Low Complexity Stopping Rule for Turbo Decoding: the Max-log Criterion

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Abstract — This paper presents a new stopping criterion for turbo decoding. It is based on the selection of the maximum log-alphas calculated by the log-MAP algorithm. The sum of these maximum alphas is compared with a threshold value. Then, a decision on the end of decoding is taken. Simulation results show that the max-log criterion offers the same performance as the sum-α and sum-log criteria, while maintaining the same complexity level. The max-log criterion uses only the max operator to select maximum alphas and a summation. Therefore, the proposed criterion is faster and offers lower complexity.

Keywords* — *iterative decoding, log-MAP, stopping criteria, sum-α, sum-log criterion

1. Introduction

The main goal of the communication theory has always been to approach the Shannon limit with low complexity. Turbo codes [1] are one of the most powerful techniques offering performance close to the Shannon limit by using iterative decoding. After a few decoding iterations, their performance may be considered acceptable. However, there is no need to reprocess a frame if it has been decoded correctly. Therefore, turbo decoding needs to be stopped to reduce computational complexity.

Research on stopping techniques for modern codes which ensure the best performance has become a top priority to reduce the complexity of iterative decoding. Several criteria have been proposed for turbo codes in [2]–[10]. Most of them exploit the log-likelihood ratios (LLRs) calculated by the decoders or the decisions taken on these ratios. Recently, the sum-α [11] and sum-log [12] techniques exploiting the probabilistic alpha or log-alpha quantities calculated by the maximum a posteriori (MAP) or log-MAP [13] algorithms have been proposed. The sum-log stopping criterion is the most attractive solution, since the log-MAP algorithm is the most frequent approach used in practice. The great particularity of the sum-log criterion is that it can provide information on the quality of the decoding even before the calculation of the LLRs. This makes this criterion a low complexity solution, as it is, in fact, a cumulative sum and is highly suitable for practical uses. However, the sum-log criterion calculates, at each instant, the sum of the log-alpha quantities. If the trellis decoder contains M_{tr} states, the sum-log criterion will cover $M_{tr} - 1$ summations at every instant t. In this article, another way of applying this criterion is proposed.

The proposed criterion uses the first version of the sum-log rule [13]. It is based on the maximum value of the logarithms of alpha. The selection of maximum log-α replaces the $M_{tr} - 1$ summations used by the sum-log-1 criterion, thus reducing complexity. This criterion is called max-log. A performance comparison is made with the sum-α, sum-log and cross-entropy (CE) criteria.

2. System Model and Coding

The frame of information bits $[s(t)]$, $t = 1, \ldots, N$ is coded by a turbo encoder of rate $R = 1/3$ to give a frame of $3N$ bits. Each information bit, after coding, gives a systematic bit $s(t)$, and two redundant bits $c_1(t)$ and $c_2(t)$. After transmission over a Gaussian channel using binary phase shift keying (BPSK) modulation, the received samples are represented by $[y_s(t), y_1(t), y_2(t)]$. Each received sample is the emitted noisy symbol by the additive Gaussian noise sample. The received frame is turbo decoded by a log-MAP turbo decoder using a maximum of 10 iterations.

3. Sum-α Stopping Criterion

Let us start with the phenomenon of concurrent alphas [11]. During MAP decoding, instead of having a maximum $\alpha_{max}(t)$ which has a significant value and other very low alphas, we probably observe, at the erroneous instants, a decrease in $\alpha_{max}(t)$ and an increase in the levels of the others alphas. The sum-α criterion proposed for the MAP algorithm uses concurrent alphas i.e., less than $\alpha_{max}(t)$ at every time t. This means that when the sum of the concurrent alphas is below a predefined threshold, the frame is considered to be correct. This feature is used in [11] to detect correct frames and stop the turbo process. It works in the manner described below.

