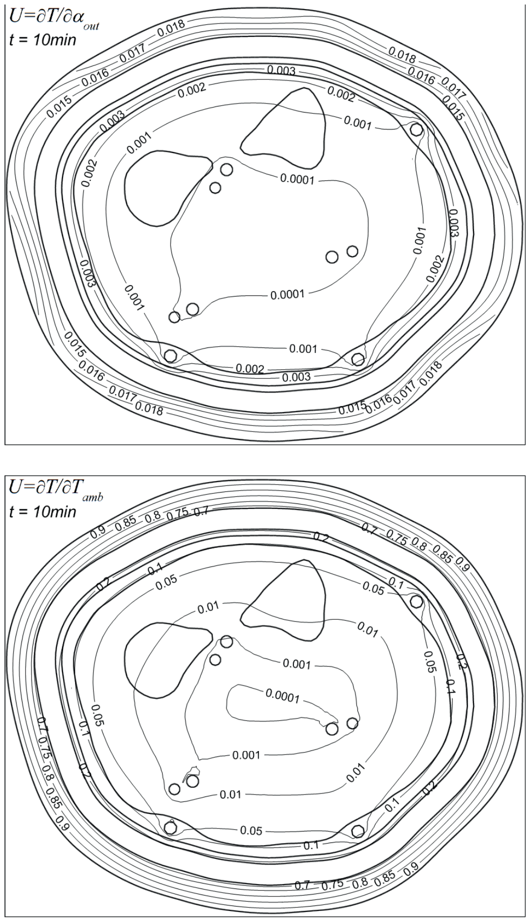

Fig. 7. Distribution of sensitivity with respect to α_{out} and T_{amb} for time 10 min.

$U_e(x, t) = \partial T_e(x, t)/\partial \alpha_{out}$ are shown in Fig. 5, while changes of $U_e(x, t) = \partial T_e(x, t)/\partial T_{amb}$ are marked in Fig. 6. Temporary distributions of sensitivity with respect to perturbations of heat transfer coefficient and ambient temperature are shown in Fig. 7.

Application of Taylor formula (only the first derivative is taken into account) gives

$$\begin{aligned} &T(x, t, p_1, ..., p_k \pm \Delta p_k, ..., p_n) = \\ &= T(x, t, p_1, ..., p_k, ..., p_n) \pm U_k(x, t) \Delta p_k \end{aligned} \tag{39}$$

Using this equation one can find temporary and local changes of temperature due to the perturbations of the parameter discussed. In Fig. 8 the results concerning the value of $\Delta \alpha_{out} = \pm 10 \text{W/m}^2\text{K}$ are shown, while in Fig. 9 – the results corresponding to $\Delta T_{amb} = \pm 7\text{K}$.

It is also possible to estimate the changes of temperature due to the simultaneous perturbations of both parameters using formula [21]

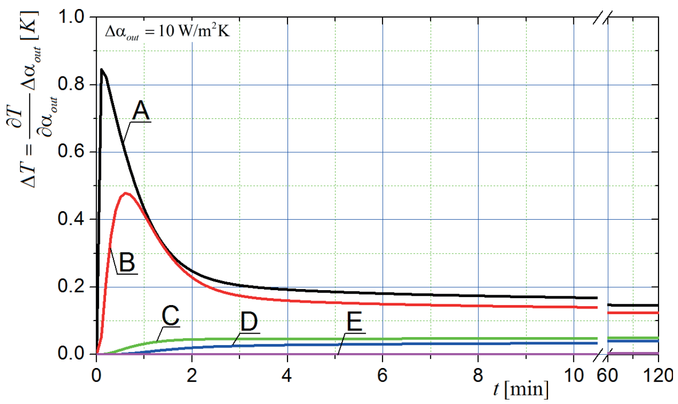

Fig. 8. Changes of temperature due to the perturbations of α_{out}

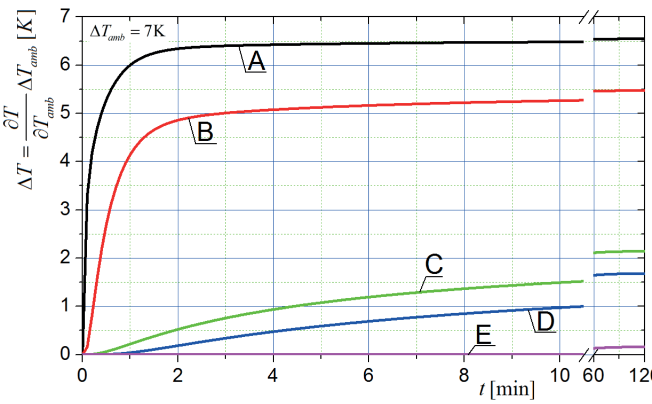

Fig. 9. Changes of temperature due to the perturbations of T_{amb}

$$\Delta T = \sqrt{\left(\frac{\partial T}{\partial \alpha_{out}}\Delta\alpha_{out}\right)^2 + \left(\frac{\partial T}{\partial T_{amb}}\Delta T_{amb}\right)^2} \tag{40}$$