After determining the alphas of each state of the trellis, the sum of the concurrent alphas is calculated by:

$$Sum_c(t) = \sum_{m=1}^{M_{tr}} \alpha_m(t) - \alpha_{max}(t) \; . \tag{1}$$

The sum of $Sum_c(t)$ across instants t normalized by the frame length before coding N is:

$$Sum\text{-}\alpha = \frac{\sum_{t=1}^{N} Sum_c(t)}{N} . \quad (2)$$

If sum-α is below a determined threshold T_{SA}, then the turbo process is stopped.

4. Sum-log Stopping Criterion

The sum-log criterion applies the same idea as sum-α but in the logarithmic domain. This means that after the calculation of quantities $A_m(t)$, where $A_m(t)$ is the logarithm of $\alpha_m(t)$, $A_m(t) = \log \alpha_m(t)$ by the log-MAP algorithm, the same principle is applied. A simple summation is used after dismissing of $A_{max}(t)$, where $A_{max}(t)$ is the maximum of $A_m(t)$ at time t, $A_{max}(t) = \log \alpha_{max}(t)$.

$$SumL(t) = \sum_{m=1}^{M_{tr}} A_m(t) - A_{max}(t) . \quad (3)$$

The quantity $SumLA$ is calculated as:

$$SumLA = \frac{\sum_{t=1}^{N} SumL(t)}{M_{tr}^2\, N} . \quad (4)$$

If $SumLA < T_{SL}$, then the frame is considered to be correct and decoding is stopped. T_{SL} is a threshold determined by simulation depending on size N of the interleaver.
The above considerations define the first version of the sum-log criterion. It is called as sum-log-1 in [12]. The second version of this criterion ignores the use of the max operator and applies the sum in Eq. (3) including $A_{max}(t)$ for each instant t as:

$$SumL(t) = \sum_{m=1}^{M_{tr}} A_m(t) . \quad (5)$$

5. Cross-Entropy Stopping Criterion

The cross-entropy (CE) [10], [14] criterion uses the LLRs at the output of the two decoders to indicate whether the decoded frame is correct. If so, decoding may be stopped before reaching the maximum number of iterations.
Let $L_m^{(i)}[s(t)]$ be the LLR of the information bit $s(t)$ at the output of the m-th decoder of iteration i at time t, and let $L_{em}^{(i)}[s(t)]$ be its extrinsic information for $m = 1, 2$. Then, CE of iteration i is given by [10]:

$$CE(i) \approx \sum_{N} \frac{\left|\Delta Le_2^{(i)}\left[s(t)\right]\right|^2}{e^{\left|L_1^{(i)}\left[s(t)\right]\right|}} , \quad (6)$$

where:

$$\Delta Le_2^{(i)} = Le_2^{(i)}\left[s(t)\right] - Le_2^{(i-1)}\left[s(t)\right] , \quad (7)$$

is the difference between the extrinsic information of two successive iterations.

The frame is considered to be correct if the following condition is met:

$$CE(i) < T_{CE} , \quad (8)$$

where T_{CE} is the threshold used, given by:

$$10^{-2}\, CE(1) \leqslant T_{CE} \leqslant 10^{-4}\, CE(1) . \quad (9)$$

6. Max-log Stopping Criterion

The criterion we propose uses only the maximum log-alphas $A_{max}(t)$ and is derived from the sum-log criterion. Since the possible correct instants are characterized by a large maximum $A_{max}(t)$ and very low and negative concurrent log-alphas, then this phenomenon can be used to detect that the frame is corrected and to stop turbo decoding. The new idea consists in calculating the sum of the maximum alphas $A_{max}(t)$ (cumulative sum) at each moment t, instead of calculating the sum of all log-alphas, and then in normalizing this sum by the size of frame N. A frame is considered correct if this normalized sum is significant, i.e, if it is greater than the T_{ML} threshold. The proposed technique is explained in detail below.
Instead of minimizing the sum of the log-alphas, it is possible to reduce complexity by avoiding the summation and attempting to select the maximum alphas $A_{max}(t)$ only. By applying Eqs. (3) and (4), $SumLA$ becomes:

$$SumLA = \frac{\sum_{t=1}^{N}\left(\sum_{m=1}^{M_{tr}} A_m(t) - A_{max}(t)\right)}{M_{tr}^2\, N} . \quad (10)$$

For the sum-log-1 criterion, the frame is correct if $SumLA$ is less than threshold T_{SL}:

$$\frac{\sum_{t=1}^{N}\left(\sum_{m=1}^{M_{tr}} A_m(t) - A_{max}(t)\right)}{M_{tr}^2\, N} < T_{SL} \quad (11)$$

which gives:

$$\sum_{t=1}^{N}\left(\sum_{m=1}^{M_{tr}} A_m(t) - A_{max}(t)\right) < T_{SL}(M_{tr}^2\, N) . \quad (12)$$

By isolating the sum of the maximum log-alphas, we get:

$$\sum_{t=1}^{N} A_{max}(t) > \sum_{t=1}^{N}\sum_{m=1}^{M_{tr}} A_m(t) - T_{SL}(M_{tr}^2\, N) . \quad (13)$$

The frame is considered to be correct if:

$$\frac{\sum_{t=1}^{N} A_{max}(t)}{N} > T_{ML} \quad (14)$$

with

$$T_{ML} = \frac{\sum_{t=1}^{N}\sum_{m=1}^{M_{tr}} A_m(t) - T_{SL}(M_{tr}^2\, N)}{N} = \frac{\sum_{t=1}^{N}\sum_{m=1}^{M_{tr}} A_m(t)}{N} - T_{SL} M_{tr}^2 . \quad (15)$$

The new threshold T_{ML} can be determined by simulation and it depends on size N of the interleaver. After the calculation

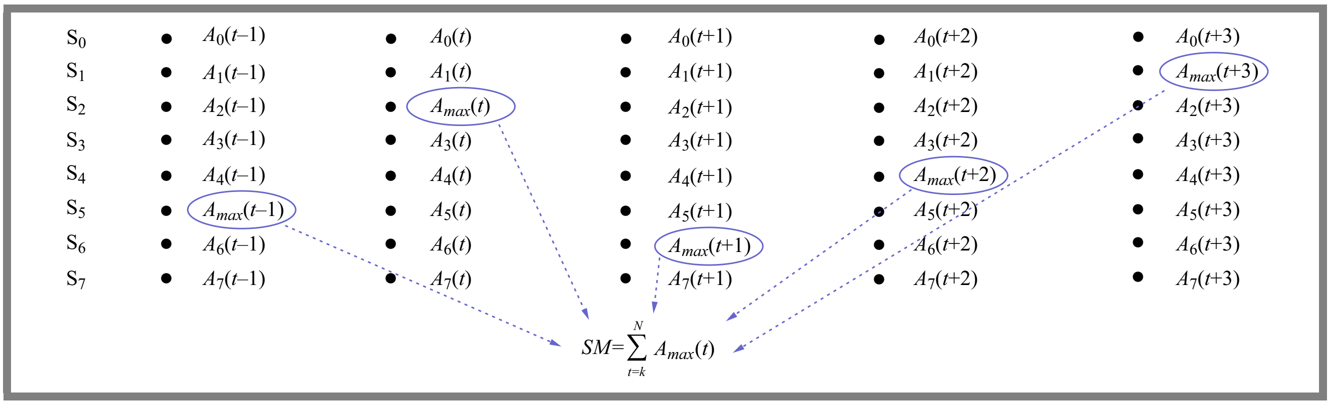

Fig. 1. The purpose of applying the max-log criterion.

of the M_{tr} log-α quantities $A_m(t)$ by the log-MAP algorithm at time t, the maximum log-alpha $A_{max}(t)$ is determined:

$$A_{max}(t) = \max_{m=1,\ldots,M_{tr}} \left[A_m(t)\right]. \tag{16}$$

For $t = [k, \ldots, N]$ calculate the sum SM of the maximum log-alphas as:

$$SM = \sum_{t=k}^{N} A_{max}(t) \ . \tag{17}$$

We use the condition $t \geqslant k$ to get away from the initialization of log-α $A_m(0) = -\infty$ of the log-MAP algorithm [12]. The logarithms of the initial alphas ($t = 0$) which are null are initialized by large negative values. In the simulations, the value of $k = 4$ is used.