In Fig. 10 the results for $\Delta\alpha_{out} = \pm 10\text{W/m}^2\text{K}$ and $\Delta T_{amb} = \pm 7\text{K}$ are shown.

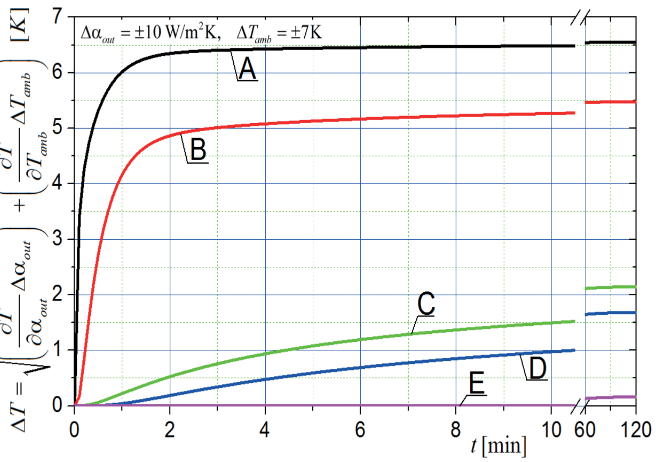

Fig. 10. Perturbations of both parameters

6. Final remarks

The variant of CVM with the use of Voronoi polygons has been discussed. This very effective approach to numerical modeling of thermal processes has been adapted to the needs of bioheat transfer problems. Discretization of 2D domain using Voronoi polygons has many advantages. A complex homogeneous or heterogeneous shape can be exactly reconstructed and the mutual position of polygons assures the correct form of the energy balances. Additionally, the possibility of arbitrary distribution of central nodes enables local refinement of the mesh (e.g. close to the boundary). It should be pointed out that the modeling of different types of boundary conditions is very simple. The most significant disadvantage of the approach proposed lies in considerable difficulties at the stage of computer program preparing. The algorithm proposed can constitute the base for the inverse problems solution using the gradient methods (e.g. [22]).

Although the example of numerical simulation presented in this paper does not allow for formulating general conclusions, the number of the information concerning the task discussed is significant. One can see that temperature increase in the domain of fabric proceeds essentially faster as compared to the tissue domain, and temperature stabilization time is considerably shorter. Fabric temperature close to the internal boundary is high (ca. 60°C). In the case of accidental contact between fabric and tissue, such a situation is not preferable. The temperature of skin tissue layer reaches the value more than 45°C. It is said that the critical value of skin tissue temperature is equal to 43°C and above this temperature, tissue burns can take place. There is also the problem of residence time at elevated temperature – see: Henriques integral [16].

Sensitivity analysis provides useful information as well. At first, one can see that the sensitivity of temperature field in a system considered with respect to ambient temperature is much greater than the sensitivity with respect to heat transfer coefficient. The courses of both functions are quite different. Sensitivity with respect to T_{amb} is an increasing function for the relevant period of time, while sensitivity with respect to α_{out} reaches its maximum value at the initial stages of heating and then rapidly decreases. In the tissue domain, sensitivity $\partial T_e(x, t)/\partial\alpha_{out}$ is practically close to zero. The perturbations of ambient temperature $\Delta T_{amb} = \pm 7\text{K}$ (possible in the real world) causes essential changes of fabric temperature (as in Fig. 9). Summing up, the insulating properties of the protective clothing considered are too weak. It results from the thermophysical parameters of the fabric and the considerable value of external heat source. Directly after the start of heating process, the boundary heat flux is equal to about 4 kW/m^2.

Further works in this field will focus on expanding the algorithm presented for the case in which the parameters of biological tissue are treated as the interval numbers [23]. Such an approach results from the fact that tissue parameters are individual features dependent on the gender, age, profession etc., and the differences are noticeable.

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References

[1] H.H. Pennes, "Analysis of tissue and arterial blood temperatures in the resting human forearm", *Journal of Applied Physiology*, 1, 93–122 (1948).