Next, the results are normalized by the size of N the interleaver, using Eq. (14):

$$SumMaxN = \frac{\sum_{t=4}^{N} A_{max}(t)}{N} = \frac{SM}{N} \ . \tag{18}$$

Turbo decoding is stopped if $SumMaxN$ is greater than the threshold:

$$SumMaxN > T_{ML} \ . \tag{19}$$

The mechanism is illustrated in Fig. 1. It shows the log-α calculated by the Log-MAP algorithm of an 8-state trellis.

After the selection of the maximum log-α at each moment t, the max-log criterion calculates their sum from $t = k$ to N. Then, after normalization, it compares the result with the T_{ML} threshold for the stop decision. Transitions between the states of the trellis are not shown in Fig. 1. The threshold level may be set by testing each frame to find its best threshold value T_{ML}, where N is the length of the interleaver. For this purpose, we compared the performance in terms of frame error rate (FER) of log-MAP turbo decoders using the max-log stopping criterion with several thresholds $[-0.022, -0.018, -0.014, -0.010, -0.006, -0.002, 0.002]$ and interleavers of size $N = 5120$ bits. We chose an average signal-to-noise ratio of SNR = 0.4 dB. The results obtained show that very low T_{ML} thresholds values (below –0.022) degrade BER and FER, because the max-log criterion forces the log-MAP turbo decoder to use a low number of iterations and, therefore, it stops processing before obtaining a correct frame.

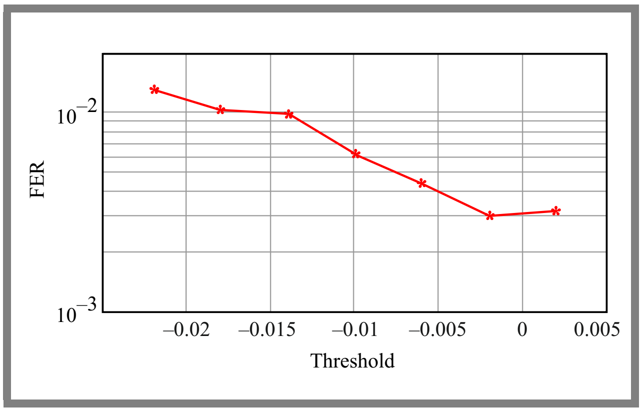

Fig. 2. FER of a turbo decoder supervised by the max-log criterion, according to different thresholds.

Thresholds greater than or equal to 0.002 result in using the maximum number of iterations (here, 10 iterations). Consequently, we chose to use a threshold of $T_{ML} = -0.0015$ for the interleaver of size $N = 5120$. This threshold value strongly depends on the size of the interleaver N. Figures 2 and 3 show this behavior. FER and the average number of iterations are plotted as a function of the different thresholds at the signal-to-noise ratio $\frac{E_b}{N_0} = 0.4$ dB and using 5000 frames. E_b is the average energy per information bit and N_0 is the power spectral density of the noise.

7. Results and Discussion

In the course of the simulations, we used a parallel convolutional turbo code with a rate of $R = 1/3$. It consists of two systematic recursive convolutional coders of generator polynomials $[1, 35/23]_{\text{oct}}$ separated by a pseudo S-random interleaver of size 5120. The coded bits are transmitted with a BPSK modulation over a Gaussian channel.

At the receiver, the log-MAP turbo decoder uses a maximum of 10 iterations. The number of transmitted frames is equal to 5000.

The threshold values are: $T_{ML} = -0.0015$ for the proposed max-log criterion, $T_{CE} = 10^{-3}\ CE(1)$ for the CE criterion, $T_{SA} = +0.001$ for the sum-α [11] criterion and $T_{SL} = -1.6$ for the recent sum-log criterion [12] (T_{SA} and T_{SL} are the

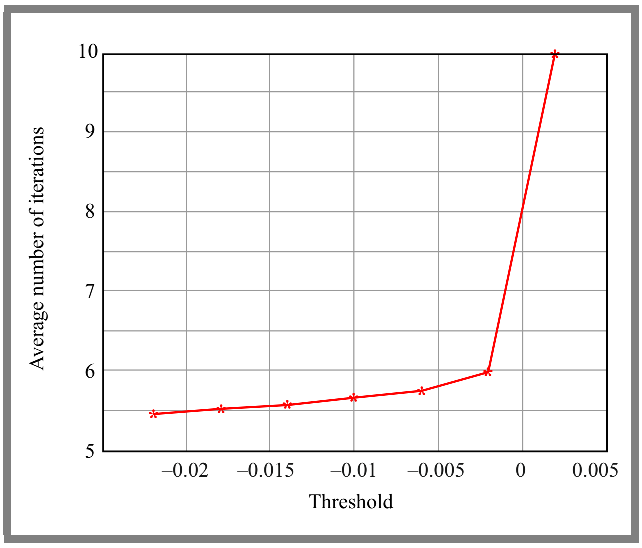

Fig. 3. Average number of iterations of a turbo decoder supervised by the max-log criterion, according to different thresholds.

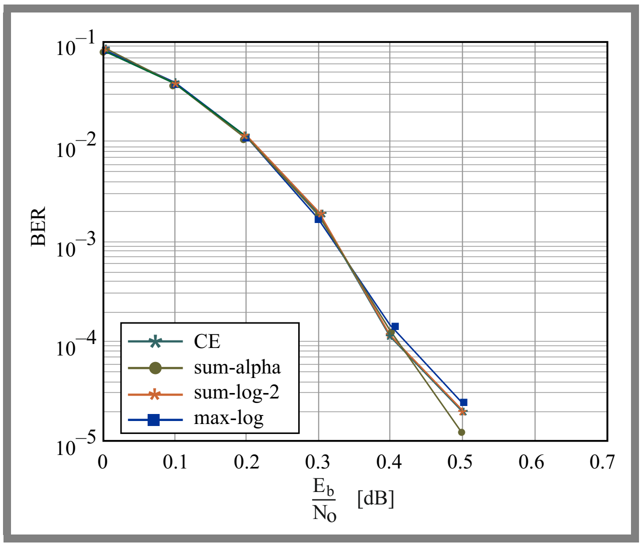

Fig. 4. BER of the log-MAP turbo decoder supervised by the max-log stopping criterion and its comparison with CE, sum-α, and sum-log criteria.

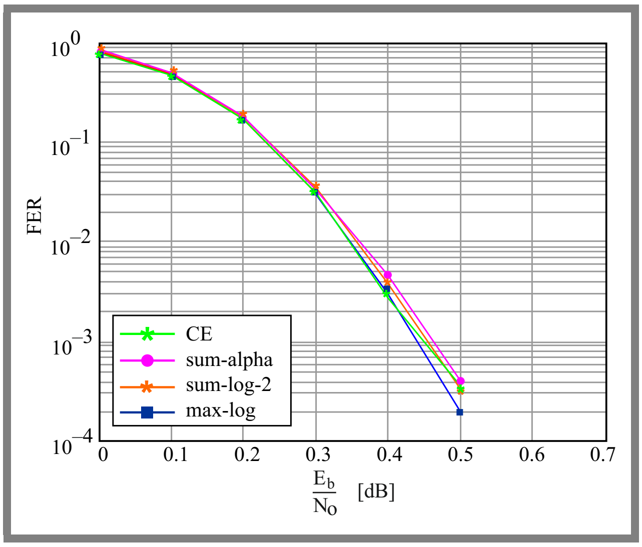

Fig. 5. FER of the log-MAP turbo decoder supervised by the max-log stopping criterion and its comparison with CE, sum-α, and sum-log criteria