[2] H. Arkin, L.X. Xu and K.R. Holmes, "Recent development in modeling heat transfer in blood perfused tissues", *IEEE Trans Bio-Med. Eng.* 41, 97–107 (1994).

[3] E. Majchrzak, "Application of different variants of the BEM in numerical modeling of bio-heat transfer processes", *MCB: Molecular & Cellular Biomechanics* 10, 3, 201–232 (2013).

[4] E. Majchrzak, B. Mochnacki, M. Dziewonski and M. Jasinski, "Numerical modeling of hyperthermia and hypothermia processes", *Advanced Materials Research* 268–270, 257–262 (2011).

[5] E. Majchrzak, B. Mochnacki and M. Jasinski, "Numerical modeling of bioheat transfer in multi-layer skin tissue domain subjected to a flash fire", *Computational Fluid and Solid Mechanics* 1–2, 1766–1770 (2003).

[6] E. Majchrzak and M. Dziewonski, "Heat transfer in biological tissue subjected to the action of cylindrical cryoprobe", *Computational and Experimantal Methods*, 1, 29–36 (1999).

[7] M. Dziewonski, B. Mochnacki and R. Szopa, "Sensitivity of biological tissue freezing process on the changes of cryoprobe cooling rate", *Mechanika Kaunas University of Technology,* 82–87 (2011).

[8] F. Xu, K.A. Seffen and T.J. Lu, "Non-Fourier analysis of skin biothermomechanics", *International Journal of Heat and Mass Transfer* 51, 2237–2259 (2008).

[9] Y. Zhang, “Generalized dual-phase lag bioheat equations based on nonequilibrium heat transfer in living biological tissues”, *International Journal of Heat and Mass Transfer* 52, 4829–4834 (2009).

[10] E. Majchrzak, “Numerical solution of dual phase lag model of bioheat transfer using the general boundary element method”, *CMES: Computer Modeling in Engineering and Sciences* 69, 1, 43–60 (2010).

[11] D. Fiala, K.J. Lomas and M. Stohrer, “A computer model of human thermoregulation for a wide range of environmental conditions: the passive system”, *Journal of Applied Physiology* 87, 5, 1957–1972 (1999)

[12] P. Chitrphiromsri, *Modeling of Thermal Performance Firefighter Protecting Clothing During the Intense Heat Exposure*, Mechanical Engineering, Doctoral Theses, Raleigh, (2004).

[13] M. Schuenke, E. Schulte, et al., *General Anatomy and Musculoskeletal System (Atlas of Anatomy)*, Thieme, Stuttgart – New York 2010.

[14] M. Kleiber*, Parameter Sensitivity in Nonlinear Mechanics*, J. Willey & Sons Ltd., London, 1997.

[15] K. Dems and B. Rousselet, “Sensitivity analysis for transient heat conduction in a solid body – Part I”, *Structural Optimization* 17, 36–45 (1999).

[16] M. Jasinski, “Investigation of tissue thermal damage process with application of direct sensitivity method” *MCB: Molecular & Cellular Biomechanics* 10, 3, 183–199 (2013).

[17] B. Mochnacki and E. Majchrzak, “Sensitivity of the skin tissue on the activity of external heat sources”, *CMES: Computer Modeling in Engineering and Sciences*, 4, 3–4, 431–438 (2003).

[18] D.F. Watson, “Computing the n-dimensional Delaunay tessellation with application to Voronoi polytopes”, *The Computer Journal* 24, 2, 167–172 (1981).

[19] Z. Domanski, M. Ciesielski and B. Mochnacki, “Application of control volume method using the Voronoi tessellation in numerical modelling of solidification process”, *AIP Conference Proceedings*, 1220, 17–26 (2010).

[20] M. Ciesielski and B. Mochnacki, “Application of the control volume method using the Voronoi polygons for numerical modeling of bio-heat transfer processes”, *Journal of Theoretical and Applied Mechanics* 52, 4, 927–935 (2014).

[21] E. Majchrzak and B. Mochnacki, “Sensitivity analysis of transient temperature field in microdomains with respect to dual-phase-lag-model parameters”, *Journal for Multiscale Computational Engineering* 12(1), 65–77 (2014).

[22] B. Mochnacki and E. Majchrzak, “Identification of macro and micro parameters in solidification model”*, Bull. Pol. Ac.: Tech* 5(1), 107–113 (2007).

[23] B. Mochnacki and A. Piasecka Belkhayat, “Numerical modeling of skin tissue heating using the interval finite difference method”, *MCB: Molecular and Cellular Biomechanics* 10, 3, 133–144 (2013).