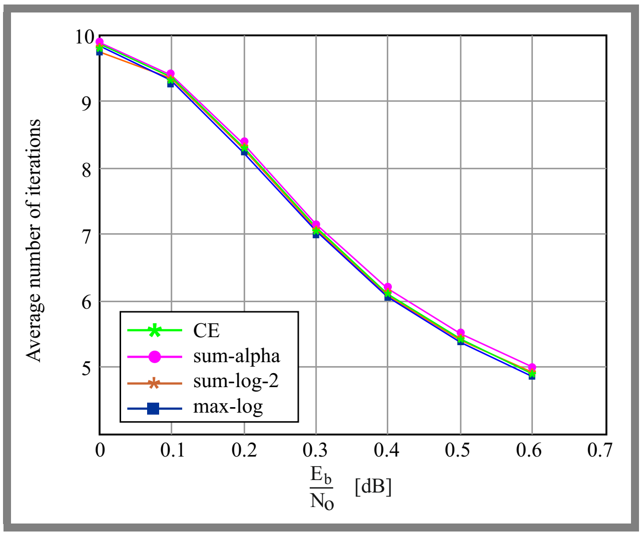

Fig. 6. Average number of iterations of the log-MAP turbo decoder supervised by the max-log stopping criterion and its comparison with CE, sum-α, and max-log criteria.

same thresholds of [11] and [12]). The value $CE(1)$ of threshold T_{CE} is the cross entropy of the first iteration.

Simulation results have shown that the max-log criterion offers the same decoding quality as those of the cross-entropy criteria, sum-α and sum-log. Figures 4 and 5 show BER and FER, respectively, of the CE, sum-α , sum-log and the proposed max-log criterion. The four criteria offer roughly the same performance in terms of BER and FER.

Note that the proposed max-log criterion uses only a selection of the maximum of $A_m(k)$ i.e., $A_{max}(k)$ and their summation. This makes it a fast and low complexity technique. However, it must be triggered from time $t = k$ to avoid infinity initialization of the log-MAP algorithm [13].

To assess the level of complexity, Fig. 6 plots the average number of iterations. It shows that the max-log criterion uses the same average number of iterations for the threshold of 0.0015. At SNR = 0.6 dB, the CE, sum-log-2 and max-log (proposed) criteria require an average of 4.9 iterations, while the sum-α criterion uses an average of 5 iterations. This family of criteria i.e., sum-α, sum-log (sum-log-1 and sum-log-2 [12]), and the criterion proposed here (max-log), are called as the sum-x family. The fundamental principle of these criteria is the sum of the probabilistic quantity's alphas (calculated by the MAP algorithm) or log-alphas (calculated by the log-MAP algorithm).

8. Conclusion

In this paper, we proposed a new stopping technique for log-MAP turbo decoding – the max-log criterion. It uses the maximum values of the alphas calculated by the log-MAP algorithm to make a stop decision. The new max-log criterion provides the same performance in terms of BER and FER as that achieved when using sum-α, cross-entropy, and sum-log criteria, with the same complexity of decoding (i.e., average number of iterations) maintained. The max-log criterion ensures low complexity, because it calculates only a cumulative sum of the maximum alphas. Moreover, it does not require any memory for data storage. The practical use of this simple criterion allows to increase the processing speed. The family of sum-α, sum-log and the proposed max-log criteria is considered to be of the early variety, as they work in the forward direction of the MAP and log-MAP algorithms and do not wait for the calculation of the LLRs i.e., the end of the decoding phase. This is an important property which makes this family the fastest in making the decision, even before the end of the iteration.

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